THE TEXT IS LIGHT IN THE BOOK

THE DESIGN OF DAMS

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SECOND EDITION

THIRD IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.

NEW YORK AND LONDON

1938

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE TO THE SECOND EDITION

In preparing the second edition of this work, the authors have attempted to bring it strictly up to date, recognizing that the science of engineering, like the other sciences, has steadily advanced during the years since the first edition was published. New material has been added covering earthquake stresses in single- and multiple-arch dams, stresses due to holes such as inspection galleries of various shapes through dams, the use of models in dam design, and the use of the hydraulic jump as a means of dissipating energy. Changes have been made where improvement would thereby result, throughout the text, and all known errors have been corrected.

It is believed that the new edition represents an entirely modern and complete treatise on this very important subject.

FRANK W. HANNA,
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OAKLAND, CALIFORNIA,
November, 1938.

PREFACE TO THE FIRST EDITION

The increased development of water supplies for irrigation, generation of electric power, domestic consumption, and industrial use during the past two decades has intensified the need for complete and scientific methods of design of dams; and a desire to meet this need has led the authors to prepare this volume.

All types of modern dams have been given full treatment, including the various types of arch dams, which in recent years have attained merited prominence in America. The treatment of both single- and multiple-arch dams is original. The formulas presented have been developed by the elastic method and have had practical application in the design of some of the most important arch dams. The relative advantages and disadvantages of curved and straight gravity dams are discussed, and original formulas for stress analysis are developed. Forms for tabulating the computations of the more complicated structures are presented in the text.

While extraneous subjects have been omitted for the purpose of reducing the size of the volume to a minimum, subjects related to the design of dams have been included so that the engineer and student may have complete information for proper design in one treatise. Pursuant to this plan, there have been included in the text, in addition to the general design of dams of all types, chapters covering accessories for dams, reservoir and dam-site surveys, rainfall and runoff of streams, and hydraulics and measurements of stream flow. Under accessories for dams there is given a particularly complete discussion of gates and hoists for control of outlets through dams.

All the fundamental equations used in this work have been rigorously developed so the engineer or student who

desires to follow through the development can do so. On the other hand, the important equations used in design have been plainly identified so that they may be used without spending the time required to follow through their derivation. In this way the work has been made useful from both a technical and a practical viewpoint.

It has been the intention of the authors to present a treatise on the design of dams that will include the important advancements of recent years in this field, that will be useful both to students and practicing engineers, and that will lead to more efficient designs and safer structures in this important branch of engineering. If they have succeeded in these intentions, they will feel highly repaid for the great amount of work involved in the preparation of this book.

Much credit is due to Mr. T. L. E. Haug, member of the American Society of Civil Engineers, and to Mr. J. W. Trahern, associate member of the American Society of Civil Engineers, for checking the more important equations of this work and for valuable suggestions that have been incorporated in the text. Acknowledgment is also made for the use of related technical literature, especially to the publications of the United States Bureau of Reclamation and the American Society of Civil Engineers.

FRANK W. HANNA,
ROBERT C. KENNEDY.

OAKLAND, CALIFORNIA,
August, 1931.

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CHAPTER I

INVESTIGATIONS AND SURVEYS

INTRODUCTION

1-1. Character of Investigations and Surveys.—Before expenditures are made on the details of a water-supply study, foundation investigations, and designs of the dam for a proposed project, a reconnaissance survey should be made to ascertain the quantity and quality of the water supply, foundation materials, and the general feasibility of the possible reservoir and dam sites. This reconnaissance can usually be accomplished by general inspection and with the use of a hand level. A rough geological examination should be made of the general geology of the reservoir and dam sites for the purpose of determining the water tightness of the reservoir and the suitability of the foundation materials. If these general investigations are encouraging then more accurate engineering and geological surveys should be made.

LEVELS AND TRIANGULATION

1-2. Primary Levels.—Leveling done in connection with reservoir and dam sites should be of two types, namely: primary leveling and secondary leveling. The former should cover all important lines and the latter only those of minor importance. In the running of primary levels, great care should be used and all circuits should close with an error not exceeding $0.05\sqrt{M}$ ft., where M is the length of the circuit in miles. In primary leveling it is customary to adhere to the following principles:

a. Lengths of back sights and fore sights should be limited to 300 ft. and should be equal.

b. Lines of sights should be kept 2 ft. or more from the ground surface at all points, particularly in the heat of the day.

c. Bench marks should be established for each mile run of line.

d. Firm turning points should be used.

e. The level man and rod man should keep their notes strictly independently of one another.

f. The notes should give complete information regarding the source from which the initial elevation was taken.

1-3. Secondary Levels.—In the running of secondary levels, back sights and fore sights need not be limited as to length or equality. Turning pins are not required and every effort may be made to save time. Bench marks should, of course, be left at frequent intervals and should be well described and carefully established. The elevation of survey monuments, summits, depressions, water levels, and bridges should be carefully recorded. The limit of error prescribed for this character of work should be closely within a fifth of the contour interval used in the topographic work.

1-4. Triangulation Control.—In triangulation work, observations on Polaris for azimuth should be made at the beginning and end of the work and the entire triangulation system should be adjusted to agree therewith. Each triangulation point should be permanently preserved by means of a stone or concrete monument. In the measurement of the angles of a triangulation system for the control of the topography of a large reservoir site, the following method should be used:

First set: Telescope direct—read on A B C–A.
Telescope reversed—read on A–C B A.
Shift the limb 90 deg.
Second set: Telescope reversed—read on A B C–A.
Telescope direct—read on A–C B A.

If more than two sets of readings are necessary the limb should be shifted between sets, 180 deg. divided by the number of sets, and both verniers read.

RESERVOIR SURVEYS

1-5. Accuracy Required.—The survey of a reservoir site should be made with sufficient accuracy to determine the approximate capacity of the reservoir within a limiting error of about 10 per cent. Two or more permanent bench marks should be established near the proposed dam site and their elevation determined with reference to river bed or water level or both. If there is a permanent bench mark of known elevation above mean sea-level in the near vicinity, these bench marks should be connected with it by a checked line of levels. This being done, a topographic survey of the reservoir site should be made either by the plane-table or the transit-stadia method, and a map of the site prepared.

1-6. Plane-table Surveys.—If the topography is to be taken with a plane table, a line of levels should be run up the valley of the reservoir and the points of contour crossings noted. A base line of suitable length and location for establishing a triangulation system should be laid out, and a careful plane table triangulation made from it. Before completing the triangulation, all triangulation stations should be marked by permanent monuments of stone or concrete. Where the reservoir site on the scale adopted covers more than three consecutive plane-table sheets, all of the angles of the triangulation system should be measured with a transit reading to 20 or 30 sec. and the relative positions of the triangulation stations computed and plotted on the plane-table sheets before beginning the plane-table survey. Secondary points for the control of the sketching should then be located by the transit-stadia method. These should be skillfully distributed so as to establish all controlling points in the topography such as changes in slope, important junctions or bends in drainage lines, hills, passes, buildings, lone trees, or other recognizable objects. If the reservoir site has been covered by a government survey, a careful search should be made for public land survey corners and all of those found should be located accurately on the plane-table sheets.

1-7. Stadia Surveys.—If the reservoir site is so densely wooded as to interfere seriously with field sketching and as to necessitate the clearing of lines of sights, the transit-stadia method should be used. Generally a traverse line should be run up the valley of the reservoir site and another along the top contour of the survey. If the reservoir site is not too large, points on these lines may be connected by traverse lines, the difference of elevation being obtained by hand level, and distance determined by careful pacing; but where the reservoir site is large, it will be necessary to run these traverse lines with transit and stadia.

1-8. Reservoir Maps.—In the preparation of the map of the reservoir, a scale of 400 ft. to the inch should be used as the standard scale for plotting the surveys if the reservoir area can be plotted on one plane-table sheet; if not, then a scale of 1,000 ft. to the inch should be used as the standard scale. The contour intervals should ordinarily be 10 ft., though in exceedingly steep country it may be increased to 20 ft. or more where this interval will correctly portray the topography. Sheets of similar scale should be used when maps are made from notes taken by the transit-stadia method.

DAM-SITE SURVEYS

1-9. Map Scales and Contour Intervals.—The surveys of dam sites should in general conform to those for the reservoir sites except that the points for contour plotting should be more frequent over the sites, the accuracy should be greater, and the map scale larger. The final survey should, in general, be made with sufficient accuracy to determine the volume of the dam within a limiting error of 5 per cent. Scales for mapping the dam site should be 40 ft. to the inch where the plotted survey of the site would occupy more than one entire plane-table sheet on this scale, or where the detail of the topography is not great a scale of 100 ft. to the inch may be used. Contour intervals

should ordinarily be 5 ft., but a smaller interval should be employed for comparatively level ground.

QUALITY OF WATER

1-10. General Statement.—Whether water is considered good from an engineering viewpoint depends on the use to which it is to be put. A water suitable for irrigation may or may not be useful for domestic or industrial purposes. Chemically pure water is a mixture of hydrogen and oxygen, but such water rarely exists in natural conditions. Natural waters generally contain either inorganic or organic impurities, or both, and whether these impurities are detrimental in the use to which the water is to be put must be determined by the engineer before development of works for its use.

Water containing less than 55 parts per million (p.p.m.) of inorganic minerals in solution is generally classed as soft. Those containing from 56 to 100 p.p.m. are usually classed as medium, while those containing from 101 to 200 p.p.m. are generally classed as hard. The chemicals usually found in solution in water are iron, sulphuric acid, a group of incrustants consisting of calcium chlorides, calcium nitrates, calcium sulphates, magnesium chlorides, magnesium nitrates, and magnesium sulphates, a group of alkalines consisting of calcium carbonate, calcium bicarbonate, sodium carbonate, and sodium bicarbonate. The total hardness of water as expressed numerically in this paragraph is caused by the amount of calcium bicarbonates, magnesium bicarbonates, calcium carbonates, and calcium sulphates.

1-11. Water for Domestic Purposes.—Water suitable for domestic use should not contain sufficient dissolved inorganic matter to give it an objectionable taste or odor or to make it unsuited for household purposes. Salts give an objectionable taste to water when more than 250 p.p.m. of chlorine are present, and generally speaking, water with a hardness greater than 200 p.p.m. is objectionable for a public water supply unless softened, thus involv-

ing an expensive process. A domestic water supply should also be free from objectionable colors, turbidity, microscopic plants, and injurious bacteria. Practically all of these objectionable qualities can be removed from a domestic water supply by purification processes. The cost of softening and purification has to be taken into consideration in connection with the economic feasibility of the use of any water-supply development.

1-12. Water for Industrial Purposes.—Water proposed for industrial use must be given careful consideration in connection with the character of the industry. The needs of the various manufacturing industries vary greatly as to the quantity, as well as to the quality of the water required. Some industries require a small quantity of water with the highest degree of quality. Others require a large quantity of water with little discrimination as to the quality.

Such industries as the manufacture of agricultural implements, automobiles, boots and shoes, clothing, electrical machinery, fertilizers, foundry products, furniture, glass, lumber, metal products, oil products, and rubber are practically independent of the quality of the water. Such industries, however, as the manufacture of chemicals, cotton goods, druggists' preparations, dyeing processes, knit goods, tannery goods, paper goods, silk goods, and woolen goods are quite dependent on the quality of water. A study of the various industries of the United States will show that the latter mentioned are located where soft waters are obtainable.

1-13. Water for Irrigation.—Water used for irrigation must be free of chemicals in quantities that are injurious to the growth of plants. Broadly speaking, these injurious chemicals are alkalies consisting principally of sodium chloride ($NaCl$), common salt; sodium sulphate (Na_2SO_4), Glauber salt; and sodium carbonate (Na_2CO_3), sal soda. In general, plants are differently susceptible to injury from an excess of salts in water and the different soluble salts comprise different properties with reference to

plant life. The most harmful, both as to kind and quantity, is sodium carbonate (Na_2CO_3) known as black alkali; the second most harmful is sodium chloride ($NaCl$); the third most harmful is sodium sulphate (Na_2SO_4). The latter two are the principal ingredients of white alkali. In general, plants do not grow successfully in soil where sodium carbonate is in excess of 0.10 per cent; where sodium chloride is in excess of 0.25 per cent; or where sodium sulphate is in excess of 0.50 per cent.

The discussion of the quality of water here given is very general and brief and the reader should, for more complete information, consult special technical works on the requisite qualities of domestic, industrial, and irrigation water.

CHAPTER II

RAINFALL AND STREAM FLOW

INTRODUCTION

2-1. General Statement.—Dams are usually constructed for the purpose of restraining or diverting water for domestic, industrial, irrigation, mining, flood-control, or hydraulic-power purposes. The supplying of water for all of these purposes requires a knowledge of the quantity of water available for storage or diversion and the supplying of many of them requires also a knowledge of the quality of the water. The feasibility of the construction of any diversion or storage dam is, therefore, dependent on the sufficiency of water supply. Prior, therefore, to the designing of a dam, it is necessary to make a study in all cases of the available quantity of water to be restrained or diverted by the dam. In making this study, streams are generally divided into three classes as to the character of runoff.

The first class is that in which the bulk of the runoff is derived from accumulated snows; the second class is that in which the bulk of the runoff is derived mainly from surface flows from heavy rains; and the third class is that from which the bulk of the runoff is derived mainly from underground or seepage sources. Each of these classes overlaps the other and each may obtain a considerable portion of runoff from the principal source of runoff of the other two classes. The percentage of precipitation yielded by runoff decreases from the first to the third class.

Representative of the first class are mountain streams having their sources at high elevations, and streams in high latitudes that are subject to heavy snows in their drainage basins in the winter months. These snows melt in the

spring and early summer months, thus causing a concentrated flow during this period. Many of these streams discharge a large percentage of their entire annual runoff during the snow-melting season. Their drainage basins are, relatively speaking, poor absorbers of rainfall and only a small proportion of the water discharged by them is derived from underground sources. Representative of this type in the United States are the large streams heading in the Sierra Nevada, the Cascade, and Rocky Mountains and, in a lesser measure, those in the higher latitudes in the northern part of the United States. It is relatively easy to predict in a general way the regimen of streams of this character, particularly where the summer seasons are usually dry.

Streams of the second class are represented by a large majority of streams on rather impervious drainage basins in the arid regions of the United States, that do not include the higher altitudes of the adjacent mountains, and in the southeastern part of the United States where the rainfall is heavy. The matter of predicting the discharge of such streams is more difficult than that of the first class and resolves itself largely into the prediction of the time and occurrence of heavy rainfalls. The discharge of streams of this character is intermittent when considered in comparison with those of the first class.

The third class of streams is fed by seepage into their drainage basins, through cracks, fissures, and pervious ground soils. While this class of streams is affected intermittently by surface runoff, the runoff is rather steady due to the fact that a large amount of rainfall is stored in the soil of the drainage area and released through underground flow to the stream channels. The characteristics of the drainage area of such a stream are perviousness of surface soils, underlying beds of sand and gravel, marshes, swamps, ponds, lakes, general flatness of topography, and possibly, to a certain extent the vegetable covering. Drainage basins of this character lose a large amount of the precipitation upon them by evaporation, plant transpira-

tion, deep-seated percolation. Streams of this character are found in the more recently glaciated areas of the United States and Canada, such as Michigan, Wisconsin, Minnesota, and Ontario.

STREAM-FLOW RECORDS

2-2. Government Stream-flow Data.—The United States Geological Survey has for a number of years maintained stream-gaging stations on a great majority of the important rivers of the United States, and the results obtained at these stations are to be found in the various water-supply papers issued by this bureau of the national government. The data accumulated and published in these papers consist, in general, of the location and description of the gaging station; the drainage area in square miles of the stream on which the gaging station is located; discharge measurements made at the station; daily gage heights; gage-height discharge-rating tables; and maximum, minimum, and mean daily discharges in second-feet for the drainage basin and per square mile thereof; and the runoff in inches of depth over the drainage area per square mile, and in acre-feet, classified by months of the calendar year, together with other information relative to the reliability of the published data. These data are reliable and should be used by engineers interested in securing accurate stream-runoff information. An engineer who is considering the designing of a dam is fortunate if the dam is to be located on a stream that has been measured by the United States Geological Survey, and particularly so if the dam is located sufficiently close to one of these gaging stations to give long-time, accurate records of discharge.

2-3. Additional Stream-flow Data.—If a dam is to be designed for construction on streams for which government records are not available, gaging stations should be established as soon as feasible and measurements started in order to obtain data on which to base early comparisons with those on adjacent streams and to furnish reliable information for future needs. This information should be used in connection with making estimates of the runoff,

which estimates should so far as practicable be based on adjacent streams that have been covered with gaging stations of the United States Geological Survey and that should have similar runoff characteristics on account of equality of drainage areas; similar and equal precipitation; like topography, geologic structure, and surface soil; and vegetation-covering and evaporating conditions. Where these conditions are not the same, proper corrections should be made according to the judgment of the engineer.

The designer of dams must in various designs or in the same design have information of the minimum, mean, and maximum discharge of the stream affecting the dam. The minimum flow affects the continuity of the supply; the mean, the total quantity of the supply; and the maximum, the wasteway capacities required.

2-4. Minimum-flow Records.—The minimum flow of streams in the arid regions is generally zero and this condition usually lasts a considerable portion of the year. Even in the humid parts of the country the minimum may become a very small percentage of the mean. It is, therefore, important to investigate the minimum-flow conditions of any stream from which it is proposed to divert water for any purpose, particularly where a continuous flow is needed and where storage is not to be provided for equalization of the flow.

2-5. Mean-flow Records.—The mean flow of streams is of extreme importance in all cases in the design of dams for reservoirs and water-supply systems, irrigation systems, and power plants. This flow not only varies greatly with the year but also from month to month and from day to day within the year. There is given in Table I a list of some of the more important rivers of the United States, with their drainage areas, average runoff in second-feet, second feet per square mile of drainage area, and annual depth in inches on the drainage area. The figures in this table give only rough ideas that must be particularized by specific study of the data relating to the stream under consideration for development for engineering purposes.

TABLE I.—LIST OF LARGE RIVERS OF THE UNITED STATES

(Arranged according to size of drainage area, and estimated average flow in second-feet[1])

Stream	Drainage area, sq. mi.	Runoff from drainage area		
		Average flow, sec.-ft.	Average sec.-ft. per sq. mi.	Average depth, in. per annum
Mississippi	1,238,000	695,000	0.56	7.6
Missouri, St. Charles, Mo.	527,000	100,000	0.19	2.6
St. Lawrence, Ogdensburg, N. Y.	383,300	252,000	0.66	8.9
Yukon	330,000	160,000*	0.49	6.6
Columbia	259,000	253,000	0.98	13.3
Rio Grande	248,000	7,100	0.03	0.4
Colorado	244,000	23,500	0.10	1.3
Ohio	202,000	300,000	1.48	20.2
Arkansas	177,500	65,500*	0.37	5.0
Mississippi, above Missouri River	171,500	127,000	0.74	10.1
Snake	108,000	57,000	0.53	7.2
Red	90,000	26,000*	0.29	3.9
Platte	83,800	11,000*	0.13	1.8
Yellowstone	69,700	13,000	0.19	2.5
Kansas	61,400	8,000*	0.13	1.8
Gila	56,500	1,500*	0.03	0.4
Pecos	56,000	600*	0.01	0.1
Green, Utah	44,400	7,900	0.18	2.4
Brazos	41,700	11,500*	0.28	3.8
Tennessee	40,700	63,000	1.55	21.0
Colorado of Texas	39,000	3,600	0.09	1.3
Red, of the North, at boundary	35,500	3,500*	0.10	1.4
Wabash	32,800	24,000*	0.73	10.0
Sacramento	31,400	33,000	1.05	14.3
North Platte	28,500			
Illinois	27,900	22,000†	0.79	10.7
Susquehanna	27,400	41,300	1.51	20.5
Milk	26,700	800	0.03	0.4
Colorado, above Green	25,900	9,500	0.37	5.0
Little Colorado	25,900	500*	0.02	0.3
Republican	25,840	1,250	0.05	0.7
San Juan	25,800	3,200*	0.12	1.7
Clark Fork	25,800	28,000	1.09	14.8
South Platte	24,000			

[1] Compiled from records of the United States Geological Survey and other sources.
* Flow estimated from insufficient data and may be subject to considerable error.
† Flow of Illinois River not adjusted for diversion from Lake Michigan.

TABLE I.—LIST OF LARGE RIVERS OF THE UNITED STATES.—(*Continued*)

Stream	Drainage area, sq. mi.	Runoff from drainage area		
		Average flow, sec.-ft.	Average sec.-ft. per sq. mi.	Average depth, in. per annum
Mouse	23,540			
James, Dakotas	21,820			
Big Horn	21,730	5,000*	0.23	3.1
Sabine	20,700	16,700	0.81	11.0
Smoky Hill	20,480			
Apalachicola	18,800	30,300	1.61	21.9
Cumberland	18,600	26,000	1.40	19.0
Trinity	17,600			
Nueces	16,800			
San Joaquin	16,500	15,000	0.91	12.4
Minnesota	16,100			
Klamath	15,500	16,500	1.07	14.5
Alabama	15,400	27,700	1.80	24.4
Osage	15,300			
Santee	14,800	24,500	1.66	22.5
Des Moines	14,700			
Potomac	14,140	14,850	1.05	14.3
Altemaha	14,100	17,700	1.26	17.1
Loup	13,540			
Hudson	13,400	21,000	1.57	21.3
Delaware	13,100	22,000	1.68	22.8
Neosho	12,660			
Iowa	12,400			
Wisconsin	12,280			
Concho	12,000			
Rainy, United States only	11,500			
Willamette	11,150			
Connecticut	11,100	18,000	1.62	22.0
Savannah	11,100			
Allegheny	11,100			
Owyhee	11,100			
Kanawha	11,000			
Rock, Illinois	10,970			
White, Indiana	10,800			
Pedee	10,600	16,900	1.60	21.7
Guadalupe	10,500			
James, Virginia	10,400	12,500	1.20	16.3
Jefferson	10,000			

* Flow estimated from insufficient data and may be subject to considerable error.

2-6. Maximum-discharge Records.—It is generally necessary for designing and construction purposes to have detailed information as to the instantaneous maximum runoff likely to occur from the drainage area tributary to a dam. Table II[1] gives a list of maximum-discharge or stream-runoff records for a large number of streams in the United States that may be useful for this purpose. This information is taken from a table compiled by C. S. Jarvis, and published in Volume 89, pages 1003 to 1024, of the *Transactions* of the *American Society of Civil Engineers*.

2-7. Maximum-flow Formulas.—Maximum instantaneous discharges of streams have been made the subject of a large number of empirical formulas. These formulas have generally been based on an equational relation between runoff and drainage area, modified with constants to fit the particular localities involved. Some of these formulas are as follows:

$$Q = 200M^{\frac{5}{6}} \quad \text{(Fanning).} \tag{1}$$

$$Q = \left(\frac{127{,}000}{M + 370} + 7.4\right)M \quad \text{(Kuichling).} \tag{2}$$

$$Q = \left(\frac{46{,}790}{M + 320} + 15\right)M \quad \text{(Murphy).} \tag{3}$$

In these formulas, Q is the maximum runoff from the total drainage area in cubic feet per second and M is the drainage area in square miles. The Fanning formula was designed particularly for the streams of New England and is given in his treatise on "Water Power Engineering." The Kuichling formula was designed particularly for rare floods in the Mohawk River and is given in the report on the Barge Canal for 1901. The Murphy formula is designed to have general application throughout the United States and is given in *Water Supply Paper* 147 of the United States Geological Survey.

The application of these runoff formulas to other localities should not be made without modification of the constants

[1] See Appendix, p. 401.

to suit the conditions involved. Moreover, the formulas are generally not accurate for small drainage areas. They should be used only for the purpose of giving general ideas as to the amount of runoff and should be checked by actual records or other means of determining runoff.

ESTIMATES OF RUNOFF FROM PRECIPITATION

2-8. Limitations.—It may be necessary in some cases to base estimates of runoff on its relation to rainfall. A number of attempts have been made to devise empirical equations or curves for this purpose, but no very encouraging results have been obtained. The measurements of both runoff and rainfall are approximations, and even when they are reasonably accurate the relations shown by comparisons are not satisfactory on account of lag of flow for seepage waters, differences in evaporation and plant transpiration at different times and the effects of natural and artificial storage. The formulas proposed are devised for estimating annual, monthly, and instantaneous maximum runoff.

In these formulas the annual and monthly runoffs are expressed in terms of the annual or monthly rainfall on the watershed, the temperature of the air or relative water-saturation requirement, and, in some instances, the slope of the drainage basin. These formulas are generally applicable to restricted sections of the country and lack, therefore, wide useful application.

2-9. Runoff Formulas Based on Precipitation Records.—Perhaps the best known and most used of these formulas is that of Vermuele for the eastern United States. Meyer[1] has developed a rational method of computing the monthly runoff from rainfall and other physical data, which is more particularly applicable to the north central states. This method involves a thorough knowledge of the plant transpiration, soil and water surface evaporation, soil

[1] MEYER, A. F., "Computing Runoff from Rainfall and Other Physical Data," *Trans. Am. Soc. Civil Eng.*, Vol. 79, pp. 1056–1224, 1915.

storage, surface flow and seepage flow on the drainage basin, and use of the empirical curves relating to them worked out by the author of the method. The method offers a scientific plan of analysis based on all of the principal factors affecting stream flow and can be used to supplement stream-flow records.

The Vermuele formula given in the *Journal of New England Water Works Association* may be written as follows:

$$F = R - (11 + 0.29R)(0.35T - 0.65), \qquad (4)$$

in which R equals the annual rainfall in inches, T the mean annual temperature in degrees Fahrenheit, and F the annual runoff in inches depth on the drainage basin. This formula is assumed to be general in application but does not differentiate as to physical conditions of watersheds or for variations in seasonal distribution of rainfall. Its use should be for obtaining rough estimates for initial use, to be checked by the rational method given in the preceding paragraph.

UNDERGROUND FLOW

2-10. Estimating Underground Flow.—In addition to surface runoff on watersheds of streams, there is also a large amount of underground flow that appears as springs or stream-bed flow increment. Such flow is an important element in the low water runoff of all streams, particularly in the humid areas.

It is often important for one reason or another to know the quantity of water flowing through the soil. Formulas for computing such flow have been developed, that known as the Slichter formula being most generally used. This formula reads as follows:

$$Q = 11.3\frac{H\,d^2A}{L\,P}[1 + 0.0187(t - 32°)] = \frac{K\,H\,A}{L}, \qquad (5)$$

in which Q equals the discharge in cubic feet per minute;

H the pressure head or drop in feet; d the effective size of the soil grains, A the area of the cross-section of the soil under consideration, t the temperature of the flowing water in degrees Fahrenheit, L the length of the path of percolation, P the constant of soil porosity, and K a general constant involving d, P, and t. Of these values H, A, t, and L can be obtained directly from data relating to the problem. d and P must be obtained from an analysis of the soil materials.

2-11. Effective Size of Soil Grains.—The effective size or diameter of soil grains is based on the assumption that they are spheres of equal volume. The effective size or diameter of a composite soil sample is such a size that 10 per cent by weight of the particles are smaller than it. The method of determining the effective size or diameter consists in running the material through sieves with a reasonable amount of shaking. Detailed descriptions of the requirements for determining the effective size will be found in *Water Supply Papers* 67 and 140 of the United States Geological Survey.

2-12. Determination of Soil Porosity.—In the determination of the percentage of porosity, let it be assumed that w_1 is the weight of a cubic foot of the materials when dry, w_2 is the weight thereof when fully saturated, and S is the specific gravity of the particles of the material determined in the usual manner. Now, $w_1 \div S$ is the weight of a cubic foot of the materials in terms of the weight of water, and $w_2 - w_1$ is the weight of the water in the pores of a cubic foot of the material when fully saturated. Then $(w_1 \div S) + w_2 - w_1$ equals the combined weight of a cubic foot of the particles and the weight of the water in the pores thereof all expressed in terms of water weight. Evidently, then, the percentage of porosity, P, of the material may be expressed as follows:

$$P = \frac{100(w_2 - w_1)}{\dfrac{w_1}{S} + w_2 - w_1}. \tag{6}$$

TABLE III.—TRANSMISSION CONSTANTS K FOR VELOCITY OF WATER IN SANDS

For use in Slichter's formula $Q = \frac{KHA}{L}$.

(Table computed for temperature of 60°F.; results for other temperatures can be found by the use of Table IV)

Diameter of soil grains, millimeters	Porosity, per cent						Kind of soil
	30	32	34	36	38	40	
0.01	0.000033	0.000040	0.000050	0.000060	0.000072	0.000085	
0.02	0.000131	0.000162	0.000198	0.000239	0.000286	0.000339	Silt
0.03	0.000296	0.000364	0.000446	0.000538	0.000645	0.000763	
0.04	0.000527	0.000648	0.000794	0.000958	0.001145	0.001355	
0.05	0.000822	0.001012	0.001240	0.001495	0.001790	0.002120	
0.06	0.001182	0.001458	0.001784	0.002150	0.002580	0.003050	Very fine sand
0.07	0.001610	0.001983	0.002430	0.002930	0.003510	0.004155	
0.08	0.002105	0.002590	0.003175	0.003825	0.004585	0.005425	
0.09	0.002660	0.003280	0.004018	0.004845	0.005800	0.006860	
0.10	0.003282	0.004050	0.004960	0.005980	0.007170	0.008480	
0.12	0.004725	0.005830	0.007130	0.008620	0.010320	0.012200	
0.14	0.006430	0.007940	0.009720	0.011720	0.014040	0.016620	Fine sand
0.15	0.007390	0.009120	0.011150	0.013450	0.016110	0.019100	
0.16	0.008410	0.010360	0.012680	0.015310	0.018350	0.021700	
0.18	0.010640	0.013110	0.016050	0.019400	0.023200	0.027450	
0.20	0.013150	0.016200	0.019830	0.023900	0.028650	0.033900	
0.25	0.02050	0.02530	0.03100	0.03740	0.04480	0.05300	
0.30	0.02960	0.03640	0.04460	0.05380	0.06450	0.07630	Medium sand
0.35	0.04025	0.04960	0.06075	0.07330	0.08790	0.01039	
0.40	0.05270	0.06480	0.07940	0.09575	0.11450	0.13550	
0.45	0.06650	0.08200	0.10050	0.12110	0.14500	0.17180	
0.50	0.0822	0.1012	0.1240	0.1495	0.1780	0.2120	
0.55	0.0994	0.1225	0.1500	0.1810	0.2165	0.2565	
0.60	0.1182	0.1458	0.1784	0.2150	0.2580	0.3050	
0.65	0.1390	0.1710	0.2095	0.2530	0.3030	0.3580	Coarse sand
0.70	0.1610	0.1983	0.2430	0.2930	0.3510	0.4155	
0.75	0.1850	0.2278	0.2785	0.3365	0.4030	0.4770	
0.80	0.2105	0.2590	0.3175	0.3825	0.4585	0.5425	
0.85	0.2375	0.2925	0.3580	0.4325	0.5175	0.6125	
0.90	0.2660	0.3280	0.4018	0.4845	0.5800	0.6860	
0.95	0.2965	0.3650	0.4470	0.5400	0.6460	0.7650	
1.00	0.3282	0.4050	0.4960	0.5980	0.7170	0.8480	
2.00	1.315	1.620	1.983	2.390	2.865	3.390	Fine gravel
3.00	2.960	3.640	4.460	5.380	6.450	7.630	
4.00	5.270	6.480	7.940	9.575	11.450	13.550	
5.00	8.22	10.12	12.40	14.95	17.90	21.20	

By means of this equation the percentage of porosity of the materials can be determined.

2-13. Determination of Constant *K*.—Slichter has made extensive studies of the constants in Eq. (5) and has worked out values for a single constant to replace them, which he calls K and which is related to the effective soil diameter d, the porosity P, and the viscosity as changed by water temperature t. Slichter has given the following formula for determining this constant:

$$K = 11.3\frac{d^2}{P}[1 + 0.0187(t - 32°)]. \qquad (7)$$

By use of Eq. (7), the value of K for soils of different effective diameters and porosities can be computed. In Table III the values of K are taken from *Water Supply Paper* 140 of the United States Geological Survey.

It will be noted that Table III is compiled for temperatures of 60°F. The results for other temperatures can be found by the use of Table IV, which Slichter also compiled in connection with his transmission constant values.

TABLE IV.—EFFECT OF TEMPERATURE CHANGE ON FLOW OF WATER THROUGH SANDS

(Flow at 60°F. taken as standard)

Temperature, degrees Fahrenheit	Relative flow	Temperature, degrees Fahrenheit	Relative flow
32	0.64	70	1.15
35	0.67	75	1.23
40	0.73	80	1 30
45	0.80	85	1.39
50	0.86	90	1.47
55	0.93	95	1.55
60	1.00	100	1.64
65	1.08		

RAINFALL

2-14. General Statement.—The only practical means now available for determining the rainfall to be expected in the future is by reference to records of the past. To be

dependable, these records must be over a long period and must have been kept by standardized and accurate methods. Such records have been maintained in more or less detail at a few stations in the eastern parts of the United States for over a hundred years, and in some western states, for over eighty years, while in European countries several records have been maintained for over two hundred years. Recent studies of tree rings in the southwestern parts of the United States have extended rainfall records, of a general character, back to the year 700 A.D.[1] These, of course, are not of quantitative value in computing future rainfall.

Many of the older records lose much of their value for engineering design by omitting the figures for maximum precipitation over one-day or shorter periods, giving only the monthly and yearly maximums and means. In regions of flashy runoff it is particularly important to obtain an accurate indication of the duration of intense rainfalls, as the resulting stream flow will usually determine the spillway capacity required for safety of the structure. Where these short-time records are not available, the judgment of the designer must be used in the assumptions made.

2-15. United States Weather Bureau Records.—The United States Weather Bureau maintains a large number of rain-gaging stations scattered throughout the country and prepares monthly and annual bulletins setting forth the data gathered for each of the 106 sections into which the country has been divided. The annual bulletins of each of these sections summarize the information obtained since the beginning of the records. These reports should be utilized when studying a project as they are the most authoritative records available.

Figure 1 is a chart prepared by the United States Weather Bureau and shows the mean annual precipitation for all parts of the United States by the use of lines

[1] DOUGLASS, ANDREW ELLICOTT, "The Secret of the Southwest, Solved by Talkative Tree Rings," *National Geographic Magazine*, Vol. 56, No. 6.

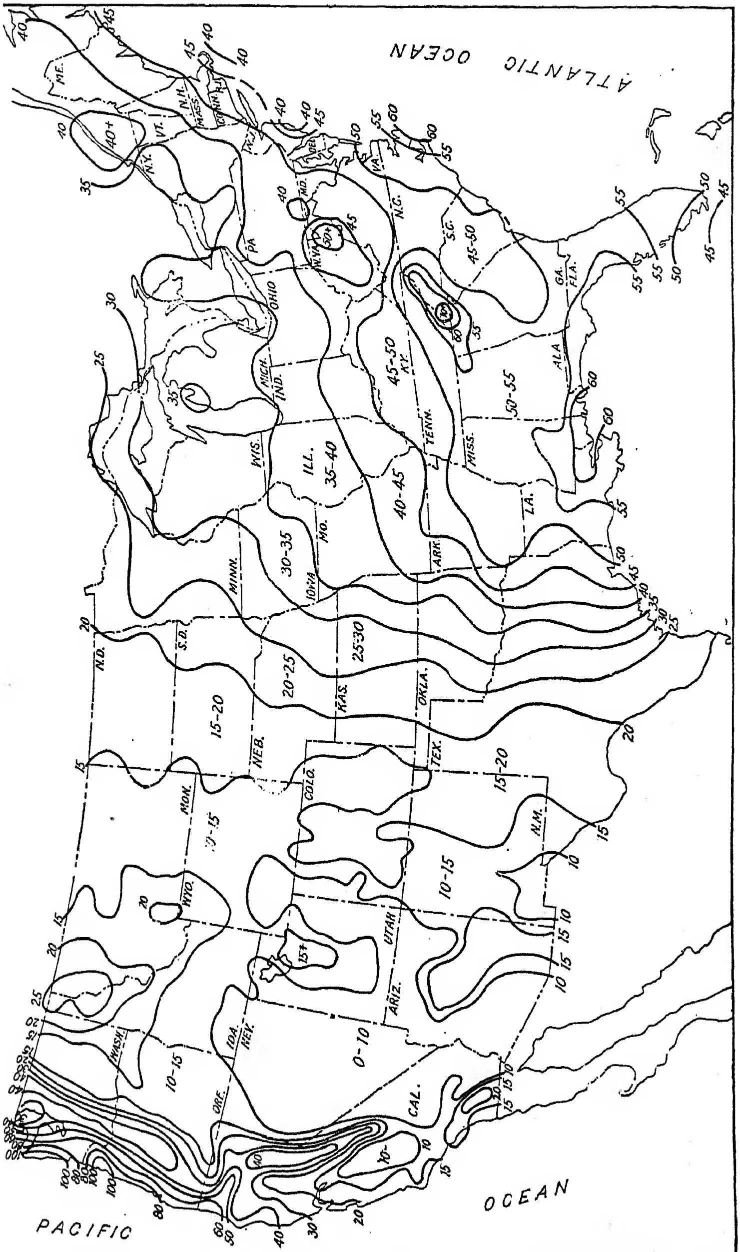


FIG. 1.—Rainfall chart for the United States. (*U. S. Weather Bureau.*)

connecting points having the same mean records. In the absence of accurate local records, this chart may be utilized, but local records are to be preferred, as the Weather Bureau chart is drawn to such a small scale that valuable information is necessarily omitted.

A summary prepared by C. S. Jarvis showing the rainfall records of 820 stations located throughout the inhabited parts of the world is given in Table V (see Appendix, page 424).

EVAPORATION

2-16. General Statement.—Evaporation is of interest in the design of dams because it is the cause of a very large part of the difference between the rainfall upon a watershed and the runoff from the area. Also, the evaporation of water from the surface of reservoirs causes a shrinkage of stored water too great to overlook.

2-17. Evaporation from Watershed.—Of the total precipitation upon an area, the water is dissipated in three general ways: (1) surface runoff into streams of the watershed, (2) percolation into the soil, from whence it later emerges (possibly into a neighboring watershed), and (3) evaporation from the surface of the ground and from the leaves of plant growth.

The relative importance of these three factors in accounting for the total precipitation is extremely variable according to the local topographic and climatological conditions. In regions of short heavy downpours and steep, nonabsorbent soil, practically all of the rainfall is accounted for as runoff. On the other hand, a region of flat contour and with a good cover of vegetation may show but slight change in runoff even after heavy storms, as the water is held by standing in pools and soaking into the soil, and by wetting the enormous total surface of the foliage. From these latter reserves much of the water will be returned to the air by evaporation. Even that which has percolated into the soil is not out of reach of the vaporizing tendency of the climatic conditions above, as capillary action of the soil may return it to the surface, and the

deep-searching roots of vegetation may absorb it and send it to the leaf surfaces where it is evaporated, this latter process being known as transpiration.

The variations in the factors governing evaporation are obviously so great in different areas and climates that only general statements are possible concerning them. A study of these factors and their effects may be a guide in comparing a watershed whose rainfall and runoff are known, with another watershed whose rainfall only has been recorded, with a view toward ascertaining the probable runoff of the latter. The effects of the more important factors are as follows:

1. *Rate of Rainfall.*—The percentage of evaporation loss will decrease with increased rainfall rate.

2. *Slope of the Watershed.*—The steeper the contour of the ground, the lower will be the evaporation loss.

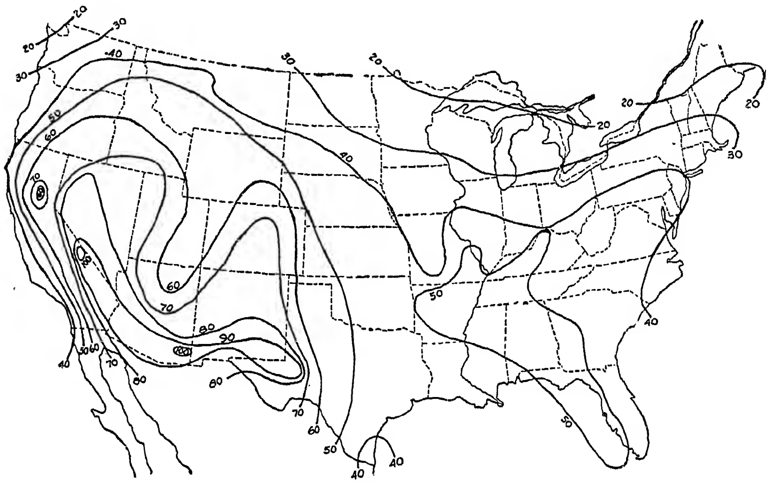
3. *Porosity of the Soil.*—High porosity, especially if accompanied by sloping contour, tends to decrease the evaporation loss.

4. *Vegetation on the Watershed.*—In general vegetation of small character tends to increase the evaporation loss, while forests tend to decrease it, except where the water table is high in which case forests will cause an increase in the loss.

5. *Temperature of the Air.*—Since the vapor pressure of a liquid increases with the temperature of the liquid, the mean annual or seasonal temperature is a gage of the evaporation loss to be expected. Evaporation occurs, however, even from ice and snow.

2-18. Evaporation from Reservoir Surfaces.—A loss of stored water ranging from 20 to over 100 in. in depth per year will occur due to evaporation from the surface of the reservoir. The amount of loss will depend upon the area of surface exposed at different times of the year, the temperature of air and water, the winds, the elevation above sea-level, the amount of vegetation projecting above the surface, etc. The only one of these factors which can be controlled for the purpose of decreasing evaporation is that of vegetation standing in the water. A considerable

increase in the loss of water will result from rank growths. For this reason, and for others in the case of water-supply reservoirs, it is important to keep the edges of the reservoir clear of vegetation.



ANNUAL EVAPORATION DEPTHS FROM FREE WATER SURFACES
DEPTHS EXPRESSED IN INCHES

FIG. 2.—Mean evaporation chart for the United States. (*U. S. Weather Bureau.*)

Figure 2 shows the average evaporation per year to be expected from water surfaces in the United States, as determined by studies of the United States Weather Bureau. Although the quantities given on this chart are not claimed to be accurate, they will suffice for the usual preliminary investigations concerning the project. If closer results are desired, actual experiments may be performed at the site of the reservoir. These experiments consist in exposing water in pans, adding thereto as required, and noting the amount of loss. Two types of pans are utilized by the United States Weather Bureau, namely, land pans, or those situated on or in the ground; and floating pans, or those which are mounted on floats in larger bodies of water.

2-19. Land Pans.—The standard land pan of the United States Weather Bureau is of galvanized iron, 4 ft. in diam-

eter and 10 in. deep. These pans are mounted above the ground surface, so that the sides and bottom are exposed to the air. A pointed rod is attached to the bottom so that the point is slightly below the level of the top of the pan. This rod forms a gage, the water level being maintained close to the point by filling with a measuring cup. The amount of water necessary to maintain the required elevation is carefully recorded, as well as the precipitation into the pan, which is measured by an adjacent rain gage.

Land pan records always show evaporation rates higher than would occur in a reservoir or lake at the same location. This is due to (1) the exposure of the sides and bottom to the air, which increases the temperature of the water, (2) the increased wind velocity above the ground surface, (3) the decrease of air humidity above the ground surface, (4) the superelevation of the water around the edges of the pan due to surface tension, thus increasing the area exposed and permitting of a higher temperature in this small volume, and (5) other minor causes.

2-20. Floating Pans.—Pans floating on the surface of a reservoir or lake give more accurate indications than land pans, of the evaporation which is taking place from that body of water. A floating pan is usually mounted on a raft in such a way that the water surface therein is close to the level of the water in which it is floating. Difficulty is experienced, especially in large reservoirs or lakes, in preventing water from splashing into the pan. To reduce this danger, the float should be relatively large and heavily constructed, 16 ft. square usually being a satisfactory size. Also, since occasional mishaps are likely to occur in spite of precautions, it is well to have a land pan situated near by, so that records may be kept simultaneously, and if a correct reading is accidentally lost on the floating pan, that of the land pan may be corrected to supply its place. Both types of pans should be protected by wire mesh from birds, etc.

The size of the evaporation pan has a large effect upon the accuracy of the record; the larger the pan, the more accurate the result. It has been shown that the rate

indicated by a 2-ft.-diameter pan is 30 per cent greater than that of a 12-ft.-diameter pan.

2-21. Relation of Evaporation Rate Indicated by Land Pans, Floating Pans, and Open Water Surfaces.—The relation of the results to be expected by means of land pans and floating pans of different diameters, is set forth in Table VI.

TABLE VI.—RATIO OF RATE OF EVAPORATION OF LAND PANS TO THAT OF FLOATING PANS FOR PANS OF SIMILAR SIZE AND AREA[1]

Shape of pan	Area of exposed surface, square feet	Relative evaporation*		Ratio, land to water pans
		Land	Floating	
2.0 ft. square	4.0	128.4	112.2	1.144
3.39 ft. circular	9.0	120.2	106.0	1.134
4.0 ft. circular	12.57	116.5†	103.5	1.125
6.0 ft. circular	28.27	108.9	97.1	1.121
3.0 ft. square	9.0	125.9	112.5	1.119

[1] SLEIGHT, "Evaporation on Reclamation Projects," *Trans. Am. Soc. Civil Eng.*, Vol. 90, p. 311, 1927.

* In terms of percentage of evaporation from land tank 12 ft. in diameter, 3 ft. deep with 2.75 ft. depth of water.

† Interpolated.

Table VII presents the results of experiments to determine the relation of evaporation rates indicated by both land and floating pans, to those from open water surfaces. It will be noted that as the size of pan increases, the accuracy of results also increases.

2-22. Evaporation Records at Various Stations in the United States.—In Tables VIII and IX are recorded the results of many experiments covering evaporation at widely scattered points in the United States. These records are from a variety of sources as indicated.

2-23. Evaporation as Determined by "Insolation."—An attempt has been made by Burt Richardson[1] to establish a relation between the exposure to the rays of the sun, *i.e.*, the insolation of a body of water, and the evaporation

[1] RICHARDSON, BURT, "Evaporation as a Function of Insolation," *Proc., Am. Soc. Civil Eng.*, pp. 945–960, May, 1930.

TABLE VII.—COEFFICIENTS OF REDUCTION; PAN EVAPORATION RECORDS TO OPEN WATER SURFACES[1]

Depth of tank or pan, feet	Depth of water in tank or pan, feet	Dimensions of tank or pan, feet	Coefficient of reduction to open water surfaces	Remarks
		Land Tanks and Pans		
		Circular diameter		
3.0	2.75	2.0	0.77	Set in ground 2.75 ft.
3.0	2.75	4.0	0.84	Set in ground 2.75 ft.
3.0	2.75	6.0	0.90	Set in ground 2.75 ft.
3.0	2.75	9.0	0.98	Set in ground 2.75 ft.
3.0	2.75	12.0	0.99	Set in ground 2.75 ft.
2.5	2.25	6.0	0.89†	Set in ground 2.17 ft.
2.0	1.75	6.0	0.88	Set in ground 1.75 ft.
0.83*	0.62	4.0	0.66	Pan set on timbers on ground. This pan is used at Class A evaporation stations, U. S. Weather Bureau.
		Cubical		
3.0	2.75	3 by 3, square	0.80	Tank set in ground 2.75 ft.; Fort Collins type.
		Floating Pans		
1.5	1.25	3 by 3, square	0.91	Pan set in water and protected by raft to reduce wave action. This is U. S. Geol. Survey floating standard.
		Circular diameter		
0.83*	0.58	4.0	0.92*	Pan set in water submerged to a depth of 7 in.
1.83	1.50	3.5	0.91†	

[1] MEEKER, "Evaporation on Reclamation Projects," *Trans. Am. Soc. Civil Eng.*, Vol. 90, p. 294, 1927; data from Denver Field Laboratory located at elevation 5,346 ft., open prairie land. Long-time climatic records at U. S. Weather Bureau in downtown of Denver, located on roof of office buildings, are:

Mean annual temperature, degrees Fahrenheit	50.1	53 years' record.
Wind in miles per hour	7.4	
Relative humidity, per cent	53.	
Mean annual precipitation, inches	14.27	

* 10-in. depth.

† Approximate interpolation by writer from Sleight data.

TABLE VIII.—EVAPORATION FROM LARGE OPEN WATER SURFACES AT STATIONS IN THE WEST[1]

Station	State	Pan, kind	Pan, size,	Date of record	Elevation of station, feet	Period of record	Annual evaporation, inches	Coefficient assumed[2]	Probable evaporation from large open water surfaces, inches
			in. ft.	month day					
Avalon Reservoir	New Mexico	Floating	19 by 3, square	1–12	3,188	1914 to 1923	73.37	0.91	66.70
Agricultural College	New Mexico	Land	3, cubical	1–12	.....	1908 to 1913	86.56	0.80	69.30
Mesilla Park	New Mexico	Land	10 by 4, circular	1–12	3,863	1918 to 1923	92.07	0.66	60.80
Elephant Butte	New Mexico	Land	10 by 4, circular	1–12	4,475	1917 to 1923	101.80	0.66	67.20
Roosevelt	Arizona	Land	10 by 4, circular	1–12	2,175	1916 to 1923	87.21	0.66	57.60
Yuma Citrus	Arizona	Land	10 by 4, circular	1–12	181	1921 to 1923	122.81	0.66	81.10
Yuma Evaporation	Arizona	Land	10 by 4, circular	1–12	127	1917 to 1923	75.81	0.66	50.10
West End Tunnel	Utah	Land	17 by 3, square	5–10	7,500	1910 to 1915	25.79	0.80	20.65
East End Tunnel	Utah	Land	17 by 3, square	4–11	7,600	1905 to 14, 15	21.51	0.80	17.20
Provo	Utah	Land	17 by 3, square	4–11	4,600	1910 to 1917	29.46	0.80	23.60
Provo	Utah	Land	10 by 4, circular	4–11	4,600	1918 to 1923	37.25	0.66	24.60
Fallon Experiment Farm	Nevada	Land	24 by 6, circular	1–12	3,960	1908 to 1923	64.30	0.90	57.90
Lake Tahoe	California	Floating	10 by 4, circular	1–12	6,230	1916 to 1923	40.50	0.92	37.30
East Park	California	Floating	10 by 4, circular	1–12	1,130 to 1,200	1911 to 1923	62.39	0.92	57.40
Adam's Boat Landing	Oregon	Floating	10 by 4, circular	5–10	4,070	1913 to 1919	33.92	0.92	31.20
Coppocks	California	Floating	10 by 4, circular	5–11	4,035	1916 to 1923	36.62	0.92	33.70
Klamath Falls	Oregon	Floating	22 by 3½, circular	1–12	4,100	1921 to 1924	50.90	0.90	45.80
Cold Springs	Oregon	Land	10 by 4, circular	1–12	623	1914 to 1923	52.02	0.66	34.30
Hermiston Experiment Farm	Oregon	Land	24 by 6, circular	1–12	450	1912 to 1923	43.98	0.90	39.60
Arrowrock	Idaho	Land	10 by 4, circular	4–10	3,220	1916 to 1923	44.33	0.66	29.28
Deer Flat	Idaho	Land	10 by 4, circular	4–12	2,510	1916 to 1923	42.91	0.66	28.30
Lake Kachess	Washington	Land	10 by 4, circular	5– 9	2,270	1917 to 1923	20.85	0.66	13.75
Nelson Reservoir	Montana	Floating	18 by 3, square	5–10	2,215	1917 to 1920	35.77	0.91	32.60
Nelson Reservoir	Montana	Floating	10 by 4, circular	5–10	2,215	1921 to 1923	36.21	0.92	33.30
Nelson Reservoir	Montana	Land	10 by 4, circular	5–10	2,215	1921 to 1923	52.89	0.66	34.90
Sherbourne Reservoir	Montana	Land	10 by 4, circular	6– 9	4,865	1922 to 1923	18.62	0.66	12.30
Sun River Diversion	Montana	Floating	10 by 4, circular	4–10	4,474	1919 to 1923	38.40	0.92	35.35
Willow Creek	Montana	Floating	10 by 4, circular	4–10	4,130	1919 to 1923	37.24	0.92	34.25
Shoshone Dam	Wyoming	Land	10 by 4, circular	1–11	5,390	1915 to 1924	44.49	0.66	29.35
Osborn Experiment Farm	Montana	Land	24 by 6, circular	4– 9	3,000	1911 to 1923	32.85	0.90	29.60
Shoshone Dam	Wyoming	Land	10 by 4, circular	5–10	5,390	1919 to 1922	42.00	0.66	27.70
Powell	Wyoming	Land	8 by 3, circular	3–10	4,390	1911 to 1919	56.62	0.60	34.00
Ralston	Wyoming	Floating	3, circular	5– 9	4,590	1911 to 1914	27.33	0.90	24.60

[1] HOUK, "Evaporation on Reclamation Projects," *Trans. Am. Soc. Civil Eng.*, Vol. 90, pp. 266–278, 1927.
[2] Ninth column from discussions of same paper by Follansbee, p. 287, and others.

TABLE VIII.—EVAPORATION FROM LARGE OPEN WATER SURFACES AT STATIONS IN THE WEST.—(*Continued*)

Station	State	Pan, kind	Pan, size	Date of record	Elevation of station, feet	Period of record	Annual evaporation, inches	Coefficient assumed[1]	Probable evaporation from large open water surfaces, inches
Pathfinder Reservoir......	Wyoming	Land	10 by 4, circular	5–10	5,900	1914 to 1923	48.64	0.66	32.10
Mitchell Experiment Farm.	Nebraska	Land	24 by 6, circular	4–10	4,080	1911 to 1923	40.32	0.90	36.30
Lake Minatare............	Nebraska	Land	10 by 4, circular	4–10	4,120	1918 to 1919	46.61	0.66	30.80
Dutch Flats Camp........	Nebraska	Land	10 by 4, circular	4–10	4,110	1909	52.17	0.66	34.50
Sunflower Camp..........	Nebraska	Land	10 by 4, circular	1–11	4,070	1910 to 1917	47.79	0.66	31.60
Newell Experiment Farm..	SouthDakota	Land	24 by 6, circular	4– 9	2,880	1908 to 1923	36.55	0.90	32.90
Jackson Lake.............	Wyoming	Land	10 by 4, circular	7– 9	6,770	1924	21.06	0.66	13.90
Wagon Wheel Gap........	Colorado	Land	10 by 4, circular	1–12	.....	1920 to 1924	.....	....	21.77
Santa Fe.................	New Mexico	Floating	18 by 3, square	1–12	.....	{1913 to 1914} {1916 to 1925}	.....	....	44.42
Myton....................	Utah	Land	10 by 4, circular	1–12	....	1918 to 1925	.....	....	39.85
Farmington...............	New Mexico	Floating	18 by 3, square	1–12	.....	1915 to 1925	.....	....	46.10
Piute Dam...............	Utah	Land	10 by 4, circular	1–12	.....	1918 to 1925	.....	....	46.70
Willcox..................	Arizona	Land	10 by 4, circular	1–12	.....	1917 to 1925	.....	....	61.05
Mesa.....................	Arizona	Land	10 by 4, circular	1–12	.....	1917 to 1925	.....	....	51.68
Deming...................	New Mexico	Floating	18 by 3, square	1–12	.....	1915 to 1925	.....	....	61.20
Lees Ferry...............	Arizona	Land	10 by 4, circular	1–12	.....	1922 to 1925	.....	....	58.38
Yuma Reservoir..........	Arizona	Floating	18 by 3, square	1–12	.....	1903	.....	....	75.39
Fort Collins..............	Colorado	Land	3, cubical	1–12	5,000	1887 to 1925	41.98	0.80	33.60
Santa Fe Field...........	New Mexico	Land	10 by 4, circular	1–12	6,975	1917 to 1925	64.55	0.66	42.60
Deming...................	New Mexico	Floating	18 by 3, square	1–12	4,300	1914 to 1924	66.29	0.91	60.40
Farmington..............	New Mexico	Floating	18 by 3, square	1–12	5,300	1915 to 1924	50.36	0.91	45.80
North Platte.............	Nebraska	Land	30 by 6, circular	4– 9	3,000	1907 to 1925	41.88	0.90	37.70
Garden City.............	Kansas	Land	30 by 6, circular	4– 9	2,900	1908 to 1925	52.66	0.90	47.40
Lake Cushman...........	Washington	Floating	10 by 4, circular	1–12	760	1924 to 1925	26.27	0.92	24.20
Tulare Lake..............	California	....	....	1–12	.....	1906 to 1916	.....	....	55.20
Lake Elsinore............	California	....	....	1–12	.....	1920	.....	....	54.00
Owens Lake..............	California	....	....	1–12	.....	1906 to 1914	.....	....	60.80
Upper Otay Lake.........	California	Land	18 by 3, square	1–12	.....	3-year	66.10	0.94	62.30
Sweetwater Reservoir......	California	Land	18 by 3, square	1–12	.....	3-year	58.70	0.97	56.90
University, North Dakota.	NorthDakota	Floating	18 by 3, square	4–11	820	1905 to 1920	28.90	0.91	26.30

TABLE IX.—OBSERVED EVAPORATION AT 32 STATIONS IN THE UNITED STATES, IN INCHES PER MONTH[1]

Station	Relative location, deg. No. Lat.	January	February	March	April	May	June	July	August	September	October	November	December	Total for 1 year
Boston, Mass.	42.5	0.90	1.20	1.80	3.10	4.61	5.86	6.78	5.49	4.09	2.95	1.63	1.20	39.61
Rochester, N. Y.	43.0	0.86	0.86	1.67	2.39	3.45	4.38	4.80	4.54	3.54	2.60	1.46	1.21	31.76
Cincinnati, Ohio	39.0	1.00	1.50	2.50	4.12	5.07	6.21	7.20	7.26	5.63	3.00	1.50	1.00	45.99
Birmingham, Ala.	33.6	1.50	1.50	2.25	4.45	5.91	7.28	7.36	7.34	6.00	4.00	2.25	1.50	51.34
Great Lakes	....	0.66	0.84	1.03	2.12	2.76	3.67	4.38	3.98	3.21	2.56	1.67	1.08	27.96
Mitchell, Nebr.	41.0	1.75	1.75	3.00	4.50	6.25	8.05	10.95	9.39	7.44	5.59	4.00	3.00	65.67
Snake River, Idaho	42.5	2.25	2.50	4.00	7.00	11.21	12.31	15.00	13.50	11.00	8.50	5.75	3.50	96.52
Boise, Idaho	43.6	2.00	2.75	4.25	6.00	7.90	9.59	10.59	12.16	9.25	5.42	5.52	2.00	77.43
Fallon, Nev.	39.0	1.75	1.75	2.25	3.25	5.25	7.86	9.86	8.70	5.13	3.35	2.50	2.00	53.65
Hermiston, Ore.	45.8	1.25	1.25	3.00	7.28	7.89	9.54	12.04	11.07	7.35	3.88	2.00	1.50	68.05
Klamath, Ore.	42.2	0.50	1.25	3.57	6.64	7.15	6.99	8.01	9.21	6.13	2.50	1.00	0.50	53.45
North Yakima, Wash.	46.8	1.75	2.50	6.25	7.91	8.36	8.90	10.74	9.41	5.51	3.15	2.00	1.50	67.88
Tahoe Lake, Calif.	39.2	1.75	1.75	1.75	2.00	3.00	4.25	6.19	7.08	6.22	3.60	2.62	2.00	42.21
Salton Sea, Calif.	33.4	3.41	5.09	5.95	8.75	10.50	13.00	14.03	12.19	12.08	9.24	5.96	5.25	105.45
Indio, Calif.	33.4	3.18	5.08	7.50	12.05	15.84	16.11	16.34	13.78	12.37	8.91	5.17	3.00	119.33
Mecca, Calif.	33.4	2.92	5.00	8.07	10.87	12.72	14.23	15.21	13.22	10.29	8.17	4.13	2.98	107.81
Brawley, Calif.	33.0	3.05	5.00	8.00	10.74	13.79	13.68	14.14	11.26	10.15	6.99	4.09	2.66	103.55
Mammoth, Calif.	33.0	4.24	5.67	8.99	12.02	15.52	16.75	18.00	13.73	12.16	9.49	5.26	3.70	125.53
Phoenix, Ariz.	33.6	4.25	4.40	5.25	7.00	9.50	12.00	12.75	12.50	11.00	8.31	6.56	4.22	97.74
Lee's Ferry, Ariz.	....	1.74	3.52	5.87	7.16	11.70	13.70	13.70	11.33	8.85	6.29	3.98	1.91	89.75
Roosevelt Dam, Ariz.	....	2.29	3.10	5.33	7.32	10.37	12.67	12.34	10.56	8.60	5.78	3.52	2.13	84.02
Mesa Experiment Station	....	2.78	3.68	5.74	7.78	10.27	11.17	10.55	8.29	6.39	4.73	3.36	2.62	77.36
Willcox, Ariz.	....	3.59	4.73	7.29	10.10	11.24	12.20	10.64	9.03	8.10	6.70	4.67	3.41	91.70
Yuma Evaporation Station, Ariz.	....	3.09	3.89	5.72	7.23	8.24	8.91	10.24	9.96	7.85	5.39	3.40	2.62	76.54
Carlsbad, N. Mex.	....	4.50	4.50	5.51	7.45	10.12	11.05	12.88	12.00	9.50	7.00	5.75	4.50	94.76
Agriculture College, N. Mex.	....	2.87	4.50	7.41	9.37	11.10	11.91	11.15	9.79	8.09	5.93	3.63	2.52	88.27
Elephant Butte, N. Mex.	....	2.78	4.49	7.56	10.44	13.43	14.48	12.22	10.83	9.07	7.76	4.32	3.05	100.43
Lake Avalon, N. Mex.	....	2.34	3.26	5.49	7.49	7.87	8.70	10.13	9.60	8.59	6.49	3.17	2.42	75.55
Santa Fe Field, N. Mex.	....	1.53	2.13	3.96	6.15	8.51	10.14	8.93	8.06	6.55	4.81	2.55	1.93	65.25
Spur, Tex.	....	2.79	3.54	4.82	5.48	6.66	8.68	9.33	7.88	5.59	4.48	3.42	2.34	65.01
Hill Ranch, Tex.	....	2.47	3.43	5.29	6.01	6.81	7.95	8.97	8.72	6.36	4.96	3.18	2.44	66.59
Beeville, Tex.	....	2.69	3.26	4.26	4.79	6.21	7.35	8.14	8.14	5.89	4.72	3.06	2.17	60.68

[1] RICHARDSON, "Evaporation as a Function of Insolation," *Proc. Am. Soc. Civil Eng.*, p. 956, May, 1930.

from the surface thereof. The insolation varies with the latitude, the time of the year, the cloudiness of the sky, the elevation, the return radiation of the earth, the existence of sun spots, and other factors. The resulting heat reaching the surface of the earth, however, may be automatically recorded by an instrument known as a pyrheliometer. From these records, together with records of water temperature, humidity, etc., may be computed rates of evaporation, which, from the brief experimentation thus far conducted, seem to give results quite close to those found by direct evaporation tests.

2-24. Evaporation Formulas.—Several formulas have been devised for computing the rate of evaporation from a

TABLE X.—FORMULAS FOR RATES OF EVAPORATION FROM WATER SURFACES

Name of originator	Reference	Formula
Fitzgerald.....	*Trans. Am. Soc. Civil Eng.*, Vol. 15, p. 581, 1886	$e = 12(V - v)\left(1 + \frac{w}{2}\right)$
Meyer........	*Trans. Am. Soc. Civil Eng.*, Vol. 79, p. 1074, 1915	$e = 15(V - v)\left(1 + \frac{w'}{10}\right)$
Vermuele.....	Report of the New Jersey Geol. Survey	$e = (0.00417T - 0.123)$ $(15.5 + 0.16R)$

In the above formulas the terms used are defined as follows:

e = the monthly evaporation in inches of water.
V = the maximum vapor tension for the temperature of the water surface (expressed in inches of mercury).
v = the actual vapor tension in the air (expressed in inches of mercury).
w = the wind velocity in miles per hour close to the water surface.
w' = the wind velocity in miles per hour by the Weather Bureau report from the nearest station.
R = the yearly rainfall in inches.
T = the mean annual temperature in degrees Fahrenheit.

water surface. Each of these formulas, however, involves variables and factors which require experimental records for their determination, thus affording no great advantage over direct experimentation, except for approximate estimates. The best known of these formulas and the definition of terms used are given in Table X.

CHAPTER III

SELECTION OF TYPE OF DAM

INTRODUCTION

3-1. General Statement.—After making surveys of the various alternative dam sites and of any storage-reservoir sites involved in the project, making a study of the available water supply through consideration of rainfall, evaporation, and runoff in the drainage basin and carefully investigating the foundation conditions at the dam sites, the engineer is ready to consider the selection of type of dam to be constructed. The type to be selected will, in general, be based on considerations of the availability of suitable construction materials, safety of property and life below the dam, foundation conditions, and height of structure.

3-2. Availability of Materials.—The construction of a dam in a location remote from railroads or highways may, for economical reasons, require its construction with materials available in the immediate vicinity. The foundation conditions being satisfactory for such construction, it is probable that either an earth or rock-fill dam in remote locations may be found most suitable from an economical viewpoint. On the other hand, where the dam site is convenient to transportation facilities, a concrete dam may be more desirable from an economical viewpoint. Again, economy may dictate, where conditions are otherwise suitable for a concrete dam, whether the dam shall be of the solid-gravity type, the deck type, or the multiple-arch type.

3-3. Safety of the Structure.—In some locations, the permanence of the dam is not so important because its destruction might not cause damage to life or property in the watershed below. In other locations the permanence

of the dam is of utmost importance and no possible risk can be taken in regard to its safety under any and all conditions. Consequently, the consideration of the question of safety or permanence of the dam is of extreme importance in selecting the type to be used.

3-4. Suitability of Foundations.—The foundations of the structure necessarily have important relations to the type to be constructed. Other things being equal, a flexible type of dam should be constructed on a flexible foundation, and a rigid type of dam on a rigid foundation. This principle leads to the general practice of building earth dams and to a lesser extent rock-fill dams on earth, sand, or gravel foundations, and the building of masonry dams on solid rock foundations. The perviousness, the bearing power, the erodability, homogeneity and continuity of the foundation material all enter into the question of the proper type of dam to be used, and require careful judgment and wide experience for wise determination of their influence on cost and permanence of the dam to be built.

3-5. Height of Dams.—The height of the dam to be built has a great deal to do with the type of dam to be selected for a given site. Low masonry dams of 25 ft. or less are often successfully founded on silt and fine sand, of 30 ft. or less on clay, of 50 ft. or less on compact gravel or thick beds of clean, coarse sand, and of 400 ft. or less on solid rock foundations. The highest dams existing or under construction at the present time are as follows:

Type	Height above foundation, feet	Name	Location
Earth-fill	245	Cobble Mountain	Massachusetts
Rock-fill	330	Salt Springs	California
Deck	240	Rodriguez	Mexico
Multiple arch	256	Lake Pleasant	Arizona
Straight gravity	307	Kensico	New York
Single arch	389	Diablo	Washington
Arched gravity	405	Owyhee	Idaho
Arched gravity	707	Boulder	Nevada-Arizona

These figures represent the present extremes of practice, and indicate that the arch and the arched-gravity types are in the lead as to height. The second highest gravity dam of any type now in existence is the Pardee Dam, 358 ft. high from base to top, and the second highest arch dam is the Pacoima, 365 ft. from base to top.

TYPES OF DAMS

3-6. Buttress Types.—The buttress types consist of the deck dams and the multiple-arch dams. The Coolidge multiple-dome dam constructed by the United States Bureau of Indian Affairs on the Gila River in Arizona[1] may be classed as a buttress type. These types require less concrete than the solid-gravity types and may generally, therefore, be constructed in shorter time, and may in many cases be less expensive, depending on the relative cost of reinforcing steel and cement. The buttress dams also have an advantage on a pervious foundation where uplift may be serious, inasmuch as the uplift pressure may be relieved between the buttresses. There is also an advantage in that the slope of the upstream face may be made such as to bring the resultant pressure on the foundation nearer the middle of the buttresses, thus better distributing the load over the foundation. Thin sections requiring reinforcing steel near the upstream face of high buttress dams should be avoided because thin sections are poor resisters of seepage and frost, and reinforcing steel is liable to rust in such positions. For such dams the recent Schorer cantilever-deck type[2] and the Noetzli round-head buttress type,[3] each with buttresses of uniform strength, are applicable. The older types of continuous deck dams with thin sections and reinforcing steel near the water surface are being replaced by such types having thicker sections with more deeply imbedded steel reinforcement or entirely without steel reinforcement.

[1] *Eng. News-Record*, Vol. 101, pp. 396–399, September 13, 1928.
[2] *Proc., Am. Soc. Civil Eng.*, Vol. 56, pp. 1947–1969, November, 1930.
[3] *Proc., Am. Soc. Civil Eng.*, Vol. 57, pp. 176–180, January, 1931.

3-7. Solid-gravity Types.—Except where narrow canyon widths make the arch-type dams preferable, the solid-gravity types are most popular. Otherwise, where cutoff walls can be made available to prevent serious uplift, and where the strength of foundation materials is ample, high solid gravity dams are most feasible and desirable. The limiting height of solid-gravity masonry dams on hard impervious rock foundations will be determined by the allowable working stresses in good concrete. Undoubtedly, better concrete will permit the use of stress allowances much beyond those at present practiced and still higher dams may be built. Even where the foundation materials are pervious and of low-pressure resistance, low solid-gravity masonry dams are successfully and economically built by using sheet piling cutoff walls and pile foundations.

As to the relative superiority of the straight-gravity masonry dam and the arched-gravity masonry dam, the latter is superior only where the arch length or the radius of curvature is not too great, and where the canyon walls are resistant to high pressure, or where the foundation contours make it cheaper to follow a curved plan. Where the height from foundation to crest decreases from the center towards the abutments, as is usually the case, the greater deflections of the longer cantilevers throw torsional stresses, sliding tendencies, and overturning moment into the shorter cantilevers towards the ends of the dam. Grouting the contraction joints will help these shorter cantilevers to absorb such loads, whereas in the straight-gravity dam they can be resisted only by increasing the section towards the abutments.

3-8. Single-arch Type.—Where the canyon span is relatively narrow, and the canyon walls consist of sound rock capable of taking arch thrusts, the arch dam will require less concrete and can consequently be constructed at a lower cost than the gravity type. Moreover, due to the fact that the water load is carried laterally to the abutments, uplift becomes unimportant in the arch type, as there is always sufficient weight in the arch itself to oppose

the uplift. The factor of safety of a gravity dam against sliding may be only a little over 1 and is seldom over 2, whereas that of a well-designed arch dam made of good concrete is 4 or 5 or more. Since the arch dam has this additional factor of safety, it is now, and always will be, a popular type of high masonry dam.

3-9. "Flexible" Types.—Under the classification of flexible dams are rock-fill and earth-fill dams, the latter being further subdivided into hydraulic-fill and rolled-fill types. These dams are suited for use where the foundation material is of earth, sand, or other relatively unstable and pervious materials, or where the materials for concrete production are so high in cost that the flexible-type dam is more economical. A very important consideration in the selection of rock-fill or earth-fill dams is that they be placed in locations where there is no possibility of water ever overtopping them. Overtopping would generally result in the destruction of the whole dam, as the crest would erode away faster than the water level dropped. Spillways should not be built over the tops of such dams, as even though the surface be paved, settlement may result in cracks in the pavement, which could easily cause the destruction of the dam. For these reasons, flexible-type dams are not suited to some locations where economy would indicate their desirability.

CHAPTER IV

FOUNDATIONS FOR DAMS

INTRODUCTION

4-1. General Statement.—The usual cause of failure of dams is defective foundations. The foundation of a dam may fail through crushing, sliding, piping, scouring, or uplift. When the bearing power of the material on which the dam is founded is exceeded, the foundation is crushed and the dam fails for want of support. If the coefficient of friction between the laminations of a laminated foundation or between the dam and the top layer thereof is exceeded, a portion of the foundation may slide upon itself or the dam may slide on the foundation, thus causing failure of the structure. In a foundation through which water may pass in a concentrated stream with an erosive velocity, piping results and the foundation ultimately wears away and leaves the dam unsupported. In an overflow dam or a dam having spillways discharging near the base, erosion of the material adjacent to the foundation may weaken it and cause it to give way under the dam through sliding, crushing, or piping. Hydrostatic uplift due to water getting into porous layers within the foundation itself or between the dam and the foundation may so increase the overturning moment or so decrease the coefficient of friction as to cause failure of the dam. For these reasons, the materials on which the dam is to be founded must be carefully determined, their density and continuity explored, their bearing power ascertained, and their surface properly prepared for the reception of the dam.

FOUNDATION MATERIALS

4-2. General Statement.—A dam may be founded on any kind of material from silt to the most solid rock by

giving the proper relative base width to height of structure and the proper treatment to the foundation material.

4-3. Silt and Fine Sand Foundations.—Silt and fine sand are successfully used as foundations for low masonry dams, usually of the diversion-weir type, generally not more than 25 ft. in height, and are used for foundations of earth dams of all heights. Seepage through such foundations is expected, but the base lengths and cutoff wall depths are made sufficient to reduce the velocities of the percolating water to a non-erosive rate. The bearing power of such soils may be increased through artificial compression, the use of concrete or wood piles, the employment of sand piles, the addition of coarse sand or gravel to the natural foundation material, the increasing of the depth of the foundation or otherwise confining the foundation material, and, in some cases, the drainage of the foundation material. The bearing power of such soils, untreated, may be as low as 1 to 2 tons per square foot but by treatment as suggested the bearing power may be greatly increased.

4-4. Clay Foundations.—Clay makes a permissible foundation for a masonry dam of not more than 30 ft. height, if properly treated; but such treatment is not always easy to secure. The bearing power of clay is greatly reduced by moisture and the securing of a dry foundation for a dam in such material is impracticable. For this reason, confining the material to prevent spreading, or loading it artificially, may be resorted to. The use of piling may be of advantage, or drainage may be applied in some cases with beneficial results. Foundations on clay soils may generally be considered capable of carrying from 2 to 4 tons per square foot without treatment. Rock-fill or earth-fill dams are more frequently built on clay foundations than are masonry dams. The same care should be given, however, in both types, as saturation or overloading of such foundations may result in failure of either type of dam.

4-5. Gravel and Coarse Sand Foundations.—Compact gravel or clean coarse sand in beds of considerable thickness makes an excellent foundation for a masonry dam of not

more than 50 ft. height, particularly if the material is well cemented, and they are well suited for the foundation of either an earth-fill or a rock-fill dam of any desirable height with proper cutoff walls. Considerable seepage is likely to occur in such foundation material and provision must be made to prevent damage from this source. Such foundations may normally be expected to carry from 4 to 8 tons per square foot with safety, without artificial treatment.

4-6. Limestone and Lava Foundations.—Limestone and lava in thick layers as a rule have ample bearing power for dam foundations, but they are both likely to be cavernous and require careful explorations and treatment to make them water-tight and otherwise useful. Often these rocks exist in thin strata or sheets and may be unsatisfactory and unreliable foundation material for this reason.

4-7. Shale and Sandstone Foundations.—Shales and sandstones of the softer quality must be used with caution, not so much on account of lack of bearing power as on account of their tendency in some cases to disintegrate on exposure to air or to water or to both.

4-8. Solid Rock Foundations.—The solid rock formations such as granite, gneiss, and schist make the most satisfactory foundations for dams of any type. They generally have high bearing power, have few seams and open pockets, and are difficult to erode.

4-9. Non-uniform Foundations.—In addition to the construction of dams on foundation materials of uniform character as outlined in paragraphs 4–3 to 4–8, inclusive, it is often necessary to build dams on materials of non-uniform character. In many cases such foundations involve the most difficult problems of dam design. Inasmuch as foundations involve individual problems of design due to many conceivable mixtures of foundation materials, only general principles relating to the most important and common types can here be considered. Generally speaking, the foundation materials are stratified and the strata may be classed as pervious or impervious, hard or soft. Com-

binations of impervious and hard strata give little trouble; but strata of pervious or soft materials often involve serious problems of design.

Where impervious foundation materials overlie pervious foundation materials, the problem is sometimes made difficult, particularly if the pervious stratum lies at such a depth or is of such a thickness as to make a complete solution by means of a cutoff wall impracticable. In such cases, the path of percolation is lengthened by use of a cutoff wall of reasonable depth, together with impervious aprons both upstream and downstream of the dam or a wide-based dam, or a combination of these.

One of the most difficult foundation problems involved in dam design is caused by the occurrence of a soft stratum in the foundation either adjacent to the base of the dam or at considerable depth below it. If the soft foundation is adjacent to the base of the dam, it is generally considered best practice to remove the soft material if possible. In some cases drainage may help to solve the problem. In such a case an apron may be extended upstream from the dam to prevent percolation into the soft stratum, a cutoff wall being placed at the upper end of this apron and drainage being provided under the body of the dam. If this method is not feasible the bearing capacity of the soft material may limit the height of the dam, and experiments should be made to determine this capacity. A more serious case consists, however, in a soft stratum lying at considerable depth below the ground surface. In such a case no means is available for determining the bearing capacity of the foundation and no treatment of the soft stratum can be applied, but the foundation of the dam may be so spread as to prevent a flow of the underlying soft material either upstream or downstream, thus making it possible for the overlying stratum of hard material to carry the weight of the dam. The tendency of the soft material to flow may cause a bulging of the ground surface either upstream or downstream of the dam. This may be prevented by loading this surface with earth, gravel, or

rock. Such loading not only prevents the bulging upward of the impervious stratum, but it so loads the soft material as to prevent it from flowing beyond the loaded area.

4-10. Seepage.—The matter of seepage through foundation materials for dams is a most serious consideration and it must be limited to non-erosive velocities and non-objectionable quantities in order to make the dam safe and useful for the purpose intended. The resistance to percolation of water through foundation materials depends upon the character of the materials forming the foundation. Stated in the descending order of resistance, these materials may be listed as granite, gneiss, schist, shale, limestone, sandstone, gravel, sand, silt, and clay. It is of interest to note that this series also closely follows the bearing resistance of the various materials named; with the exception that sandstone follows shale, and clay follows silt. Of course, it is self-evident that massive granitic rocks are less apt to transmit water than are stratified rocks, and that stratified rocks are more likely to transmit water in the direction of the stratification than normal to it. Consequently, in a foundation composed of stratified rocks the worst condition is where the layers are horizontal and the best condition is where they are vertical.

4-11. Examination of Foundations.—It is hardly necessary to state that the foundation of any dam should be thoroughly investigated before the structure is designed or built. This should be done by laboratory tests of the strength and permanency of the materials by tunneling and boring the site; by expert examination of the geological formations in the vicinity of and at the dam site; and, in many cases, by percolation tests on the foundation materials.

PREPARATION OF ROCK FOUNDATIONS

4-12. Precautions Necessary in Rock Excavations.—Rock foundations for masonry dams should consist of clean, rough, unweathered, unshattered material. To secure these conditions the foundation should be excavated

to sound ledge rock, free from open seams or other objectionable defects. If the rock is stratified in thin layers, the excavation should be carried deep enough to insure resistance to sliding of one stratum on another. In order to preserve the solid ledge rock, both within and outside of the lines of excavation, explosives of only moderate quantities and power should be permitted, and blasting that is liable to injure the natural rock should in no case be permitted. Excavation of the rock in places where injury is liable to occur should be performed by means of wedging and barring, or other approved methods of removal.

The rock foundation should be left in such condition as to resist both compressive and sliding forces and to form suitable contact with the masonry of the dam. Before laying any masonry, the foundation should be scrupulously freed from all dirt, loose rock fragments, or other objectionable substances. This may be accomplished by means of a jet of water, air, or steam under effective pressure, or by means of stiff brooms and other tools.

4-13. Cutoff Trench.—A cutoff trench near the upstream face of the dam should be excavated along the entire length of the foundation and to a suitable depth to insure resistance to percolation.

4-14. Grouting of Foundation.—Grouting should be resorted to in case of seamy or cavernous foundations for low dams, and for all foundations for high dams. The grout holes should be drilled to suitable depths at intervals in the bottom of the cutoff trench. These holes are usually 3 in. in diameter, spaced 10 to 15 ft. apart and drilled to a depth of from 25 to 50 ft. It is also desirable in many instances to grout the entire foundation of the dam. The grout should be composed of neat cement and water, and should be forced into the foundation under pressure. Care must be exercised that the pressure is not sufficient to disturb the natural position of the rock in the foundation, but it should be high enough to thoroughly fill all of the interstices of the rock. The pressure may be increased by delaying the grouting until an overburden of masonry has

been placed on the foundation, or by providing a suitable means in the grouting pipe for releasing pressure at various depths in the grout hole.

PREPARATION OF SOIL FOUNDATIONS

4-15. Stripping of Surface.—Soil foundations should usually be prepared by stripping the surface to firm material and to uniform grades and lines so far as practicable, over the entire dam site, including whatever upstream and downstream aprons are necessary for protection against erosion from overflowing and percolating water. No grass, sod, or roots of any kind should be permitted in the foundation, as such organic material will disintegrate and leave holes for water percolation. In the case of earth dams, it is well to plow longitudinal furrows across the site at close intervals in order to obviate definite and continuous division planes between the fill and the foundation.

4-16. Cutoff Walls and Aprons.—A cutoff wall of either concrete or sheet piling, or a combination thereof, should usually be placed under the dam and extended into the foundation to relatively impervious material or to a sufficient depth to effectively aid in preventing an undue rate of percolation. Under earth dams the cutoff wall is usually placed near the center, but in concrete dams it is usually placed near the heel. The direction and nature of the paths of flow of water under a dam vary with the character of the dam and the perviousness of the foundation material. While the paths of percolation are generally curvilinear, due to the varying pressure on the foundation caused by the weight of the dam, they are usually assumed to be rectilinear. If the foundation is impervious and there is a porous connection between the dam and its foundation, the water will travel from the heel to the toe of the dam along this connection and around the face of the cutoff wall. If the foundation is quite pervious, the water will travel approximately from the heel of the dam in a series of curved lines (assumed to be straight) to the bottom of

the cutoff wall and thence in a straight line to the toe of the dam. For intermediate conditions the travel will be intermediate between these two lines and on the average it is assumed to be along the base of the dam for a distance, thence downward and upward past the end of the cutoff wall at angles of 45 deg. with the horizontal. By reference to Figs. 3, 4, and 5, where L is the length of percolation

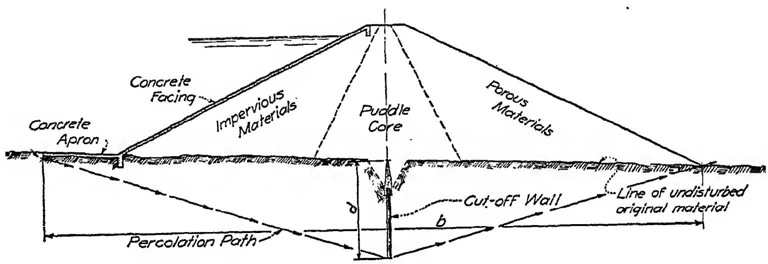


FIG. 3.—Seepage under earth dam with cut-off walls on pervious foundation.

path; x the increased length of travel caused by the cutoff wall; b, the length of the base of the dam; and d, the depth of the cutoff wall, it will be seen that for the first case,

$$x = 2d,$$

and for the second case that

$$x = L - b = \sqrt{b^2 + 4d^2} - b$$

and that for the intermediate case

$$x = 2\sqrt{d^2 + d^2} - 2d = 2.83d - 2d = 0.83d = d \text{ (approximately)}.$$

In general, for average conditions, practice is divided between assuming the increased length of the percolation path to be either one or two times the depth of the cutoff wall.

Figure 3 represents the usual practice in the construction of earth dams on earth foundations. Both center and upstream cutoff walls are desirable, and, if required, a concrete apron may be laid as shown upstream from the dam to increase the percolation path. Figure 4 illustrates a similar treatment for a low overflow masonry dam,

built on an earth foundation. The downstream apron is designed to protect the stream bed from erosion, and, by its weight, to overcome the uplift pressure of the percolating

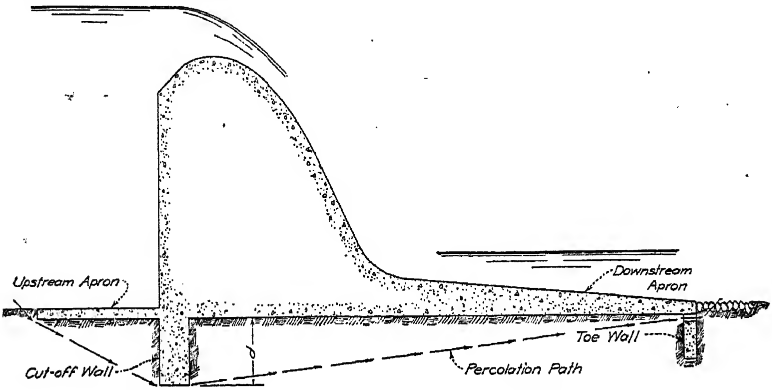


FIG. 4.—Seepage under overflow dam on earth foundation.

water. Figure 5 shows the proper treatment for a low overflow masonry dam built on a gravel or other foundation not subject to erosion by percolating water. In this case

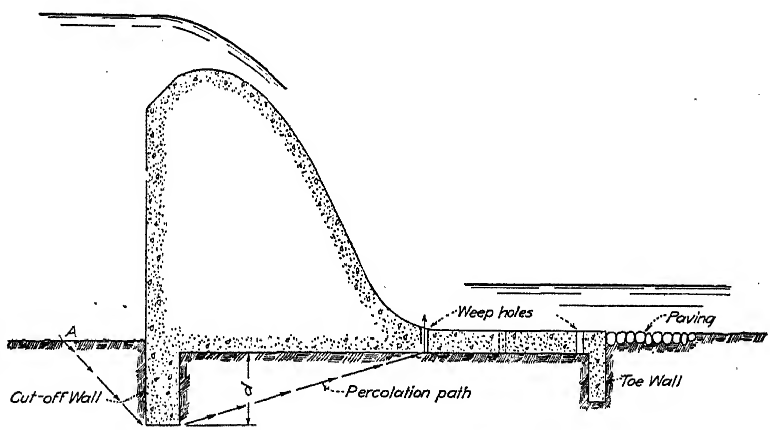


FIG. 5.—Seepage under overflow dam on gravel foundation.

the uplift pressure under the downstream apron is relieved through weep holes, and thus the thickness of the apron may be reduced.

CHAPTER V

FORCES ACTING ON A GRAVITY MASONRY DAM

INTRODUCTION

5-1. General Statement.—The forces acting on a gravity dam consist of the vertical weight of the masonry, the vertical weight of the water or earth on the battered faces and top, the horizontal thrust of the water or earth on the faces, the dynamic water pressure, the water pressure on the base, the reaction of the foundation, ice pressure, air pressure, and earthquake shocks. Of these forces, dynamic water pressure, ice pressure, wind pressure, and earthquake shocks are of minor importance except under unusual conditions. The reaction of the foundation is usually resolved into vertical and horizontal forces.

5-2. Units of Measurement and Assumptions Used in Design.—In determining the effects of the acting forces on a gravity dam, a vertical slice of the dam 1 ft. thick measured perpendicular to its axis and bounded by parallel planes is usually taken, and the forces and stresses computed for such a slice are usually assumed to apply for all parts of the dam of equal height and cross-section. That this is not exactly true will later be shown. In applying the equations herein developed to specific problems, due consideration must be given to the directions in which the forces and moments act. The units of measure are taken as the foot, pound, and foot-pound.

5-3. Nomenclature.—In the development of formulas for determining the forces and stresses, let the following symbols be used with the significance indicated:

H = resultant of horizontal forces above horizontal section.
W = resultant of vertical forces on horizontal section.
w = unit weight of water.

w' = unit weight of masonry.
w'' = unit weight of earth in air or in water.
h = variable depth of water or equivalent water head above horizontal section.
h_1 = depth of water or equivalent water head above horizontal section.
h_0 = depth of water or equivalent water head over the top of dam in the case of overflow dams.
b = length of base in feet on horizontal section.
ϕ = angle of batter of downstream face of vertical section.
ϕ' = angle of batter of upstream face of vertical section.
θ = angle between downstream face and the horizontal, or equals the complement of ϕ.
u = distance in feet of downstream edge to the center of pressure on horizontal section.
d = distance in feet from upstream edge of horizontal section to vertical through center of gravity of vertical forces.
d_1 = distance in feet from downstream edge of horizontal section to vertical through center of gravity of vertical forces.
e = distance in feet from center of horizontal section to center of pressure on horizontal section.
p_2 = vertical unit pressure at toe of horizontal section.
p_1 = vertical unit pressure at heel of horizontal section.
p = vertical unit pressure at any point distant x from the upstream edge of horizontal section.
p' = horizontal unit pressure at any point distant x from the upstream edge of horizontal section.
q = vertical and horizontal unit shear at any point distant x from upstream edge of horizontal section.
g = the acceleration due to gravity.
x = a variable distance measured from the upstream edge of horizontal section.

FORCES ACTING ON A GRAVITY DAM

5-4. Static Water Pressure.—It is evident that one of the primary forces acting on a dam is that of water pressure. In free water pressure existing on either face of the dam, it is customary for convenience in computations, to consider the horizontal and vertical water pressures separately.

By a well-known principle of hydrostatics, (a) the total normal pressure in pounds on any immersed surface is equal to the product of the weight of a cubic foot of water in

pounds, the area of the surface in square feet, and the head in feet on its center of gravity; and by another well-known principle, (*b*) the pressure of water in a given direction is equal to the normal pressure on a projection of the surface perpendicular to that direction.

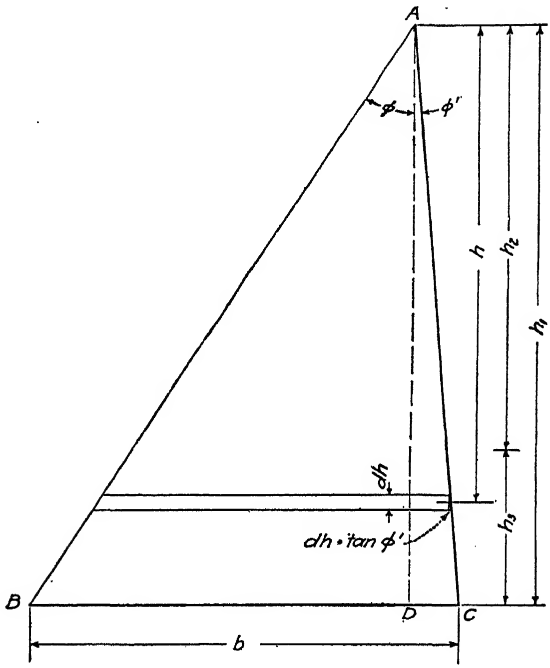


FIG. 6.—Elementary section of a gravity dam.

Figure 6 represents a vertical section 1 ft. thick of a dam. From hydrostatic principles (*a*) and (*b*) it follows that if H_1 and W_1 are respectively the horizontal and vertical pressures of water on the upstream surface of the dam, then, for an overflow dam,

$$H_1 = \int_{h_0}^{h_1} wh\,dh = \frac{w}{2}(h_1^2 - h_0^2); \tag{8}$$

$$W_1 = \int_{h_0}^{h_1} \tan\phi' wh\,dh = \frac{w\tan\phi'}{2}(h_1^2 - h_0^2); \tag{9}$$

and for a non-overflow dam,

$$H_1 = \int_0^{h_1} wh\,dh = \frac{wh_1^2}{2}; \tag{10}$$

$$W_1 = \int_0^{h_1} \tan\,\phi' wh\,dh = \frac{\tan\,\phi' wh_1^2}{2}. \tag{11}$$

From Eqs. (8) and (10), it is evident that the area beneath the pressure curve is a triangle with base wh_1 and altitude h_1 for the full depth h_1 of the water, and a trapezoid for a partial depth such as that between the depths h_1 and h_2 in which case its upper base is equal to wh_2, its lower base to wh_1, and its altitude to $h_1 - h_2$. Whence the total pressure between the limits h_1 and h_2 is

$$H_1 = w\frac{(h_1 + h_2)}{2}(h_1 - h_2) = \frac{w}{2}(h_1^2 - h_2^2), \tag{12}$$

which expression could have been derived by integrating Eq. 8 between the limits of h_1 and h_2. It is also evident that the center of pressure for the triangle of pressure is at a height $\frac{1}{3}h_1$ above the base, and for the trapezoid of pressure, at a height above the base of

$$\frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)}.$$

5-5. Dynamic Pressure from Water, Ice, and Logs.— The flowing of the water in the channel above an overflow dam or the impact of waves in a long open reservoir may cause dynamic water pressure on the structure. This pressure may be taken to be horizontal and may be found from the following formula, which gives the maximum possible value,

$$H_2 = \frac{w\,a\,v^2}{g}, \tag{13}$$

where H_2 is the pressure in pounds per square foot of area affected, a is the area affected in square feet, and v is the mean velocity in feet per second of the moving water.

The pressure due to stream current in overflow dams is generally very small except at the crest, where it will probably never exceed 200 lb. per square foot, correspond-

ing to a mean velocity of about 10 ft. per second in Eq. (13); that due to wave impact even in large reservoirs will probably never exceed a pressure of 2,500 lb. per square foot in the wave impact area corresponding to a mean velocity of about 36.0 ft. per second. An allowance of this amount will also care for impact from floating ice and logs. The impact pressure of current, waves, ice, and logs should be provided for in designing the thickness of the crest, but may be neglected in the design of the dam as a whole, except in the case of relatively low dams and all dams in extremely cold climates. The crushing strength of ice ranges from a few pounds to 1,000 lb. per square inch or more, but before this pressure is exerted on a dam in an ordinary reservoir with sloping banks and considerable reach or a river with similar characteristics, the ice will buckle or shear and move away from the dam.

5-6. Earth Pressure.—Earth pressure may exist on either face of a gravity dam, in which case its effect must be determined. The vertical component of this pressure on the battered faces of the dam is equal to the weight of the earth directly above the faces. If the earth is submerged, its weight must be decreased for flotation effect on the actual solid contents of the material. A good average value to be taken for earth in air is 100 lb. per cubic foot; and earth in water, 65 lb. per cubic foot. For earth level on top, the *horizontal* component of the thrust may be taken as,

$$E_1 = \frac{w''h''^2}{2} \frac{(1 - \sin c)}{(1 + \sin c)}; \qquad (14)$$

for earth sloping towards the dam at the angle of repose,

$$E_1 = \frac{w''h''^2}{2} \cos c; \qquad (15)$$

and for earth sloping away from the dam at the angle of repose,

$$E_1 = \frac{w''h''^2}{2} \frac{\cos c}{(1 + 2 \sin c)^2}; \qquad (16)$$

in which E_1 is the total horizontal pressure of the earth

in pounds for the depth of earth h'', and c is the angle of repose. E_1 should be added to the water pressure over the same area in case of submerged earth, thus giving a value which errs on the side of safety.

5-7. Air Pressure.—Wind pressure on a dam may be neglected, inasmuch as it may amount at most to only about 60 lb. per square foot of exposed area; but air-pressure differential due to the creation of a partial vacuum beneath the overflowing sheet of water of an overflow dam may amount to a considerable part of full atmospheric pressure, which pressure differential may exist over a large proportion of the entire upstream face of the dam. Just what part of full atmospheric pressure may become effective in such a case is a moot question, as the overflowing sheet will bend in toward the dam, thus relieving the pressure, and will break if the suction becomes too high. Another relief of suction is due to air released from the water, and the rise of water below the dam into the suction area. It is highly objectionable to have any such undeterminable force acting on the dam, and it is, therefore, necessary in the design of an overflow dam to so shape the crest that the water sheet will adhere to it in its descent; or to admit air beneath the water sheet by proper means.

5-8. Weight of Masonry.—In computing the weight of the masonry in a dam, samples of the materials to be used should be taken and the weight determined from them. The average weight per cubic foot of masonry varies with the kind of materials of which it is composed, and is somewhat less than that of these materials. The average weight in pounds per cubic foot of some of the standard materials of which dams are built are as follows: granite 155, limestone 160, sandstone 140, granite or trap concrete 155, granitic gravel concrete 150, and broken limestone concrete 145.

UPLIFT IN DAMS

5-9. Cause of Uplift.—Due to the pressure of the water on the upstream side of the dam, there is certain to be

more or less water pressure and flow through the pores of the rock or concrete, and along the contact surfaces between the concrete and rock or the construction joints in the body of the dam itself. In addition, seams in the rock strata below the base of the dam may permit entrance of water under long-continued high pressure. The water thus gaining admittance under the dam causes an upward pressure, generally known as uplift. This upward force must be taken into account in determining the stability against both sliding and overturning of the dam.

5-10. Nature of Uplift.—Under the conditions usually met in practice, the uplift pressure under the dam may be assumed to be similar to that which would exist in a small pipe carrying water from the headwater pressure to the tail-water pressure, the pressure gradient falling uniformly between these points. In this case the uplift-pressure diagram will be represented by a trapezoid as shown in Fig. 7. Since, however, the uplift pressure can exist only in the pores and interstices affecting the section under consideration, it could not extend to the full area of the section unless the dam were actually floating on the film of water. There is, therefore, a factor c to be applied to the area in computing the uplift force.

5-11. Computation of Uplift.—For a section of the dam 1 ft. thick the total uplift W_3' will be the area of the trapezoid 1–2–3–4 (Fig. 7) or

$$W_3' = \frac{cwb}{2}(h_1 + h_1'), \tag{17}$$

and the resultant of this force will be located at the center of gravity of the trapezoid, or a distance d_2 from the upstream face of the dam, whence

$$d_2 = \frac{b}{3}\frac{(h_1 + 2h_1')}{(h_1 + h_1')}. \tag{18}$$

In these equations, h' is the depth of headwater and h_1' is the depth of the tail-water, as shown in Fig. 7. In case there is no tail-water pressure against the dam, the term h_1' becomes equal to zero and the uplift-pressure diagram

becomes a triangle. The equation expressing total uplift W_3' then becomes

$$W_3' = \frac{cwbh_1}{2}, \tag{19}$$

and the distance d_2 of the resultant from the upstream face of the dam becomes

$$d_2 = \frac{b}{3}. \tag{20}$$

5-12. Methods of Reducing Uplift.—In order to reduce the factor c to the lowest possible value, modern dams of

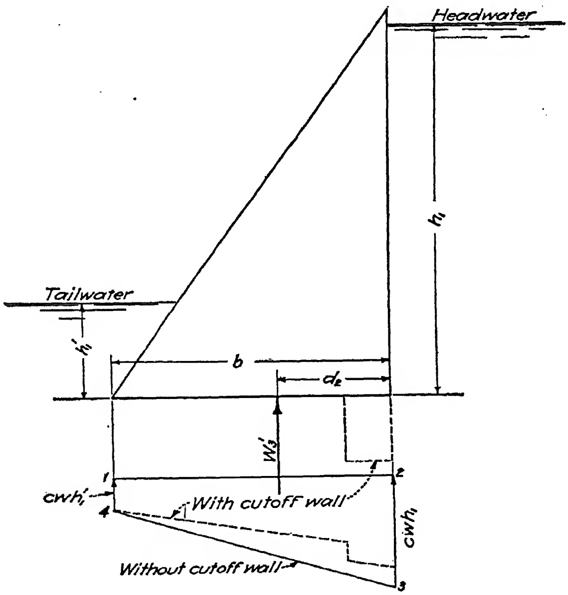


FIG. 7.—Uplift under a gravity dam.

considerable height are usually provided with cutoff walls under the upstream edge, and the rock below the base, if seamy, is sealed with cement grout under high pressures, thus making more difficult the passage of seepage water below the dam. An additional precaution against uplift is taken in properly designed high gravity dams by con-

structing drainage wells into the foundation a short distance downstream from the cutoff wall and for the full length of the dam. These wells intercept the seepage water and conduct it through channels or conduits, within the dam, to the downstream face.

Since the horizontal construction joints within the body of the dam are subject to complete control, so far as water-tightness is concerned, it is proper to adopt a lower value of c for the upper section of the dam than is assumed for the dam as a whole. This lower value, however, is conditioned upon care being exercised in stepping the surface of the concrete to increase the length of the percolation path, and in thoroughly cleaning the old surface of dirt and laitance and sprinkling it with cement before pouring the next section upon it.

As in the case of the flow of water through a pipe of uniform size, the drop in the pressure gradient through the dam will depend upon the length of the percolation path. This length is increased if a cutoff wall is placed below the base of the dam, as in Fig. 7. The hydraulic gradient will then be stepped as shown, and the area of the uplift diagram will be correspondingly reduced, thus resulting in a lower overturning effect. The area and center of gravity of the uplift diagram may be easily solved by simple rules of geometry.

5-13. Relation of Factor c to Uplift.—The amount of water gaining admittance to the pores and interstices of the foundation and the body of the dam depends, of course, upon the porosity of the material forming them, and consequently the value of c depends also on this porosity. Specimens of sound rock forming the foundation of dams, and specimens of good concrete after being soaked in water for some time, generally show an increase of from 2 to 3 per cent in weight due to the penetration of the water. This increase in weight indicates that about 6 per cent of the volume consists of voids. The most unfavorable condition would indicate that full water pressure may be developed in the voids near the upstream

face of the dam and tail-water pressure at the downstream face.

5-14. Theoretical Determination of Factor *c*.—It is probable that the voids in concrete or in rock have a multiplicity of shapes. However, for practical purposes these may be assumed to be either roughly spherical or roughly cubical. Practically the same results will be obtained by using either assumption. A surface of maximum pressure will probably be a warped section passing through the largest number of pore spaces; but for computation purposes the holes cut by this warped surface may be considered to be projected onto a horizontal plane.

In a spherical hole with a volume v and a radius r,

$$v = \tfrac{4}{3}\pi r^3; \tag{21}$$

whence

$$r = \sqrt[3]{\frac{3v}{4\pi}}. \tag{22}$$

If a equals the area of a great circle of the sphere, then

$$a = \pi r^2. \tag{23}$$

Substituting the value of r in Eq. (23) from Eq. (22), and reducing,

$$a = \sqrt[3]{\frac{9\pi}{16}}v^{\frac{2}{3}} = 1.21v^{\frac{2}{3}}. \tag{24}$$

5-15. Practical Values of Factor *c*.—If v is assumed to be 0.06 in Eq. (24), it follows that,

$$a = 0.2 \text{ (nearly)}. \tag{25}$$

This indicates that the pore spaces in concrete may be taken to be about 0.2 of the total surface in a horizontal plane. Consequently the water pressure within the pore spaces on a horizontal section may be taken as 0.2 of the reservoir head on the plane at the upstream face of the dam and of the head of tail-water at the downstream face. This indicates a value 0.2 for c. In case of more porous concrete and foundation rock a value of 0.4 should be used for c. In practice c has been varied from 0 to 1, but the usual value for good concrete on a good rock foundation is

0.25, for good concrete on medium rock foundation 0.50, and for good concrete on very porous foundation material 1.0.

EARTHQUAKE SHOCKS

5-16. Precautions Necessary in Regions Subject to Earthquakes.—In regions subject to earthquakes, attention should be given to the possible effects of shocks on the dam, and the forces produced by such shocks should be added to the other forces used in the design. A careful geologic examination should be made of the position of live faults in the vicinity of the proposed structure. The building of a dam across such a fault should in no case be done, and the location of the structure close to such a fault should be obviated if at all possible.

5-17. Nature of Earthquake Shocks.—The generally accepted theory of earthquakes is that they are caused by sudden slippage of rock surfaces under stress, the resulting waves of vibration advancing through the earth's crust in all directions from the point of disturbance.

The motion of the earth's surface under earthquake vibrations, and therefore of all particles rigidly attached to it, is generally assumed to be "harmonic." If a particle P moves with a constant velocity around the circumference of a circle, then the motion of the projection of P on a diameter of the circle is, by definition, simple harmonic, and is therefore the assumed motion of a particle under earthquake vibrations. Similar conditions exist relative to the motion of the vertical projection of P on the diameter of the circle, but observations of earthquake action indicate that this displacement is only about one-third of the horizontal displacement.

5-18. Values of Earthquake Acceleration to Be Used in Design.—Measurements taken by means of the seismograph during earthquake movements have indicated that the acceleration of points on the earth's surface may acquire values from 0 up to 32 ft. per second per second, which latter value is equal to g. The very severe earthquake

which occurred in Japan on Sept. 1, 1923, had an acceleration of $\frac{1}{3}g$. In the United States a conservative value to adopt in the design of dams would be $\frac{1}{5}g$. The value adopted should, of course, depend upon the region where the dam is to be constructed, the proximity of active faults in the earth's crust, the danger to lives and property in case of failure of the structure, the value of the structure, and the cost of providing a high factor of safety against failure due to earthquakes.

5-19. Earthquake Forces Due to Inertia of Dam.—If a dam is acted upon by an accelerating force, the amount of this force may be determined by the physical law that the force equals the product of the mass and the acceleration, or

$$F = Mk = \frac{W_1 k}{g}, \tag{26}$$

where F is the force on the mass of the dam due to earthquake action, M the mass, W_1 the weight of the dam above the section under consideration, k the acceleration of the body under the force F, and g the acceleration of gravity. The force F should be considered as being applied at the center of gravity of the body, thus producing a moment equal to Fd, in which d is the distance to the center of gravity above the section under consideration. Obviously the shear on this section is equal to F.

In the case of a slice of a dam triangular in shape and 1 ft. thick, the force F should be considered as applied at a point one-third of the height from the base or from the horizontal section above which the stability is being computed, and all points of the dam may be considered to have the same acceleration at the same time. It should be treated as an additional horizontal load, which may act either in an upstream or a downstream direction.

5-20. Earthquake Forces Due to Inertia of Water.—In addition to the force F, there will be a force acting against the dam due to the actual motion forward against the body of water in front of it, which tends to remain motionless. There is also resistance to the vertical motion of the water,

but in determining the pressure on the upstream face of the dam, any effect due to this may be neglected, and hence that due to horizontal motion, only, will here be considered.

In Fig. 8, let F be a prism of water 1 ft. long and of very small equal width and height. As the dam moves forward against it, the prism of water will transmit pressure nearly

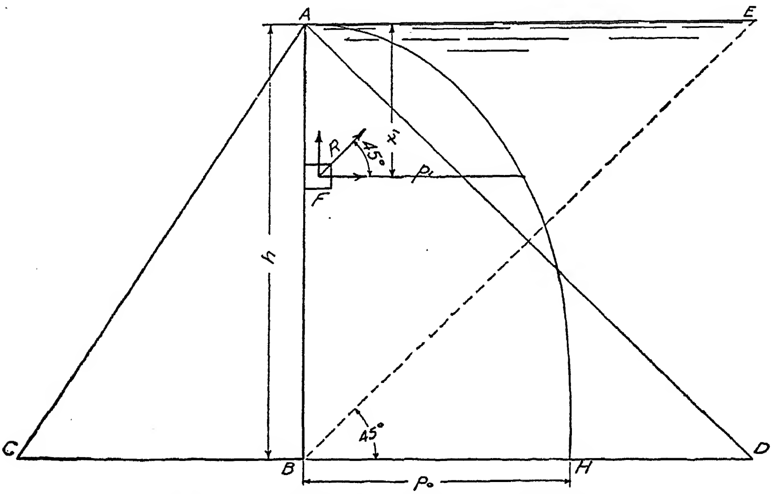


FIG. 8.—Diagram of earthquake forces on dam (forces not to scale).

equally in all directions, and will tend to move in the direction of the resultant R, which, due to the approximate equality of its components, must make an angle of about 45 deg. with the horizontal. Thus the triangle ABE represents the cross-section of the water that will be displaced, and if P is the total inertial resistance to the movement, M' the mass of the water in the prism whose cross-section is ABE, and f the maximum horizontal component of the acceleration of the dam, then, from the well-known physical law,

$$P = M'f. \tag{27}$$

Since it is evident from Fig. 8 that $M' = wh^2/2g$, and if $f = ag$ (in which a is a coefficient to be determined by observation of actual earthquakes), then

$$P = \frac{awh^2}{2}. \tag{28}$$

5-21. Distribution of Earthquake Pressures Due to Water, Based on Straight-line Variation.—It is next necessary to determine the vertical distribution of pressure intensities on the face of the dam. Obviously this pressure is zero at the water surface, and increases according to some law such that the total area enclosed by the pressure curve above the base is expressed by the equation

$$P = \frac{awh^2}{2}. \tag{29}$$

First assuming this area to be triangular, such as indicated by ADB in Fig. 8, and letting p_0 denote the maximum pressure at the base, it follows that

$$\frac{p_0h}{2} = \frac{awh^2}{2},$$

or

$$p_0 = awh. \tag{30}$$

Also, since the center of gravity of the triangle ADB is distant $h/3$ above its base, the moment M about this base will be

$$M = \frac{awh^2}{2} \cdot \frac{h}{3} = \frac{awh^3}{6}. \tag{31}$$

5-22. Distribution of Earthquake Pressures Due to Water, Based on Elliptical Variation.—If the above treatment of this problem had taken into consideration the compressibility of the water, it would have been found that the pressure-intensity diagram is a curve rather than a straight line. This curve would be tangent to the water surface at A, where the pressure is zero, and would be vertical at the level of the base. These conditions are satisfied by an ellipse. The equation of the quadrant of an ellipse AH with its vertex at point A, its vertical axis equal to h, and its horizontal axis equal to p_0, when the origin of coordinates is taken at A, is

$$p = \frac{p_0}{h}\sqrt{x(2h - x)}. \tag{32}$$

The area of the quadrant of this ellipse is $\pi p_0 h/4$, which must equal P, where

$$P = \tfrac{1}{2}awh^2. \tag{33}$$

Equating these values of P and solving for p_0 gives

$$p_0 = \frac{2awh}{\pi}, \tag{34}$$

and substituting p_0 from this equation in Eq. (32), there results

$$p = \frac{2aw}{\pi}\sqrt{x(2h - x)}. \tag{35}$$

The center of gravity of the quadrant of this ellipse is located at a distance d above the horizontal axis, where

$$d = \frac{4h}{3\pi}. \tag{36}$$

This then is the point of application of the total pressure P against the face of the dam, which pressure acts as an overturning force. Hence, M, the overturning moment at the base of the dam is

$$M = \frac{2awh^3}{3\pi}. \tag{37}$$

5-23. Use of Earthquake Formulas.—The formulas for earthquake effects developed in the preceding paragraphs are evidently applicable to arch dams as well as to gravity dams, and also to dams with sloping upstream faces, since the pressures used are assumed to be horizontal and, therefore, applicable to the projection of the upstream face on a vertical plane. It may be shown that this treatment of the problem, on the assumption of either straight-line or elliptical variation of pressure, agrees roughly with the results obtained by considering the waves of expansion caused by harmonic vibrations transmitted through the water in a manner similar to sound waves, as set forth in "Hydrodynamics," (4th ed.) by Horace Lamb, Cambridge University Press. An ingenious development by Wester-

gaard[1] of equations yielding similar results is of interest in connection with earthquake effects on dams.

STABILITY AGAINST EXTERNAL FORCES

5-24. Stability against Sliding.—The net horizontal water and earth pressure tend to slide the dam on a horizontal section, the pressures on the upstream face tending to move the dam above the section downstream, and those on the downstream face tending to move it upstream. The tendency to slide upstream is of minor importance but that downstream is usually the determining factor in the stability of the dam.

The tendency of the dam to slide on any horizontal section is resisted by the friction due to the weight of masonry of the dam and of the water and earth on its battered faces above this horizontal section, decreased by the hydraulic uplift under the section. With no factor of safety, the coefficient of friction f must be just great enough to balance these forces. If a factor of safety, s, is allowed, the value of f must be such that

$$f = s\frac{H}{W}. \tag{38}$$

Data on the actual coefficients of friction of concrete and rock under the high compressive stresses usually existing under large dams are very meager at the present time, being chiefly based on experiments conducted with crude equipment and low pressures. In view of this fact, great care should be used in selecting the value to be used in the design and, if possible, tests should be conducted to determine the actual value for the foundation materials upon which the dam is to be built. Such tests should be made with smooth specimens, the increased friction due to roughness of the natural rock providing an added factor of safety against sliding.

The coefficient of friction of concrete on masonry or masonry on rock generally varies from about 0.60 to 0.75,

[1] "Water Pressures on Dams during Earthquakes," by H. M. Westergaard, senior mathematician, U. S. Bureau of Reclamation

for concrete on concrete it is about 0.60 to 0.70, for concrete on gravel about 0.60, for concrete on sand about 0.40, and for concrete on clay or shale about 0.33. Adhesion of the concrete to bedrock, earth and rock above the plane of contact, and roughness of the foundation bringing shear into play greatly increases resistance to sliding.

In computing the sliding factor it is very important to assume the water surface fully as high as it will ever exist under extreme flood conditions. This is because the total pressure against the dam varies as the square of the water depth, as was shown in Eqs. (8) and (10), thus causing a much larger error in the force than was made in the water level. The overturning effect of the water load, which is discussed in a following paragraph, increases as the cube of the water depth, thus indicating even more strongly the necessity for conservative assumptions in the maximum water level.

5-25. Stability against Overturning.—The horizontal forces, modified by the vertical forces due to uplift, tend to overturn the dam, being resisted by the vertical downward forces. All of the horizontal forces may be reduced to a single force H, acting at the center of gravity of all of these forces at a height h_2 above the section; and all of the vertical forces may likewise be reduced to a single force W, acting at the center of gravity of all of the vertical forces at a distance d_1 from the downstream edge of the section. Then by taking moments about the downstream edge of the section and applying a factor of safety, s_1, to the overturning force H, $s_1Hh_2 = Wd_1$, and, therefore,

$$s_1 = \frac{Wd_1}{Hh_2}. \tag{39}$$

It is necessary to investigate the stability of the dam against overturning in the downstream and upstream directions with the reservoir full and empty, respectively.

When the reservoir is empty the uplift will be zero unless conditions are such that tail-water can exist. The forces due to earthquake and wind acting in the upstream direction

should be computed, however, but not as acting simultaneously, as it has been observed that earth disturbances of a major nature do not occur except under static conditions of the air.

5-26. The Factor of Safety.—It would be very gratifying if, in the design of gravity dams, a factor of safety could be adopted as is done in the case of steel or concrete structures of other types, to be applied uniformly in the design to all of the external and internal forces and stresses tending to prevent the destruction of the dam. However, in practically all true gravity dams of considerable size built at the present time, an analysis would show that the greatest weakness is in the tendency to slide downstream, only the frictional resistance between concrete or rock surfaces being sure to resist this tendency. Next in order, the greatest weakness is usually in overturning, followed by crushing of the rock or concrete at the toe and failure through too high internal stresses in the concrete.

A uniform factor of safety against all of these agencies of destruction is not possible of attainment in practice. A low factor must be assumed for sliding and overturning in order to make the dam feasible, and then every means must be adopted to insure that the assumed external forces are not exceeded.

The minimum factor of safety adopted should properly depend upon the loss which would be occasioned in event of failure of the structure. Dams whose failure would result in the destruction of towns or valuable lands and other property below, and whose replacement cost would be high, should warrant a higher expenditure of capital to insure their safety than if these conditions were not true.

CHAPTER VI

STRESSES IN STRAIGHT-GRAVITY DAMS

INTRODUCTION

6-1. General Statement.—Having determined the exterior forces acting upon a gravity dam of the dimensions assumed, it is next necessary to determine the pressures and stresses resulting from these forces. Certain maximum and minimum pressures and stresses are allowable and if it is found that the computed values fall outside of these limits, it is necessary to change the design accordingly.

FORMULAS FOR PRESSURE DETERMINATIONS

6-2. Weight of Segment of Dam.—In the design of a straight-gravity dam it is customary to proceed downward by horizontal slices with thicknesses of 10 to 20 ft. and it is desirable to devise formulas for such procedure. Let Fig. 9 be the plan and elevation of any such horizontal slice of a dam between two parallel vertical planes 1 ft. apart. Let W_1' be the weight of the segment and M_1' the moment of its weight about its downstream edge and let the other symbols be as shown on the figure.

By referring to Fig. 9 it will be evident that,

$$W_1' = \int_0^{y_2} \int_{y_2 \tan\phi - y\tan\phi}^{y_2 \tan\phi + t + y\tan\phi'} w'dz \cdot dy \cdot 1 = \frac{w'y_2}{2}(T + t). \quad (40)$$

Also referring to Fig. 9 and taking moments about the point A,

$$M_1' = \int_0^{y_2} \int_{y_2 \tan\phi - y\tan\phi}^{y_2 \tan\phi + t + y\tan\phi'} w'dz \cdot dy \cdot z \cdot 1 = \frac{w'y_2}{2}\Big(t^2 +$$

$$2ty_2 \tan\phi + t \cdot y_2^2 \tan\phi' + y_2^2 \tan\phi \cdot \tan\phi' + \frac{2}{3}\, y_2^2 \tan^2\phi$$

$$+ \frac{1}{3}y_2^2 \tan^2\phi'\Big). \quad (41)$$

Since M_1' is the moment of the weight of the segment shown in Fig. 9, there should be added to it, in determining the total moment about the point A, the moment of all the weight lying above this segment. This additional moment

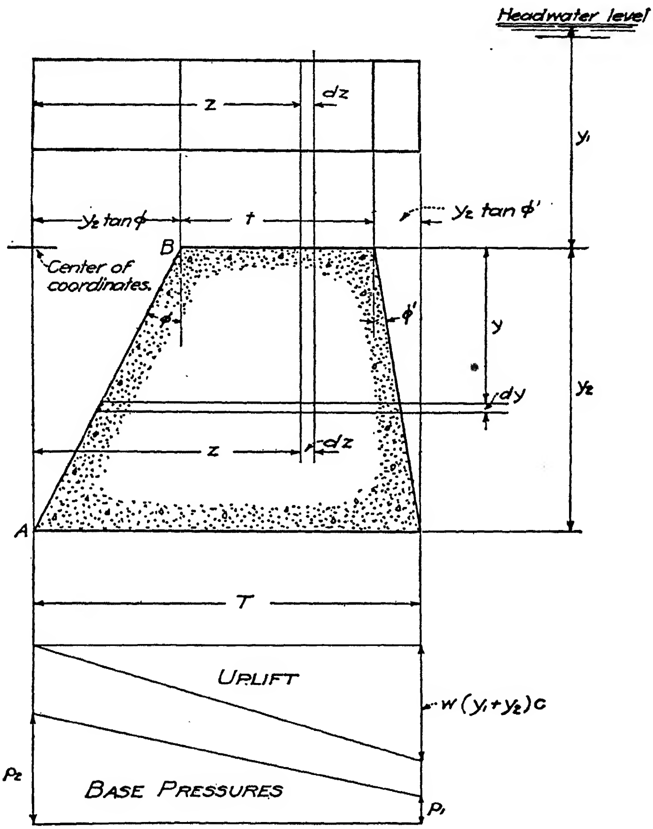


FIG. 9.—Diagram for pressure determinations on a gravity dam.

is equal to $y_2 \tan \phi$ times the total weight above the top surface of the segment, plus the moment of this weight about the point B.

6-3. Horizontal Water Pressure.—The total horizontal water pressure on the segment of Fig. 9 acting in a downstream direction on its upstream face is

$$H_1' = \int_0^{y_2} w(y_1 + y)dy \cdot 1 = wy_2\left(y_1 + \frac{y_2}{2}\right), \tag{42}$$

and the moment of this horizontal pressure about point A is

$$M_2' = \int_0^{y_2} w(y_1 + y)dy(y_2 - y) = \frac{wy_2^2}{2}\left(y_1 + \frac{y_2}{3}\right). \tag{43}$$

Since M_2' is the moment of the horizontal force acting against the upstream face of the segment shown in Fig. 9, there should be added to it, in determining the net moment about the point A, the moment of the horizontal force acting against the part of the dam above the point B. The moment of this force about point B having been previously determined, it is only necessary to add to it the product of force above point B and y_2, in order to compute its moment about point A.

6-4. Vertical Water Pressure.—The water pressure acting vertically on the upstream batter of the segment is

$$W_2' = \int_0^{y_2} w(y_1 + y)dy \cdot \tan \phi' \cdot 1 = w\left(y_1 + \frac{y_2}{2}\right)y_2 \tan \phi', \tag{44}$$

the moment about point A of this vertical water pressure is

$$M_2'' = \int_0^{y_2} w(y_1 + y)dy \cdot \tan \phi'(y_2 \tan \phi + t + y \tan \phi') = wy_2 \tan \phi'\left[y_1\left(T - \frac{y_2 \tan \phi'}{2}\right) + \frac{y_2}{2}\left(T - \frac{y_2 \tan \phi'}{3}\right)\right]. \tag{45}$$

6-5. Uplift on Base of Segment.—If there is assumed to be uplift on the base of the segment and if c is assumed to be the percentage of uplift effective over the whole area, then for trapezoidal uplift pressure on the base (water being also below the dam),

$$W_3' = \frac{cwT}{2}[(y_1 + y_2) + (y_1' + y_2')]; \tag{46}$$

$$M_3' = \frac{cwT^2}{6}[2(y_1 + y_2) + (y_1' + y_2')]; \tag{47}$$

in which y_1' and y_2' refer to backwater pressure. If there is no backwater effect, y_1' and y_2' each equal zero, and

$$W_3' = \frac{cwT}{2}(y_1 + y_2); \tag{48}$$

$$M_3' = \frac{cwT^2}{3}(y_1 + y_2). \tag{49}$$

Equations (48) and (49) apply to the usual condition of a percentage c of the hydrostatic head at the heel varying uniformly to zero at the toe.

STABILITY AGAINST OVERSTRESSING

6-6. General Statement.—In addition to failing through sliding on a horizontal section, overturning about the toe, or yielding of the foundation, a masonry dam may fail through overstressing the material composing the structure. It is, therefore, necessary not only to compute the probable stresses produced in the dam, but to have accurate knowledge of the strength characteristics of the materials of which the dam is to be built. At the present time masonry dams are largely constructed of concrete, but occasionally of masonry composed of granite, limestone, sandstone, or other equally suitable stone. The stresses liable to produce failure are compression, shear, and tension. It is generally conceded by all authorities that a masonry gravity dam should in all cases be free from tensile stress.

6-7. Allowable Working Stresses in Masonry and Rock. The allowable unit working stress for compression and shear for the principal materials of which masonry dams are constructed may be taken as follows:

Material	Stresses, pounds per square inch	
	Compression	Shear
Granite	650 to 700	150 to 175
Limestone	550 to 650	125 to 150
Sandstone	450 to 550	100 to 125
Concrete:		
1,500 lb. at 28 days	375	175
2,000 lb. at 28 days	500	250
2,500 lb. at 28 days	625	300
3,000 lb. at 28 days	750	375

The stresses given in this tabulation are those to be used for a good grade of granite, limestone, sandstone, and concrete made with gravel, granite, trap rock, or hard limestone aggregate.

6-8. Stresses in Existing Dams.—The maximum compressive toe stresses and shearing toe stresses for reservoir full, computed from published descriptions of some large American dams, are as given below. In these computations the masonry weight is taken uniformily at 150 lb. per cubic foot, hydrostatic uplift and arch action are neglected, and parallel-sided vertical slices 1 ft. wide of the dam are taken.

Dam	Water depth at section considered, feet	Toe stresses, pounds per square inch			Radius of curvature
		Vertical	Inclined compression	Inclined shear	
Croton Falls	167	137	215	107	Infinite
Wachusett	205	99	198	99	Infinite
Olive Bridge	220	128	235	107	Infinite
Lake Cheeseman	224	179	279	140	400 ft.
New Croton	238	208	266	133	Infinite
Kensico	250	149	271	135	Infinite
Elephant-Butte	260	175	284	142	Infinite
Roosevelt	260	264	381	190	400 ft.
Exchequer	272.5	362	505	252	674.7 ft.
Arrowrock	341	294	475	237	669.5 ft.
Pardee	350	329	481	240	1,200 ft.
Owyhee	405	418	582	241	500 ft.
San Gabriel*	442.5	365	567	233	1,400 ft.

* Designed but not constructed.

6-9. Vertical Stresses at Toe and Heel on Horizontal Section.—Let Fig. 10 represent the cross-section of a non-overflow gravity dam with the various acting forces and distances indicated by the symbols shown thereon. No earth pressure is assumed to act against either face. Since the vertical loads on the horizontal section of the dam must

be equal to the vertical reaction of the foundation and since the center of gravity of the vertical loads at the horizontal section of the dam must be directly over the center of

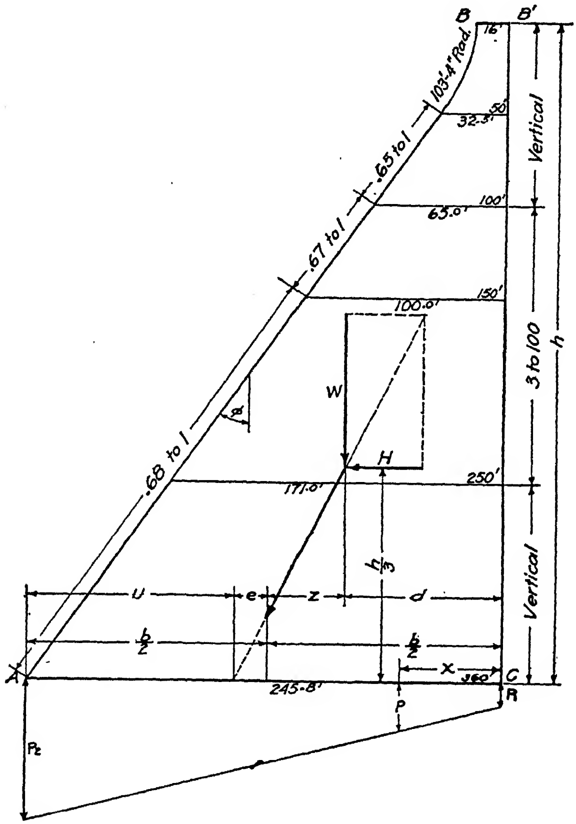


FIG. 10.—Section of gravity dam.

gravity of the reaction forces on the foundation, it is evident that, for unit width,

$$W = \left(\frac{p_2 + p_1}{2}\right)b; \tag{50}$$

$$u = \frac{(p_2 + 2p_1)b}{3(p_2 + p_1)}. \tag{51}$$

Eliminating p_1 from Eqs. (50) and (51), solving for p_2, then eliminating p_2 from these equations, solving for p_1 and reducing,

$$p_2 = \frac{2W}{b}\left(2 - \frac{3u}{b}\right); \tag{52}$$

$$p_1 = \frac{2W}{b}\left(\frac{3u}{b} - 1\right). \tag{53}$$

Noting that $u = \frac{b}{2} - e$ in Eqs. (52) and (53), the values of p_2 and p_1 become as follows:

$$p_2 = \frac{W}{b}\left(1 + \frac{6e}{b}\right); \tag{54}$$

$$p_1 = \frac{W}{b}\left(1 - \frac{6e}{b}\right). \tag{55}$$

If e is equal to $b/6$ in Eqs. (54) and (55),

$$p_2 = \frac{2W}{b} \quad \text{and} \quad p_1 = 0;$$

if e equals 0,

$$p_2 = p_1 = \frac{W}{b}.$$

If

$$e > \frac{b}{6}, \quad p_2 = \frac{2W}{3\left(\frac{b}{2} - e\right)}$$

assuming no tension possible in the masonry. It is of interest to note that Eqs. (54) and (55) are identical with the general formula of mechanics for stresses in a rectangular bar under combined compression and flexure due to an eccentric load. In the general formula for a rectangular bar,

$$p = \frac{W}{A}\left(1 \pm \frac{6e}{b}\right),$$

in which p is identical with p_2 or p_1 according to the interpretation of the signs on the right-hand side of the equation; W, e, and b are identical with the same symbols in Eqs. (54) and (55); and A is equal to $b \cdot 1$, since the horizontal cross-

section of the dam under consideration has the width of b ft. and the length of 1 ft.

6-10. Vertical Stresses at Any Point in Horizontal Sections.—The vertical pressure at any point in the base, distant x from the upstream edge of the section, can be determined from the principle of the trapezoid of pressure as follows:

$$p = p_1 + \frac{x}{b}(p_2 - p_1). \tag{56}$$

Taking the moment of the total vertical force W and the horizontal water force H about the point of intersection of the resultant of these two forces with the horizontal section, it may be shown that

$$e = \left(\frac{wh^3}{6W} - \frac{b}{2} + d\right). \tag{57}$$

Substituting the value of e from the Eq. (57) in Eqs. (54) and (55); then substituting the values of p_2 and p_1 from the resulting equations in Eq. (56), and reducing terms, Eq. (56) takes the following form for a non-overflow dam, from which p may be determined:

$$p = \frac{2W}{b}\left(2 - \frac{3d}{b}\right) - \frac{wh^3}{b^2} + \left[\frac{6W}{b}\left(\frac{2d}{b} - 1\right) + \frac{2wh^3}{b^2}\right]\frac{x}{b}. \tag{58}$$

The total vertical pressure P on the horizontal section may be obtained by integration of the pressure intensity given by Eq. (58), whence

$$P = \int_0^b \left[\frac{2W}{b}\left(2 - \frac{3d}{b}\right) - \frac{wh^3}{b^2}\right]dx + \left[\frac{6W}{b}\left(\frac{2d}{b} - 1\right) + \frac{2wh^3}{b^2}\right]\frac{x\,dx}{b} = W. \tag{59}$$

For an overflow dam,

$$p = \frac{2W}{b}\left(2 - \frac{3d}{b}\right) - \frac{w}{b^2}(h - h_0)^2(h - 2h_0) + \left[\frac{6W}{b}\left(\frac{2d}{b} - 1\right) + \frac{2w}{b^2}(h - h_0)^2(h - 2h_0)\right]\frac{x}{b}. \tag{60}$$

6-11. Shearing Stresses on Horizontal Sections.—Inasmuch as the vertical cross-sections of dams are generally of triangular or trapezoidal shape, it is somewhat difficult to determine the shearing stress on a horizontal section such as is here under consideration. It is evident that the intensity of horizontal shear on such sections will vary from point to point along the section and that the total shear will be equal to the resultant horizontal force on the dam above the horizontal section. However, here, as in rectangular beams, the horizontal and vertical shearing intensities are equal at any point in the section. E. P. Hill has published in the 1908 *Proceedings of the Institute of Civil Engineers* a formula for finding the intensity of shear on horizontal or vertical sections at any point of a horizontal section for a non-overflow dam with a vertical back and an inclined face, which formula may also be found on page 905 of "American Civil Engineers' Handbook," by Merriman and Wiggin (5th ed.). The formula may be written as follows in terms of the symbols here used:

$$q = [4W\tan\phi(b - 3d) + wh^2(3b - 2h\tan\phi)]\frac{x}{b^3} -$$

$$[6W\tan\phi(b - 3d) + 3wh^2(b - h\tan\phi)]\frac{x^2}{b^4}. \qquad (61)$$

In the case of a rectangular dam,

$$\phi = 0,\ d = \frac{b}{2}$$

and

$$q = \frac{3wh^2x}{b^2} - \frac{3wh^2x^2}{b^3}.$$

If $x = 0$ or $x = b$, $q = 0$.

$$\frac{dq}{dx} = \frac{3wh^2}{b^2} - \frac{6wh^2x}{b^3} = 0;$$

whence $x = b/2$.

Since

$$\frac{d^2q}{dx^2} = -\frac{6wh^2}{b^3},$$

a negative quantity q is a maximum at $x = b/2$, and the shear diagram is a parabola with its axis vertical through the center of the base of the horizontal section. When

$$x = \frac{b}{2}, \; q = \frac{3wh^2}{4b}.$$

For a triangular dam with the upstream face vertical,

$$d = \frac{b}{3} \text{ and } \tan \phi = \frac{b}{h};$$

whence,

$$q = \frac{wh^2x}{b^2}.$$

If

$$x = 0, \; q = 0; \text{ if } x = b, \; q = \frac{wh^2}{b}.$$

That is, there is no maximum ordinate in the shear diagram within the dam and the maximum shear intensity is at the downstream face.

The integral of Eq. (61) will give the total horizontal shear on the horizontal section, whence

$$Q = \int_0^b [4W \tan \phi(b - 3d) + wh^2(3b - 2h \tan \phi)]\frac{x\,dx}{b^3} - [6W \tan \phi(b - 3d) + 3wh^2(b - h \tan \phi)]\frac{x^2dx}{b^4} = \frac{wh^2}{2}. \quad (62)$$

6-12. Normal and Shearing Stresses on Vertical Planes. The horizontal water pressure exerts normal and shearing stresses on vertical planes in the dam. The intensity of the shearing stress on a vertical plane is equal to that on the horizontal plane at the same point, and may be found from Eq. (61). The intensity of the normal stress on

vertical planes increases from the heel towards the toe of any horizontal section. Hill, in the article already referred to, has published a formula for finding this intensity, and the same is to be found in the "American Civil Engineers' Handbook," at the page already referred to. The formula may be written as follows in terms of the symbols here used:

$$p' = wh + \left[\frac{2W\tan^2\phi}{b^3}\left(2 - \frac{9d}{b}\right) - \frac{3wh(b - h\tan\phi)^2}{b^4} + \frac{\tan\phi}{b}\right]x^2 + \left[\frac{6W\tan^2\phi}{b^4}\left(\frac{4d}{b} - 1\right) + \frac{2wh(b - h\tan\phi)(b - 2h\tan\phi)}{b^5} - \frac{\tan\phi}{b^2}\right]x^3. \quad (63)$$

For a rectangular dam, $\phi = 0$ and $d = b/2$, whence

$$p' = wh - \frac{3whx^2}{b^2} + \frac{2whx^3}{b^3}.$$

When $x = 0$, $p' = wh$, and when $x = b$, $p' = 0$, that is, the intensity of horizontal pressure on vertical planes is equal to the intensity of water pressure at the heel, and to 0 at the toe of any horizontal section.

$$\frac{dp'}{dx} = -\frac{6whx}{b^2} + \frac{6whx^2}{b^3} = 0 \text{ and } x = 0 \text{ or } b.$$

$$\frac{d^2p'}{dx^2} = -\frac{6wh}{b^2} + \frac{12whx}{b^3},$$

which is negative for $x = 0$ and positive for $x = b$. Whence, there is a maximum value of p' at the upstream face and there is no maximum or minimum value between the heel and toe of any section.

For a triangular dam with the upstream face vertical,

$$d = \frac{b}{3}$$

and

$$\tan\phi = \frac{b}{h},$$

whence

$$p' = wh - \left[\frac{2W}{bh^2} - \frac{1}{h}\right]x^2 + \left[\frac{2W}{b^2h^2} - \frac{1}{bh}\right]x^3.$$

When

$$x = 0 \text{ or } b,\ p' = wh.$$

$$\frac{dp'}{dx} = -2\left[\frac{2W - bh}{bh^2}\right]x + 3\left[\frac{2W - bh}{b^2h^2}\right]x^2.$$

For maximum or minimum,

$$\frac{dp'}{dx} = 0;$$

whence

$$x = 0, \text{ or } x = \frac{2}{3}b.$$

$$\frac{d^2p'}{dx^2} = \left[\frac{4W - 2bh}{bh^2}\right]\left(\frac{3x}{b} - 1\right),$$

which is negative for $x = 0$ and hence there is a maximum value of $p' = wh$ at the upstream face. For

$$x = \frac{2}{3}b,$$

d^2p'/dx^2 is positive, and hence there is a minimum value of

$$p' = wh - \frac{4b}{27h}\left(\frac{2W}{h} - b\right).$$

6-13. Principal Stresses within the Dam.—The intensity of the stresses on horizontal and vertical planes at any point are not the maximum or minimum intensities. There are two planes at right angles to each other through the point, on one of which the normal intensity is a maximum and on the other of which it is a minimum. The stresses on these planes are called the *principal stresses* at the point; and the planes are called the planes of principal stresses. The shearing stress intensity is 0 on the planes of principal stresses, and is a maximum on planes making angles of 45 deg. with the planes of principal stresses. Formulas

for finding the amounts of the principal stresses and directions of the planes of principal stresses and of finding the maximum shearing intensity and the direction of the planes

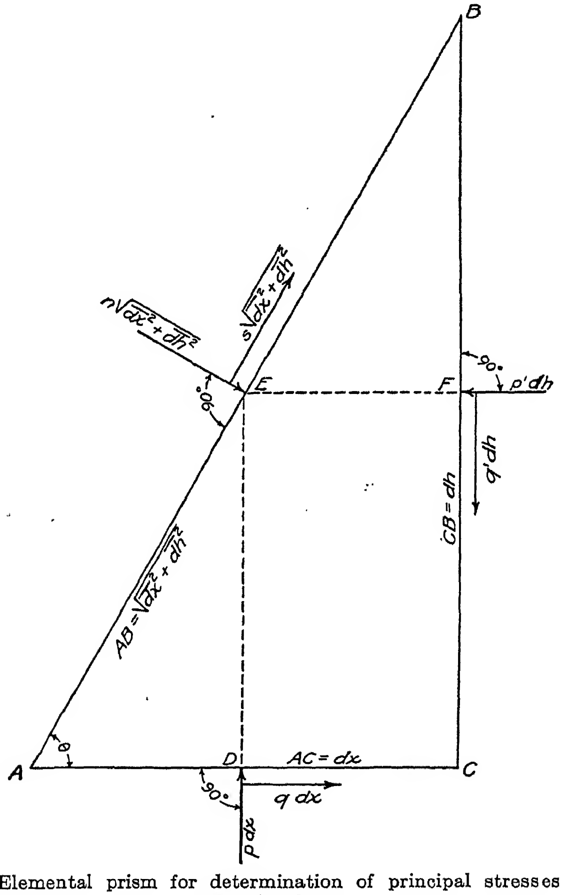


FIG. 11.—Elemental prism for determination of principal stresses within a gravity dam.

on which it occurs are developed in works on mechanics of materials.

Let ABC of Fig. 11 represent an infinitesimal prism, with faces parallel to the plane of the paper and a length of one unit perpendicular thereto, taken from the interior of the dam in which the angle of the sloping face with the

horizontal, the dimensions and the stress intensities, directions, and points of application are as shown thereon. Since the prism is very small the intensities of the forces may be considered to be constant over the faces of the prism and applied at the mid points of the sides of the figure. The weight of the prism is neglected, being a quantity of the second order. Assume as positive, compressive forces, counterclockwise angles and moments, and shears which tend to cause counterclockwise rotation of the prism. Assume opposite forces as negative. Taking moments about E, it will be found that

$$q' \cdot dh \cdot \frac{dx}{2} = q \cdot dx \cdot \frac{dh}{2},$$

whence by reduction

$$q' = q. \tag{64}$$

Placing the sum of the vertical forces and also of the horizontal forces each equal to zero,

$$p \cdot dx - q \cdot dh - n\sqrt{dx^2 + dh^2} \cos \theta + s\sqrt{dx^2 + dh^2} \sin \theta = 0,$$

and

$$-p' \cdot dh + q \cdot dx + n\sqrt{dx^2 + dh^2} \sin \theta + s\sqrt{dx^2 + dh^2} \cos \theta = 0.$$

Noting that

$$dh \div dx = \tan \theta$$

and that

$$\sqrt{dx^2 + dh^2} \div dx = \sec \theta,$$

solving successively for n and s and making the required trigonometric substitution

$$n = \frac{p + p'}{2} + \left(\frac{p - p'}{2}\right) \cos 2\theta - q \sin 2\theta; \tag{65}$$

$$s = \frac{p' - p}{2} \sin 2\theta - q \cos 2\theta. \tag{66}$$

Equations (65) and (66) furnish a means of computing the normal and shearing intensities on any plane making the angle θ with the horizontal. The value of θ that will make n a maximum or minimum and likewise that which will make s a maximum or minimum may be determined by applying the usual derivative formulas for determination of maximum and minimum quantities. From Eq. (65),

$$\frac{dn}{d\theta} = -(p - p') \sin 2\theta - 2q \cos 2\theta = 0,$$

whence

$$\tan 2\theta = \frac{2q}{p' - p}. \tag{67}$$

Equation (67) is satisfied by two values of 2θ differing by 180 deg. and, therefore, it is satisfied by two values of θ differing by 90 deg. The value of n for one of these values of θ is a minimum and for the other it is a maximum. The second derivative of n with respect to θ is

$$\frac{d^2n}{d\theta^2} = -2(p - p') \cos 2\theta + 4q \sin 2\theta,$$

from which it may be shown that the plane of minimum stress lies between 0 and 45 deg., and that of maximum stress between 90 and 135 deg.

From trigonometric relations,

$$\cos 2\theta = \frac{1}{\sqrt{1 + \tan^2 2\theta}}$$

and

$$\sin 2\theta = \frac{\tan 2\theta}{\sqrt{1 + \tan^2 2\theta}},$$

and by substituting the value of $\tan 2\theta$ from Eq. (67) in these expressions,

$$\sin 2\theta = \frac{\pm 2q}{\sqrt{(p - p')^2 + 4q^2}}$$

and

$$\cos 2\theta = \frac{\pm(p - p')}{\sqrt{(p - p')^2 + 4q^2}}.$$

Substituting these values in Eq. (65), it becomes

$$n = \frac{p + p' \pm \sqrt{(p - p')^2 + 4q^2}}{2}. \qquad (68)$$

From Eq. (68), the maximum and minimum principal stresses may be found at any point in the dam.

Since the maximum normal stresses are perpendicular to the planes of principal stresses, no tangential component and consequently no shear exists on them. It is evident, therefore, that the planes on which the maximum shearing intensity occurs are not coincident with either of the planes of principal stresses. The angle θ will then be different for a maximum value of s in Eq. (66) from that for a maximum or minimum value of n in Eq. (65). Designating the angle for the shear plane as θ', the revision of Eq. (66) becomes

$$s = \frac{p' - p}{2} \sin 2\theta' - q \cos 2\theta'.$$

Taking the first derivative of s with the respect to θ',

$$\frac{ds}{d\theta} = (p' - p) \cos 2\theta' + 2q \sin 2\theta' = 0;$$

whence,

$$\tan 2\theta' = \frac{p - p'}{2q}. \qquad (69)$$

Equation (69) is satisfied by two values of $2\theta'$ differing by 180 deg.; and, therefore, it is satisfied by two values of θ' differing by 90 deg. Since the shear on planes at right angles to one another is always equal as shown in Eq. (64), the value of s will be a maximum on both of these planes. From Eq. (67),

$$\cot 2\theta = \frac{p - p'}{2q} = \tan 2\theta' = \cot (90° - 2\theta')$$

and

$$2\theta = 90° - 2\theta',$$

whence

$$\theta' = 45° - \theta.$$

Therefore, the planes of maximum shearing intensity make an angle of 45 deg. with the planes of principal stresses. Noting that

$$\sin 2\theta' = \frac{\tan 2\theta'}{\sqrt{1 + \tan^2 2\theta'}},$$

and

$$\cos 2\theta' = \frac{1}{\sqrt{1 + \tan^2 2\theta'}},$$

substituting the value of $\tan 2\theta'$ from Eq. (69) in these expressions, and then substituting these results in the revision of Eq. (66), the following expression is found for the maximum shear,

$$s = \mp \tfrac{1}{2}\sqrt{(p - p')^2 + 4q^2}. \qquad (70)$$

6-14. Principal Stresses at Faces of the Dam.—The normal and shearing stresses on an elementary prism at the faces of the dam are different from those for an elementary prism within the dam owing to the lack of adjacent masonry on one face to offer resistance to the movement of the elementary prism used in the analysis. However, the vertical compressive and shearing stresses at the toe and heel of the dam are not the maximum stresses at these points, and it is therefore important to determine these maximum stresses so far as practicable.

Let *ABC* of Fig. 12 represent an infinitesimal prism at one of the faces of the dam with its face parallel to the plane of the paper and a length of one unit perpendicular thereto, in which the angle of the sloping face with the horizontal, the dimensions, the stress intensities, directions, and points of application are as shown thereon. Since

the prism is very small, the intensities of the forces may be considered to be constant over the faces of the prism

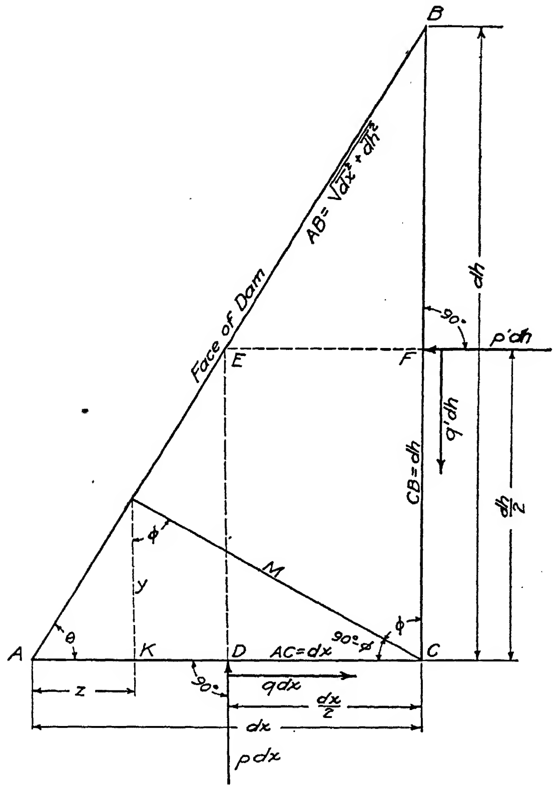


FIG. 12.—Elemental prism for determination of principal stresses at downstream face of a gravity dam.

and applied at the midpoints of the faces of the prism. The prism is held in equilibrium by external forces $p'dh$, $p\,dx$, $q'dh$, and $q\,dx$, the weight of the prism being neglected as a quantity of the second order.

Taking moments about the point E,

$$q\,dx\frac{dh}{2} = q'\,dx\frac{dh}{2}.$$

From this equality,

$$q' = q. \tag{71}$$

In Fig. 12, let CG be the general trace of a plane through C, perpendicular to ABC and making an angle of ϕ with the face BC, on which it is desired to find the maximum unit compression and unit shear. The angle ϕ will, of course, be different for compression and shear. The area M on the plane CG within the prism ABC may be found from the geometry and trigonometry of the figure to be,

$$M = \frac{dx}{\sin \phi + \frac{dx}{dh} \cos \phi} = \frac{dx}{\sin \phi + \cot \theta \cos \phi}. \quad (72)$$

The unit thrust n on the area M is that due to the forces $p\,dx$ and $q\,dx$, or its equivalent $p'dh$, resolved perpendicular to M and divided by it; whence

$$n = p(\sin \phi + \cot \theta \cos \phi)^2. \quad (73)$$

$$\frac{dn}{d\phi} = 2p(\sin \phi + \cot \theta \cos \phi)(\cos \phi - \cot \theta \sin \phi),$$

from which $\cos \phi - \cot \theta \sin \phi = 0$, or $\tan \phi = \tan \theta$, as the condition for maximum when the second derivative of n with respect to ϕ is negative since $\sin \phi + \cot \theta \cos \phi$ cannot equal zero from the evident limitations on the values of ϕ and θ. $d^2n/d\phi^2$ is negative for $\tan \phi = \tan \theta$, whence n is a maximum for $\phi = \theta$. Substituting θ for ϕ in Eq. (73) and reducing,

$$n = p \csc^2 \theta. \quad (74)$$

The unit shear s on the area M is that due to the forces $p\,dx$ and $q\,dx$, or its equivalent $p'\,dh$ resolved parallel to M and divided by it, whence

$$s = p(\cos \phi - \cot \theta \sin \phi)(\sin \phi + \cot \theta \cos \phi). \quad (75)$$

$$\frac{ds}{d\phi} = p[(1 - \cot^2 \theta) \cos 2\phi - 2 \cot \theta \sin 2\phi] = 0,$$

or

$$\tan 2\phi = -\cot 2\theta$$

for the critical value of s. Since

$$\frac{d^2s}{d\phi^2} = p[2(\cot^2 \theta - 1) \sin 2\phi - 4 \cot \theta \cos 2\phi]$$

is negative for all values of θ between 45 and 90 deg., s is a maximum when $\tan 2\phi = -\cot 2\theta$. To satisfy this equality

$$2\phi = 90° + 2\theta \text{ and } \phi = 45° + \theta.$$

Substituting $45° + \theta$ for ϕ in Eq. (75) and reducing,

$$s = \frac{-p \csc^2 \theta}{2}. \tag{76}$$

It will be noted that Eqs. (74) and (76) apply to either face of the dam, the stress p being taken for the reservoir full for the toe and for the reservoir empty at the heel; and θ in either case being taken as the angle of the slope of the face of the dam under consideration.

6-15. Torsional Stresses.—If a gravity dam is divided into a series of vertical imaginary cantilever beams by means of parallel planes, 1 ft. apart, and perpendicular to the axis of the dam, these beams will each deflect downstream under the water load differing amounts due to varying lengths or to attachment to the sides of the canyon or both. In the usual V- or U-shaped canyon each succeeding beam from the center towards the canyon walls will be shorter than the preceding one; consequently, any intervening beam will be deflected less than the preceding and more than the succeeding one. But since the cantilever beams are imaginary only, any intervening beam will be affected by the adjacent ones, the shorter one tending to hold it back and the longer one to pull it forward. The results are a torsional stress in the intervening beam, an increase in the overturning moment on the shorter, and a decrease in the overturning moment on the longer, adjacent beams. It is evident, therefore, that the lower parts of the dam should have correspondingly greater resistance against overturning, particularly if there is no

support to be gained from the abutments. Good, substantial, unyielding abutments and a curved axis of the dam will help to provide this additional needed resistance.

The deflections of the cantilever beams may be computed by means of the law of displacements or the internal-work

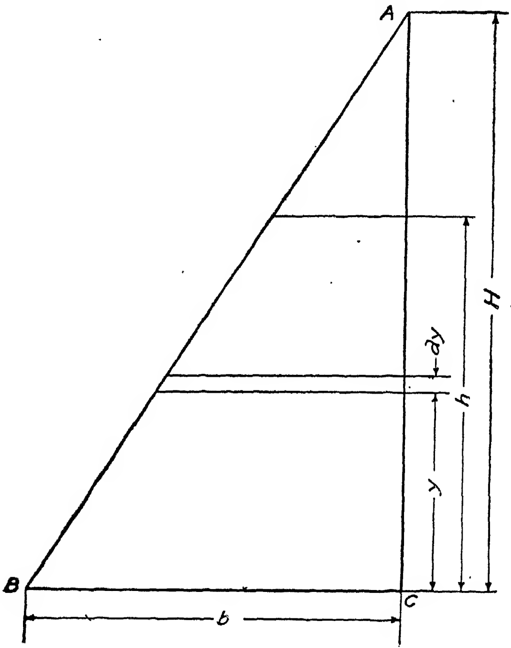


FIG. 13.—Diagram for determination of deflection in a gravity dam.

theory, the fundamental formula for which exclusive of rib shortening is,

$$\Delta_h = \int_0^h \frac{Mm}{EI}\,dy + 3\int_0^h \frac{Vv}{EA}\,dy, \qquad (77)$$

for a triangular-shaped section in which M and V are the moment and shear due to the water load acting in producing stress on the elementary strip dy, and m and v are the respective moment and shear on dy due to a unit load of 1 lb. applied at the point of deflection h, all as shown in Fig. 13. By assigning proper values to the various symbols in Eq. (77), the deflection Δ_h, for any point h, is

$$\Delta_h = \frac{wH^3h^2}{Eb^3} + \frac{3wH}{4Eb}(2Hh - h^2); \tag{78}$$

and when $h = H$,

$$\Delta_h = \frac{wH^5}{Eb^3} + \frac{3wH^3}{4Eb}. \tag{79}$$

If the cross-section of the dam is not triangular, an equivalent triangular section may generally be assumed.

By computing the deflections of sections of different heights, the influence of the various cantilevers on one another may be seen to exist, but its actual amount is difficult to determine.

6-16. Stresses Due to Changes in Volume.—The volume of concrete in a dam tends to change with variations in temperature and variations in water content. This shrinkage, if unrestrained, amounts to 0.0003 to 0.0005 of the length of each dimension of the mass during the curing process, which corresponds to a drop in temperature of from 50 to 83°F. If restrained the stress caused by the tendency to shrink, assumed to be uniform throughout the cross-section under consideration, is identical with that due to a force causing an equal deformation. A method of computing the temperature at any point in the body of a concrete mass for a known range of exterior temperature is given in paragraph 17-7. The unit stress produced in the concrete may be computed by the well-known equation

$$s = 0.0000065ET \div L,$$

where E is the modulus of elasticity of concrete, T the change in temperature, and L the length under consideration. If no provisions are made to withstand shrinkage due to temperature, cracks will be formed in the concrete. To avoid this difficulty, dams are usually constructed in independent blocks of suitable lengths. This method is discussed in paragraph 8-16. Other possible methods are the use of cements of low setting heat and the dissipation of heat in the concrete by forcing cold water through a

network of pipes distributed throughout the mass, thus preventing the concrete from reaching a high temperature and hastening the reduction to a degree at which it is permissible to pour the adjacent blocks.[1]

6-17. Classical Theory of Stress Variation on Plane Sections.—In the treatment of gravity dams in this book, it is assumed that the variation of stress on plane sections subjected to bending moment is linear. Although it is appreciated that this theory is not strictly correct, it has nevertheless been used on account of the resulting simplicity of stress equations. This use is in conformity with accepted practice in dam design and is justified by the success of the many dams designed in accordance with it. The error in this assumption has long been suspected, but was first experimentally proved by English engineers through the use of model dams made of plasticene and India rubber.[2] In the plasticene model the results were sufficient to indicate that tension might exist at the heel even though the resultant cut the base at the middle-third. In the India-rubber model, the principal stresses were found to act parallel to the inclined downstream face near that face, and perpendicular to the upstream face near it. The variation of stresses along both horizontal and vertical planes was found to be curvilinear rather than rectilinear. An approximate solution of the stress variation in gravity dams; based on the principle of least work, that appears to confirm the conclusions drawn from these tests, has been advanced by Jakobsen.[3] This solution is only a partial one because it fails to include the effects of the deformations of the foundation plane.[4] Much further experimental and mathematical work remains to be done before a satisfactory solution of this problem can be obtained. In the meantime, the designer should adhere to the classical theory.

[1] *Western Construction News*, Vol. 6, p. 10, Jan. 10, 1931.

[2] *Proc. I.C.E.*, Vol. 172, Part II, 1907–1908.

[3] *Proc. Am. Soc. Civil Eng.*, Vol. 56, September, 1930; see also following discussions.

[4] VOGT, FREDERICK, "Computation of the Deformation of Foundations"; Oslo, 1925.

CHAPTER VII

STRESSES IN CURVED-GRAVITY DAMS

INTRODUCTION

7-1. General Statement.—An arched-gravity dam will produce arch action under water load if the necessary conditions and precautions are taken. First, the canyon walls must be able to take the thrust without yielding; second, the canyon span and the radius of curvature must not be too great; third, sufficient contraction joints must be provided to prevent occurrence of shrinkage cracks; fourth, constant contact of the various sections of the dam must be secured by grouting the contraction joints in the coldest available weather. Under these conditions a curved-gravity dam will be safer than a straight-gravity dam of the same cross-section.

7-2. Methods of Computing Stresses in Curved Gravity Dams.—The stresses in an arched-gravity dam, subject to arch action, should be figured on the parallel-plane theory, that is, a slice 1 ft. wide between parallel planes, in the same manner as those for a straight-gravity dam, because such a slice is the only one that can consistently be assumed to slide, shear, or rotate out of position. Where, however, non-resistant, converging canyon walls or earth abutments or better foundation conditions on a curved plan exist, it is desirable to use the radial-plane theory, that is, a vertical slice between radial planes, 1 ft. apart, measured horizontally at the mid-height of the segment, and at its upstream face, since in such cases no added safety results from curvature of plan. Formulas for stresses based on the radial-plane theory may, therefore, be useful.

FORMULAS FOR PRESSURE DETERMINATIONS

7-3. Weight of Segment of Dam.—Let Fig. 14 represent the plan and sectional elevation of one of the segments of a vertical slice of an arched-gravity dam comprised between two horizontal planes; and let the symbols shown in the figure have the significance indicated. Consider an elementary volume and an elementary surface of the segment and extend these considerations to the entire segment

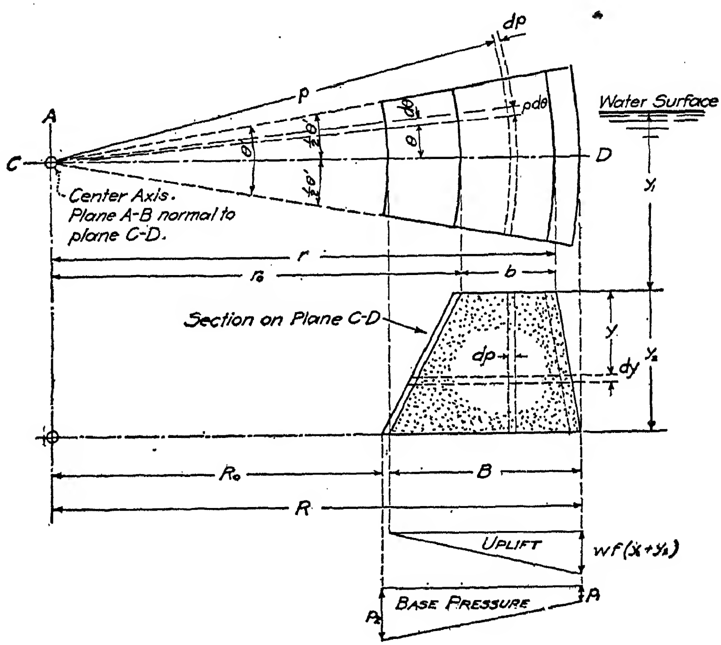


Fig. 14.—Plan and section of segment of an arched gravity dam.

by integration of the differential quantities between proper limits. Let the moments be taken about a vertical plane perpendicular to the radial plane bisecting the segment and passing through the center of curvature.

Now, from Fig. 14, the weight of an elementary volume is $w'dy\, dp\, p\, d\theta$, and the moment m about the center axis is $w'dy\, dp\, p\, d\theta\, p \cos \theta$, where w' is the weight of a cubic foot of the material in the segment and the dimensions are in feet.

Whence

$$W = w' \int_{y=0}^{y=y_2} \int_{p=r_0+\frac{y}{y_2}(R_0-r_0)}^{p=r+\frac{y}{y_2}(R-r)} \int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}} dy\, dp\, p\, d\theta; \quad (80)$$

$$W = \frac{w'y_2\theta'}{6}\left[\left(\frac{R^3 - r^3}{R - r}\right) - \left(\frac{R_0^3 - r_0^3}{R_0 - r_0}\right)\right]. \quad (81)$$

Also referring to Fig. 14, it will be evident that,

$$M = w' \int_{y=0}^{y=y_2} \int_{p=r_0+\frac{y}{y_2}(R_0-r_0)}^{p=r+\frac{y}{y_2}(R-r)} \int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}} dy\, dp\, p\, d\theta\, p \cos\theta, \quad (82)$$

$$M = \frac{w'y_2 \sin \theta'/2}{6}\left[\left(\frac{R^4 - r^4}{R - r}\right) - \left(\frac{R_0^4 - r_0^4}{R_0 - r_0}\right)\right]. \quad (83)$$

7-4. Vertical Water Pressure.—The upstream surface of the segment is acted upon by vertical water pressure in conjunction with the weight of the segment, thus increasing its stability, and it is also acted upon by horizontal water pressure tending to produce overturning about the downstream edge and sliding on the base. The vertical pressure acts on the horizontal projection of the upstream surface and the horizontal pressure acts on the vertical projection thereof. The horizontal projection of the elementary area n of the surface lies at a horizontal distance from the center axis of

$$r + \frac{y}{y_2}(R - r)$$

and has a width of

$$\frac{dy}{y_2}(R - r).$$

The water pressure on this elementary area is, therefore,

$$w_1 = w(y_1 + y)\left(\frac{R - r}{y_2}\right)\left[r + \frac{y}{y_2}(R - r)\right]dy\, d\theta, \quad (84)$$

and the total pressure W_1 on the entire projection is,

$$W_1 = W\left(\frac{R - r}{y_2}\right)\int_{y=0}^{y=y_2}\int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}}(y_1 + y)\left[r + \frac{y}{y_2}(R - r)\right] dy\, d\theta, \qquad (85)$$

whence

$$W_1 = \frac{w}{6}\theta'(R - r)[3y_1(R + r) + y_2(2R + r)]. \qquad (86)$$

The moment m_1 of the vertical water pressure on the elementary area n about the center axis, and the total moment M_1 for the entire area are, respectively,

$$m_1 = w\left(\frac{R - r}{y_2}\right)(y_1 + y)\left[r + \frac{y}{y_2}(R - r)\right]^2 dy \cos\theta\, d\theta \qquad (87)$$

and

$$M_1 = w\left(\frac{R - r}{y_2}\right)\int_{y=0}^{y=y_2}\int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}}(y_1 + y)\left[r + \frac{y}{y_2}(R - r)\right]^2 dy \cos\theta\, d\theta, \qquad (88)$$

whence

$$M_1 = \frac{2w}{3}\sin\frac{\theta'}{2}(R - r)\left[y_1\left(\frac{R^3 - r^3}{R - r}\right) + \frac{1}{4}y_2(3R^2 + 2Rr + r^2)\right]. \qquad (89)$$

7-5. Horizontal Water Pressure.—The amount of the water pressure on the vertical projection of the upstream face of the segment shown in Fig. 14 and the moment thereof about the center axis may be found in a manner similar to that used in the vertical projection by obtaining the mathematical expression of the water pressure on a vertical elementary area of the height dy across the face of the segment and of the moment thereof with respect to the center axis, and then integrating between the limits of y_2 and 0. Proceeding in this manner, it readily follows that

$$W_2 = 2w\, y_2 \sin\frac{\theta'}{2}\left[\frac{y_1}{2}(R + r) + \frac{y_2}{3}\left(R + \frac{1}{2}r\right)\right] \tag{90}$$

and

$$M_2 = \frac{w}{6}y_2^2 \sin\frac{\theta'}{2}\left[2y_1(R + 2r) + y_2(R + r)\right]. \tag{91}$$

7-6. Uplift on Base of Segment.—The intensity of hydrostatic uplift on the base of a dam varies with the permeability of its foundation and this is a rather indeterminate characteristic of foundation material *in situ*. It is generally assumed, however, that (*a*) the intensity at the heel of the dam is equal to the intensity of the water pressure at that point multiplied by some specified factor; (*b*) the intensity at the toe is equal to the backwater pressure at this point, generally zero; and (*c*) the intensity varies in a straight line between the heel and toe. Taking W_3 as the entire uplift on the base of the dam, c as the percentage of uplift effective over the whole area, and M_3 as the moment of the uplift on the base of the dam about the center axis,

$$W_3 = \int_{p=R_0}^{p=R}\int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}} dp\ p\ d\theta\ w(y_1 - y_2)c\left(\frac{p - R_0}{R - R_0}\right), \tag{92}$$

$$W_3 = c\left[\frac{w(y_1 + y_2)\theta'}{(R - R_0)}\right]\left[\frac{1}{3}(R^3 - R_0^3) - \frac{R_0}{2}(R^2 - R_0^2)\right], \tag{93}$$

and

$$M_3 = \int_{p=R_0}^{p=R}\int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}} dp\ p\ d\theta\ w(y_1 + y_2)c\left(\frac{p - R_0}{R - R_0}\right)p\cos\theta, \tag{94}$$

whence

$$M_3 = 2w\, c\sin\frac{\theta'}{2}(y_1 + y_2)\left[\frac{1}{4}\left(\frac{R^4 - R_0^4}{R - R_0}\right) - \frac{R_0}{3}\left(\frac{R^3 - R_0^3}{R - R_0}\right)\right] \tag{95}$$

7-7. Vertical Stresses at Toe and Heel on Horizontal Sections.—By reference to Fig. 14 it is evident that the downward pressure on the base of the segment is the integral of the downward pressure on an elementary area $dp\, p\, d\theta$ due to the weight of the superimposed masonry in the segment and the vertical water pressure on its upstream face plus the total pressure on the top of the segment. If p_2 is the intensity of pressure at the toe, p_1 the intensity of pressure at the heel, W_4 the entire downward pressure on the base of the segment, and M_4 the moment thereof about the center axis, then

$$W_4 = \int_{p=R_0}^{p=R} \int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}} dp\, p\, d\theta$$

$$\left[p_2 - \frac{p \cos \theta - R_0 \cos \theta'/2}{R - R_0 \cos \theta'/2}(p_2 - p_1)\right], \quad (96)$$

whence

$$W_4 = p_2\theta'\left(\frac{R^2 - R_0^2}{2}\right) - (p_2 - p_1)$$

$$\left[\frac{2 \sin \frac{\theta'}{2}\left(\frac{R^3 - R_0^3}{3}\right) - \theta' R_0 \cos \frac{\theta'}{2}\left(\frac{R^2 - R_0^2}{2}\right)}{R - R_0 \cos \theta'/2}\right], \quad (97)$$

and

$$M_4 = \int_{p=R_0}^{p=R} \int_{\theta=-\frac{\theta'}{2}}^{\theta=\frac{\theta'}{2}} dp\, p\, d\theta\, p \cos \theta$$

$$\left[p_2 - \left(\frac{p \cos \theta - R_0 \cos \theta'/2}{R - R_0 \cos \theta'/2}\right)(p_2 - p_1)\right], \quad (98)$$

whence

$$M_4 = p_2 \frac{2}{3} \sin \frac{\theta'}{2} (R^3 - R_0^3) - (p_2 - p_1) \left[\frac{\frac{1}{8}(\theta' + \sin \theta')(R^4 - R_0^4) - \frac{1}{3} R_0 (\sin \theta')(R^3 - R_0^3)}{R - R_0 \cos \theta'/2} \right]. \tag{99}$$

The pressure intensities p_1 and p_2 are found from the combinations of Eqs. (97) and (99), after the total base pressure has been found from Eqs. (81) and (90), and the moments thereof from Eqs. (83), (89), and (91). Where hydrostatic uplift is assumed to act, Eqs. (93) and (95) must be used to determine modifications of stresses resulting from this action.

CHAPTER VIII

DESIGN OF NON-OVERFLOW SOLID-GRAVITY DAMS

INTRODUCTION

8-1. General Statement.—Non-overflow solid-gravity dams are very generally used where the flow over the top would be detrimental to the structure, or where the dam is very high, or where it is more economical to divert the flow of the stream on which the dam is constructed around the dam site, as is often the case. In this chapter there will be discussed the general features relating to the design of this type of dam. First there will be presented the applicability of the so-called theoretical section to dam design with a view to pointing out its compliance with the requirements for general stability, its limitations for the higher structures, and its modifications for the lower ones. General accessories of non-overflow solid gravity dams will also be discussed.

8-2. Requirements for Stability of a Dam.—It will be recalled that the general requirements for stability of a dam are:

1. That there shall be no tension in the masonry.
2. That the maximum pressure on any plane will not exceed a certain prescribed safe working stress.
3. That there will be no sliding on horizontal sections.
4. That the dam near the top shall be able to resist shocks from waves, floating bodies, and ice pressure.

It is generally assumed that the first of these conditions will be satisfied by designing the section of the dam so that the resultant pressure on all horizontal sections between two vertical planes, 1 ft. apart and perpendicular to the axis of the dam, falls within the middle third of the base of the section both for reservoir full and reservoir empty.

The second condition is met by spreading out, if necessary, the base beyond that required to fulfill the first condition. The third condition is almost always met by the requirements of the first and second conditions. The fourth condition is met by widening the top of the dam in excess of the requirements for resistance by water pressure sufficiently to take care of the forces tending to disrupt it.

8-3. Nomenclature.—In this discussion the principal symbols used will have the following significance:

b = base length of the section of the dam.
c = part of water pressure effective as uplift.
e = eccentricity of center of pressure on horizontal section.
h = the height of the dam subjected to water pressure.
H = the resultant horizontal water pressure on the upstream face.
n = the safe compressive stress on masonry or foundation.
R = the result of H and W.
$s = w' \div w$ = the specific gravity of the masonry.
W = the weight of the masonry of the dam above any horizontal section.
w' = the weight of 1 cu. ft. of masonry.
w = the weight of 1 cu. ft. of water.

DESIGN OF THE THEORETICAL SECTION

8-4. General Statement.—The theoretical cross-section of dams subjected to water pressure is defined as a right triangle with a vertical face upstream in which the base b at a depth h below the apex of the triangle is given by the expression

$$b = h \div \sqrt{s}.$$

This section applies with slight modifications to all relatively low, non-overflow dams. Its general applicability and limitations in the design of dams will now be investigated.

8-5. Width of Base Required for Safety against Overturning.—Where triangle ABC in Fig. 15 represents the theoretical section of a dam, let there first be found the base length that will make the section stable against overturning. This condition will be satisfied when the center

of pressure is at the downstream one-third point of the base. Let D be the center of gravity of the figure, then the horizontal water pressure H and the vertical weight W of the section of the dam both act through D. From the principles of center of water pressure and of gravity, $DE = \frac{1}{3}h$

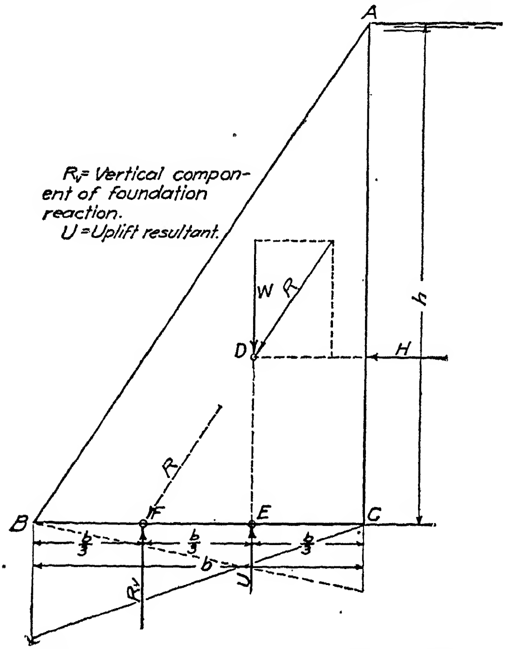


FIG. 15.—Theoretical section of a gravity dam.

and $CE = \frac{1}{3}b$. Assume F to be at the center of pressure on the base and at the downstream middle-third point. Taking moments about F,

$$\tfrac{1}{6}w'b^2h = \tfrac{1}{6}wh^3; \tag{100}$$

$$b^2 = \frac{wh^2}{w'} = \frac{h^2}{s}; \tag{101}$$

$$b = \frac{h}{\sqrt{s}}. \tag{102}$$

Equation (102) indicates that the theoretical section is stable against overturning when the base width is equal

to the height of the dam divided by the square root of the specific gravity of the masonry. This is true, however, only when there is no hydrostatic uplift.

Where hydrostatic uplift occurs there will be a force acting upward on the base of the dam. Let it be assumed that this force varies rectilinearly from cwh at the heel to 0 at the toe, then the resultant uplift will act vertically at E and will be equal to $\frac{1}{2}cwhb$. Again referring to Fig. 15 and taking moments about F as before,

$$\tfrac{1}{6}w'b^2h = \tfrac{1}{6}wh^3 + \tfrac{1}{6}cwhb^2; \tag{103}$$

$$(s - c)b^2 = h^2; \tag{104}$$

$$b = \frac{h}{\sqrt{s - c}}. \tag{105}$$

8-6. Limiting Height for Safety against Overstressing at Toe.—It is essential to find the limiting depth at which the base width of $h \div \sqrt{s}$ is no longer applicable in the theoretical section, without uplift, on account of excessive pressure at the toe of the dam. Again referring to Fig. 15 representing such a section, of height h, and base width $h \div \sqrt{s}$, D is taken as the center of gravity of the section and the point of application of both the vertical weight and the horizontal water pressure, and E and F are the one-third points of the base. Now, the vertical pressure at the toe in the theoretical profile as derived in Chap. VI and expressed by Eq. (54) is as follows:

$$p_2 = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{2W}{b}; \tag{106}$$

and the inclined pressure as expressed by Eq. (74) in the same chapter is

$$n = p_2 \csc^2 \theta = \frac{2W \csc^2 \theta}{b}; \tag{107}$$

whence

$$n = \frac{2 \cdot \frac{1}{2}w'hb(1 + \cot^2 \theta)}{b}; \tag{108}$$

or

$$n = w'h\left(1 + \frac{1}{s}\right) = swh\left(1 + \frac{1}{s}\right); \tag{109}$$

therefore,

$$h = \frac{n}{w(1 + s)}; \tag{110}$$

in which n is the maximum allowable working pressure in pounds per square foot, and h, w, and s have the significance as defined in paragraph 8-3. For a working pressure of 30 tons per square foot and a specific gravity of 2.4, $h = 280$ ft., which is the limiting height for the theoretical profile under the assumed conditions required to hold the pressure down to the required working pressure. Hydrostatic uplift has no effect on the foundation pressure of the dam, because whatever relief of pressure comes from the uplift is transmitted back to the foundation through the water reaction. Consequently the limiting height is not changed through the effect of hydrostatic uplift on the base pressure.

8-7. Friction Factor Required for Safety against Sliding. In regard to safety against sliding of a dam with the theoretical profile, Eq. (38), Chap. V, may be applied to determine this. By using S as the factor of safety against sliding Eq. (38) reads, $f = SH/W$, and hence

$$f = \frac{\frac{1}{2}wh^2S}{\frac{1}{2}w'bh} = \frac{S}{\sqrt{s}}, \tag{111}$$

If

$$s = 2.25,\ f = \frac{2S}{3};$$

whence

$$f > \tfrac{2}{3}.$$

If

$$s = 2.4,\ f > 0.64.$$

Inasmuch as the maximum friction factor allowed under normal conditions in the design of dams is 0.75, this corresponds to a factor of safety against sliding of less than 1.2. It is, therefore, obvious that the weight of the dam must be increased, or the sliding friction augmented by foundation preparation if a higher factor of safety is desired.

8-8. Top-width Requirements of Dams.—Owing to the triangular shape of the theoretical dam section, it would be too thin at the top to withstand the shocks from waves and floating bodies, ice pressure, and the possible requirements for traffic over the top. The top of the dam is, therefore, widened to take care of these conditions. For low, non-overflow dams of about 50 ft. height or less, the top widths are generally fixed at about 15 per cent of the height with a minimum width of 2 ft., unless conditions require a greater width; and for non-overflow dams of height over 50 ft., the top widths are generally fixed at a value about equal to the square root of the height up to a maximum of about 16 to 20 ft. This corresponds very well to the maximum limit in practice for higher dams, and fits very well with the roadway requirements over the top of such structures. In the case of a roadway over the top of the dam, the top width is increased, although it becomes greater than the specified 15 per cent or the square root of the height of the structure.

8-9. Methods of Securing Top Width.—The top width of a dam is usually secured by joining either a right triangle or an oblique triangle to the downstream face of the theoretical section at the top. If an oblique triangle is used its downstream face is battered about 1 in 10. Either of the triangles used may be joined to the downstream batter of the theoretical section with a curve tangent to the downstream slopes of the theoretical section and the triangle.

8-10. Effect of Top Width on Theoretical Section of Dam.—The placing of a triangle, as described above, on the top of the dam will change the directions of the lines of pressure in the theoretical section both for reservoir

full and reservoir empty. The influence of this block of masonry is to move the center of gravity of the theoretical section downstream above a certain elevation so near the top of the dam as to be negligible, and upstream for all horizontal sections below another elevation which is farther down from the top.

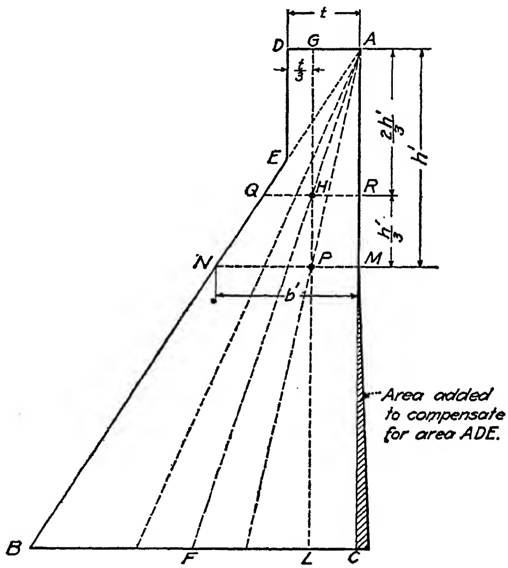


FIG. 16.—Section showing effect of top width of a gravity dam.

In order to determine the elevation at which the change in the center of gravity takes place, let reference be had to Fig. 16, which represents a theoretical section of a dam with a right triangular block used to provide the required top width. In this figure the addition of the triangle ADE does not disturb the stability of the structure as a whole so far as resistance to downstream forces is concerned and, therefore, does not involve any further changes in the downstream face. The upstream face, however, requires certain changes to compensate for the addition of this triangle. Let consideration be given to this feature of the problem. Triangle ADE is similar to triangle ABC; AF is the median of the triangle ABC, whence its center

of gravity lies on the line AF; GL passes vertically through the center of gravity of the triangle ADE and intersects the line AF at H. The line MN is located so that PH equals one-third of PG. Now it is clear that the addition of the triangle ADE to the theoretical section throws the center of gravity of the entire figure downstream for all elevations above MN; and upstream, and hence outside of the middle third of all horizontal sections for all elevations below MN. The center of gravity of the triangle ADE is two-thirds distant from A to D and the center of gravity of triangle AMN is one-third distant from M to N and the triangle AMN and APQ are similar, whence

$$b' \div 2 \cdot \tfrac{2}{3}t = h' \div 2h',$$

or $b' = 2t$. But from the definition of the theoretical triangle

$$b' = \frac{h'}{\sqrt{s}},$$

therefore,

$$h' = 2t\sqrt{s}. \tag{112}$$

Since the moving of the center of gravity of the dam upstream in the theoretical section causes the center of pressure to pass outside of the middle-third for reservoir empty for all heights of the dam greater than $2t\sqrt{s}$, sufficient batter must be given to the upstream face to correct this difficulty. A similar procedure for the oblique triangle additions may be used to find the limiting depth below crest level where upstream batter must be provided. The amount of batter is small in either case and is found by trial.

8-11. Effect of Vertical Water Pressure on Stability against Overturning.—The theoretical section is based on no vertical water pressure on the upstream face of the dam, and the consequent adoption of a vertical upstream face for this section. The effect of upstream batter and vertical water pressure on the upstream face will now be investi-

gated as to overturning, crushing at the toe, tension in the masonry, and sliding.

Assume a triangular section with base width, $b = h \div \sqrt{s}$; height, h; and batter, $x \div h$, as shown in Fig. 17. The problem is to find the value of x that will give the maximum overturning resistance about the toe of the dam. The

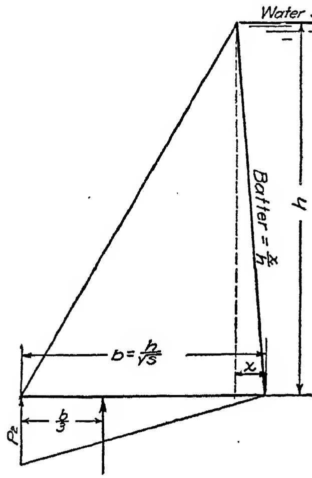


FIG. 17.—Section showing effect of upstream batter on a gravity dam.

moment M about the toe, considering vertical water pressure, is,

$$M = \tfrac{1}{6}h(2w'b^2 - w'bx + 3wbx - wx^2); \qquad (113)$$

$$\frac{dM}{dx} = \frac{1}{6}h(-w'b + 3wb - 2wx) = 0. \qquad (114)$$

Solving for x and noting that for a small upstream batter, $b = h/\sqrt{s}$, nearly,

$$x = \frac{(3w - w')b}{2w'} = \frac{h}{2}\left(\frac{3}{\sqrt{s}} - \sqrt{s}\right); \qquad (115)$$

$$\frac{d^2M}{dx^2} = -\frac{wh}{3}, \qquad (116)$$

which being negative, M is a maximum for the value of x given in Eq. (115). If the specific gravity of the masonry

is 2.4, x equals 0.3 b or 0.19 h. This shows that there is an advantage in a small upstream batter so far as overturning resistance is concerned.

8-12. Effect of Vertical Water Pressure on Pressure at Toe and Sliding.—In regard to the effect of upstream batter on toe pressure, again refer to Fig. 17, and note from Eq. (54), Chap. VI, that

$$p_2 = \frac{W}{b}\left(1 + \frac{6e}{b}\right), \tag{117}$$

whence by substituting the values of W, e, and d, and assuming $s = 2.3$ for convenience,

$$p_2 = \frac{wh}{b^2}(x^2 + 1.3\ bx + h^2), \tag{118}$$

$$\frac{dp_2}{dx} = \frac{wh}{b^2}(2x + 1.3\ b) = 0, \tag{119}$$

$$x = -0.65b, \tag{120}$$

$$\frac{d^2p_2}{dx^2} = \frac{2wh}{b^2}, \tag{121}$$

which is positive and indicates that p_2 is a minimum at $x = -0.65\ b$. This corresponds to an inclination upstream for a minimum pressure at the toe. Such an inclination is, however, not feasible as it would throw the center of pressure outside of the middle-third of the base length for reservoir empty, causing tension in the masonry at the toe.

Since the sliding factor is increased by increasing the load on the dam, it is improved by giving the upstream face a larger batter. In fact, the best condition for sliding alone would be produced by turning the theoretical section around with the right angle at the heel of the dam, which, of course, would be objectionable for other considerations.

8-13. Summary of Theoretical Requirements for Practical Design.—Taking the resistance to overturning, crushing at the toe, and sliding all into consideration at the same time, it is evident that variance of interest is involved, as the first requires positive upstream face batter of a small

degree, the second requires a negative upstream batter, and the third requires a large upstream batter. The theoretical section with vertical upstream face is thus probably the most satisfactory section except that the top must be given a proper width to resist acting forces, and that the base must be spread to meet the middle-third and working-stress requirements. It is also desirable to determine whether the dam shall be straight or curved in plan. If the average canyon width is comparatively large, it is better to select the straight-gravity type of structure; but where the canyon walls are fully capable of taking arch thrust and where the foundation contours make it cheaper, it is better practice to use the curved plan, in order to secure the benefit of arch action in case of incipient failure. Also a certain amount of arch resistance can be secured prior to such incipient failure if the dam is constructed so as to have all of its longitudinal shrinkage taken up with grouting between the construction sections.

It is customary to place parapet walls on the upstream and downstream edges of the crest of the non-overflow dams, and where flow over the top of the structure would be injurious from any cause the parapet walls are made solid so as to prevent such flow over the structure. Where, however, such overflow is not dangerous, an open parapet construction may be used for ornamental purposes.

MISCELLANEOUS FEATURES IN DESIGN OF GRAVITY DAMS

8-14. Freeboard Requirements.—It is customary in the design of a dam to have a certain amount of superelevation or freeboard over and above the estimated maximum water level to provide for wave action due to wind or earthquakes, ice jams, and unforeseen floods. As a general average the freeboard may be taken at about 5 per cent of the height of the dam for dams up to 150 ft. and then to remain constant for dams of greater height. The height required to take care of waves caused by wind will vary with the length of the reservoir and consequent

sweep of the wind. The Stephenson formula, reading as follows, is generally used for computing wave heights in reservoirs:

$$H = 1.5\sqrt{L} + (2.5 - \sqrt[4]{L}), \tag{122}$$

in which H is the maximum height of wave in feet and L is the length of reservoir in miles. A minimum height of 2 ft. is generally used regardless of the length of the reservoir. The provisions for wind waves will be more than ample for protection against waves caused by earthquakes.

8-15. Cutoff Walls and Foundation Grouting.—As already stated, the downstream batter of the dam is based, in a certain measure, on the amount of uplift assumed to occur from water pressure on the base of the dam, and provisions are usually made to prevent, as far as possible, the entrance of water under the dam to avoid such pressure. In the case of pervious earth foundations, it is customary to provide a cutoff trench near the heel of the dam and to drive sheet piling in the bottom of it for the purpose of either preventing seepage as much as possible or increasing the length of the path of percolation under the dam. The top of the sheet-piling line is usually incorporated in the concrete body of the dam. In dams built on solid foundations that are assumed to be more or less impervious, a cutoff trench under the dam near the heel is excavated to a suitable depth varying from 5 to 20 ft., or more, deeper than the general foundation excavation. This trench is then filled with a high grade of concrete to form an impervious curtain. In the most modern practice this cutoff curtain is usually supplemented with grouting, grouting holes being drilled in the bottom of the cutoff trench and the openings being carried up through the concrete in steel pipes to a sufficiently high level to permit grouting after the concrete in the lower part of the foundation is poured. The pressure used in this grouting should be the maximum that can be applied without disturbing or lifting any parts of the concrete already placed or disturbing the natural stratification of the foundation mate-

rials. Such pressure usually varies from 50 to 200 lb. or more, per square inch.

8-16. Contraction Joints and Water Stops.—Concrete, after being poured in the body of the dam, contracts in setting and cooling. The temperature and volume change may be determined as outlined in paragraph 17-7. A dam constructed as a monolith from one end to the other will, therefore, necessarily crack into sections of varying length. These cracks usually are irregular and unsightly and also provide means for water to seep through the structure. In order to avoid these cracks, it has become the custom to construct dams in sections of from 25 to 50 ft., thus providing natural construction joints. These joints should, if possible, be placed so as to meet the foundation at any sharp changes in the slopes of the transverse profile. The various sections may be constructed a considerable elevation in advance of the adjacent ones, or they may be carried up in advance in time only sufficient to permit of the setting of the concrete before pouring the adjacent blocks. In order to avoid cracking, it is believed that such contraction joints should not be placed farther than about 50 ft. apart. In order to make the passage of water more tortuous through the contraction joints, and in order to provide shearing resistance between adjacent sections, it is desirable to recess one section and to provide a key on the adjacent one. These recesses should vary in width from 5 ft. in the smaller dams to 10 to 20 ft. in the larger ones and should have a depth of about 1 ft. An illustration of typical contraction joints is shown in Fig. 25. To add to the difficulty of water percolating through the contraction joints, copper or asphaltic water stops are placed in them near and parallel to the upstream face of the dam at a distance of 2 or 3 ft. from the face. These water stops are placed in the structure in such a manner as to prevent water from passing around their ends and as to provide flexibility in expansion and contraction without rupturing the metal or disturbing its bond with the concrete. Such a water stop is shown in Fig. 26.

8-17. Grouting of Contraction Joints.—In order to take up the vacant space between the faces of the adjoining blocks at the contraction joints, a grouting system is usually provided in the best designs, so that grout can be forced into these contraction joints under high pressure. The pressure used should be about 100 lb. per square inch. Care should be taken to prevent the grout from getting under the dam near its ends where the weight is light, as there is a possibility of lifting this part of the dam off its foundation. Provisions should also be made to prevent the grout from entering the drainage wells or drainage galleries within the dam. This may be done by placing a metal strip between the general grouting area of the contraction joints and the tunnels or drainage wells. A metal strip may also be placed in the contraction joints near the downstream face to prevent the grout from escaping at the time of grouting.

8-18. Drainage and Inspection Galleries.—Every large, well-designed gravity dam is now provided with inspection and drainage galleries. These galleries usually run parallel to and at a distance of from 10 to 30 ft. from the upstream face of the dam and extend up the slopes along the abutments within the dam to elevations within 25 to 50 ft. of the top of the dam. A drainage gutter is placed in the drainage and inspection gallery to collect the water entering it and to carry it out of the structure. An entrance tunnel is placed in the dam joining the drainage and inspection gallery with the lower face of the dam at the elevation of the lowest point in the gallery. Drainage walls are connected with tile placed in the cutoff trench and brought up to the drainage gallery to relieve hydrostatic pressure at this point. Drainage wells are also carried from the gallery upward to near the top of the dam. These wells are usually placed about 10 ft. from the upstream face of the dam and brought into the gallery at the level of the drainage gutter. The drainage wells from the bottom or cutoff trench should be connected in pairs so that water can be forced in one and out the other for cleansing purposes and for keeping grout out of them

during the grouting of the dam. This provision requires that the tile in the cutoff trench shall be discontinuous between the pairs of wells. Entrances to the drainage gallery are usually provided at the upper elevations on the sides of the canyon. A plan of a drainage and inspection gallery and drainage wells is shown in Fig. 24.

8-19. Sluiceways and Penstocks through Dam.—Every dam of consequence should, if practicable, have sluiceways provided through it for draining the reservoir for various purposes. Proper controls at the upper and lower ends of such sluiceways should be provided. These controls should be of some of the types shown and discussed in Chap. XXI. The capacity of the sluiceways is determined on the basis of the quantity of water that must be discharged within a given time.

Penstocks are usually either brought through the dam or through tunnels in one or the other of the canyon walls where power-plant developments are made. These penstocks are dimensioned according to the needs of the power plant. They should be protected at their upstream ends with sufficient grillage to prevent the entrance of débris, and provided with proper gate controls at the upper end and at the downstream end adjacent to the turbines. The character of trash racks and gates used for this purpose is discussed in Chap. XXI.

8-20. Effect of Holes in Dams on Stresses.—Holes through a dam always cause disturbance in the paths and magnitudes of stresses in the structure. This disturbance is usually of greatest importance at the faces of the dam. The holes are of two general types, circular holes for sluiceways and oblong ones for inspection galleries. A method for finding the stresses around these two kinds of holes therefore should be of interest and value. The equations developed will be applicable both to gravity and arch dams.

Let consideration first be given to a circular sluiceway opening in a gravity dam as shown in Figs. 18*a* and 18*b*. Figure 18*a* represents a side view of the opening and Fig. 18*b*

its vertical cross-section. Let r_1 be taken as the radius of a circular section and p the computed mass stress in pounds per square foot at the center of the hole, predicated on the absence of the hole. The mass stress will be a maximum near the upstream face of the dam for reservoir empty. Let s_r represent the stress at M along the radius vector r,

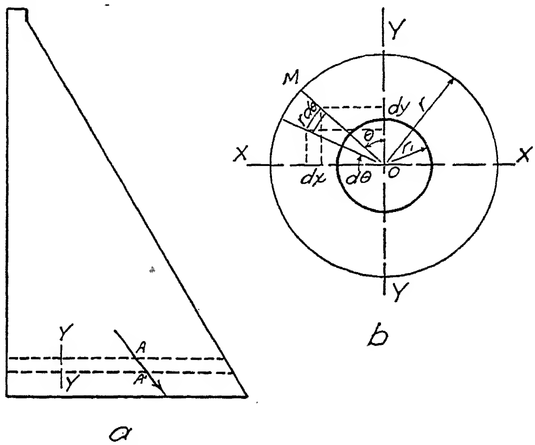


FIG. 18.—Diagrams for stresses about holes in dams.

with its pole at O and initial line YY; s_θ, the stress at M at right angles to the radius vector r; and s_s, the shearing stress at M. Under these assumptions it can be shown that

$$s_r = \frac{p}{2}\left[1 - \frac{r_1^2}{r^2} + \left(1 - \frac{4r_1^2}{r^2} + \frac{3r_1^4}{r^4}\right)\cos 2\theta\right]; \quad (123)$$

$$s_\theta = \frac{p}{2}\left[1 + \frac{r_1^2}{r^2} - \left(1 + \frac{3r_1^4}{r^4}\right)\cos 2\theta\right]; \quad (124)$$

$$s_s = \frac{p}{2}\left[-1 - \frac{2r_1^2}{r^2} + \frac{3r_1^4}{r^4}\right]\sin 2\theta. \quad (125)$$

The derivation of these fundamental equations will be found in Appendix II.

To find the mass stresses at a great distance from the edge of the hole, assume r to be very large compared with r_1 so the quotients of powers of the former divided by powers of the latter can be neglected in Eqs. (123), (124), and (125); these equations can then be reduced to the following forms:

$$s_r = p \cos^2 \theta, \tag{126}$$
$$s_\theta = p \sin^2 \theta, \tag{127}$$
$$s_s = p \sin \theta \cos \theta. \tag{128}$$

From these equations it will be observed that the radial, tangential, and shearing stresses in no other locations are greater than the stress p and are, therefore, unimportant in connection with this investigation.

To find the stresses at the edge of the hole, let $r = r_1$ in Eqs. (123), (124), and (125), in which case they will reduce to

$$s_r = 0, \tag{129}$$
$$s_\theta = p(3 - 4 \cos^2 \theta), \tag{130}$$
$$s_s = 0. \tag{131}$$

In Eq. (130), $s_\theta = -p$ for $\theta = 0°$ and $3p$ for $\theta = 90°$. s_θ must, therefore, change signs at some point between $\theta = 0°$ and $\theta = 90°$, and at that point it must become zero; *i.e.*, $p(3 - 4 \cos^2 \theta) = 0$ and, therefore, $\cos \theta = \frac{1}{2}\sqrt{3}$ and $\theta = 30°$. This shows that there is tension for 30 deg. on each side of the top and bottom points of the opening.

It is evident that the foregoing equations for stress around holes will apply in an arch dam, if p is used as the appropriate computed compressive stress in the arch ring.

The solution for combined mass and water load in a gravity dam requires somewhat different treatment. The stress p' for this case should be taken as the effect of the resultant force at and through the center of the section of the opening in the plane under consideration. Owing to the inclination of the resultant of the two forces producing this stress, the section of the circular opening will be an ellipse with its major axis in a vertical plane and parallel to the resultant of the acting forces. Its minor axis will be in a horizontal plane encompassing the center line of the sluiceway. The tangential stress at the ends of the vertical diameter will be given by the equation

$$s_\theta = -p', \tag{132}$$

and that at the ends of the horizontal diameter will be

expressed by

$$s_\theta = p'\left(1 + \frac{2b}{a}\right), \tag{133}$$

where b is the semiminor axis and a the semimajor axis of the ellipse. If $b = a$, the ellipse merges into a circle, and

$$s_\theta = 3p', \tag{134}$$

as it should.

A further discussion of stresses in elliptical holes in dams will be found in Appendix II.

Inspection galleries in dams are usually of oblong shape, consisting of a semicircle on top of a rectangle or of a circular segment on top of a rectangle. Where the full semicircle is used, the stresses for this portion of the gallery can be determined from Eqs. (123) to (131), inclusive. The stresses in the rectangular section will be essentially the same as those at the ends of the horizontal diameter of the semicircular section until points near the bottom are reached. Here difficulty is introduced by the too common use of flat-bottomed galleries. The angular corners should, at least, be obviated by rounding them with fairly liberal radii, or better still, by using a 60-deg. circular or an elliptical segment for the gallery bottom.

8-21. Reinforcing Steel Requirements around Holes in Dams.—Of the stresses most dangerous to the stability of the dam are those caused by the maximum stress in it. This may be taken as the vertical stress at the downstream face of a gravity dam and the horizontal stress at the upstream face at the crown or at the downstream face at the abutments of an arch dam. These stresses are represented by Eq. (134), showing compression s_θ to be $3p'$, and by Eq. (132), showing s_θ to be equal to $-p'$ at the side and top, respectively, of the hole for a gravity dam, and similarly at the top and the side, respectively, of the hole for an arch dam. Generally no trouble will result from the effects of the compressive stress $3p'$ in good concrete, because the usual maximum working stress for compression

will, for small areas, readily stand three times that stress, but the result from the effects of the tensile stress $-p'$ may cause cracking parallel to the axis of the hole. Consequently, reinforcing steel should be provided to prevent this trouble. From Eq. (124) the distance to which the tension will extend may be found by placing its right number equal to 0 and also letting $\theta = 0$, whence $r = r_1\sqrt{3}$. The tensile stress then extends from $r = r_1$ to $r = r_1\sqrt{3}$. The total tension may be found, therefore, by integrating the value of s_θ over this range. If F is taken as the total tension, then, from Eq. (124),

$$F = \int_{r_1}^{r_1\sqrt{3}} s_\theta dr = -\int_{r_1}^{r_1\sqrt{3}} \frac{p'}{2}\left(\frac{3r_1^4}{r^4} - \frac{r_1^2}{r^2}\right)dr = \frac{-p'r_1}{3\sqrt{3}}. \quad (135)$$

The distance of the point of application of the resultant tensile force F from the center of the hole may be found by the principle of the center of mass, *i.e.*,

$$r = \int r\,dm \div \int dm,$$

or

$$\bar{r} = \frac{\int_{r_1}^{r_1\sqrt{3}} -\frac{p'}{2}\left(\frac{3r_1^4}{r^4} - \frac{r_1^2}{r^2}\right)r dr}{\int_{r_1}^{r_1\sqrt{3}} -\frac{p'}{2}\left(\frac{3r_1^4}{r^4} - \frac{r_1^2}{r^2}\right)dr} = \frac{3\sqrt{3}}{4}(2 - \log_e 3)r_1. \quad (136)$$

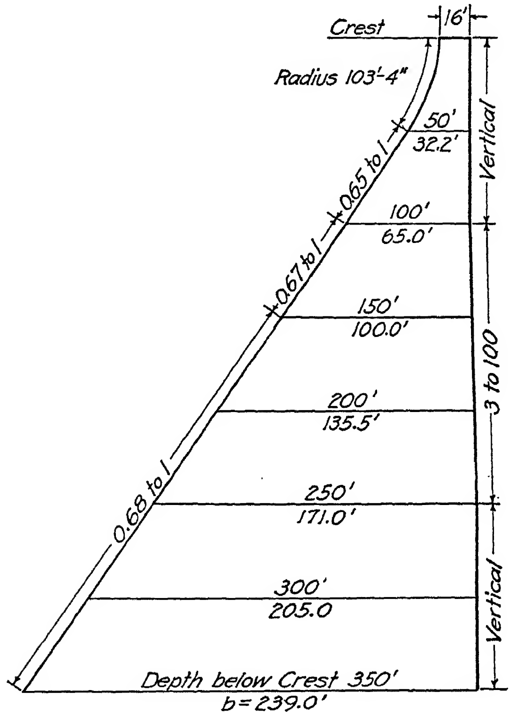
From Eqs. (135) and (136) the required amount of reinforcing steel can be determined.

8-22. Spillways.—An important factor in connection with any dam is adequate spillway capacity. It is customary to make the spillway capacity from 50 to 100 per cent in excess of the known or estimated maximum discharge to be provided for. The design of the spillway is usually merely a problem of the design of an overflow dam, which subject is treated in a later chapter.

ILLUSTRATION OF COMPUTING AND RECORDING STRESSES FOR A NON-OVERFLOW SOLID-GRAVITY DAM

8-23. General Statement.—It is a valuable aid in the design of a dam to follow systematized methods of com-

puting and recording stress analyses. In order to set forth these principles in a logical manner, a section of a high dam will be assumed and analyzed, and computations



HORIZONTAL PLANE-FEET BELOW CREST	DOWNSTREAM FACE COMPRESSION VERTICAL	DOWNSTREAM FACE COMPRESSION MAXIMUM	DOWNSTREAM FACE SHEAR MAX.	UPSTRM COMPRESSION-VERT.	ALLOWABLE UPLIFT	SLIDING FACTOR, 25% UPLIFT
50	57	81	40	66	70.9%	.615
100	99	141	71	117	29.1	.650
150	145	210	105	160	25.9	.673
200	189	276	138	206	25.8	.676
250	234	342	171	253	26.6	.674
300	280	409	205	307	26.6	.674
350	329	481	241	360	26.3	.676

FIG. 19.—Section and stress table for a gravity dam.

and recordations both on the parallel-plane theory and the radial-plane theory of stress determination will be made.

The methods of analysis and computation are especially developed to accommodate the use of the modern calculating machine which can quickly and easily carry quantities to nine or more figures. It is suggested that in the beginning of the study of a dam, a form sheet similar to Fig. 20 or 21 be prepared to a larger scale and on tracing cloth. From this tracing, black-line or blue-line prints may be obtained on transparent paper, from which blue prints can be made if desired after the computations have been tabulated thereon.

8-24. Method of Analysis by the Parallel-plane Theory. Figure 20 presents the full computations for the stress analysis on the basis of the parallel-plane theory of the dam whose maximum cross-section is shown in Fig. 19. It will be noted that the dam is 350 ft. high, approximating a triangular section except for slight changes in batter of both the upstream and downstream faces, and the addition of a curved crest to provide a road over the top of the dam. The base width is 239 ft., this being $0.693h$. The maximum water surface is assumed to be at the top of the dam.

For the purposes of computations the dam has been divided by horizontal planes into eight segments, the two top segments being 25 ft. high and the remaining six segments being 50 ft. high. The length along its axis of the part of the dam under consideration is 1 ft. Each segment is assumed to act independently of any of those below it. For simplicity in stating the equations let $y_2 \tan \phi = n$, in Eqs. (40) to (45) of Chap. VI, representing the increase in thickness due to the downstream batter, and $y_2 \tan \phi_1 = m$, the increase in thickness due to the upstream batter. Using these simplifications of notation for this special case, Eqs. (40) to (49) may be restated here for convenience as follows:

$$W_1 = \frac{w' y_2}{2}(T + t); \tag{137}$$

$$M_1 = \frac{w'y_2}{2}\left(t^2 + 2nt + mt + mn + \frac{2}{3}n^2 + \frac{1}{3}m^2\right); \quad (138)$$

$$H_1' = wy_2\left(y + \frac{y_2}{2}\right); \quad (139)$$

$$M_2' = \frac{wy_2^2}{2}\left(y_1 + \frac{y_2}{3}\right); \quad (140)$$

$$W_2' = wm\left(y_1 + \frac{y_2}{2}\right); \quad (141)$$

$$M_2'' = wm\left[y_1\left(T - \frac{m}{2}\right) + \frac{y_2}{2}\left(T - \frac{m}{3}\right)\right]; \quad (142)$$

$$W_3' = \frac{cwT}{2}(y_1 + y_2); \text{ (there being no backwater pressure)} \quad (143)$$

$$M_3' = \frac{cwT^2}{3}(y_1 + y_2). \quad (144)$$

Reference is made to Fig. 20. After tabulating the dimensions of the eight segments in Cols. 1 to 7, there are recorded on the computation sheet in Cols. 8 to 28 the derived properties of the segments which are required for the solutions of Eqs. 137 to (144) above. The values in the ensuing columns are then obtained as follows:

Column 29 is the product of Cols. 8 and 16.

Column 30 is the cumulative totals of Col. 29

Column 31 is the product of Cols. 15 and 16.

Column 32 is the sum of Col. 31 plus the value for the next above segment in Col. 32 plus the product of n and the weight of all segments above as given in Col. 30 for next higher segment. Thus, for segment depth 200 to 250, ft. Col. 32 is computed as follows:

From Col. 31	107,250,000
From Col. 32 (seg. 150 to 200 ft.)	182,589,062
2,034,375 (from Col. 30) × 34	69,168,750
Result for Col. 32	359,007,812

Segment Depth	n	m	y_2	y_1	t	T	T+t	t^2	2nt	mt	nm	$\frac{2}{3}n^2$	$\frac{1}{3}m^2$	Sum of Cols.(9) to (14) incl.	$w \cdot \frac{y_2}{2}$	$62.5 y_2$	$y_1+\frac{y_2}{2}$	$62.5 m$	$31.25 y_2^2$	$y_1+\frac{y_2}{3}$	$y_1[T-\frac{1}{2}m]$	$\frac{y_2}{2}[T-\frac{1}{3}m]$	Sum of Terms (7) and (8)	T^2	$\frac{T^2}{6}$	$31.25(y_1+y_2)$
0-25'	4.25	0	25	0	16.0	20.25	36.25	256	136	0	0	12.0416	0	404.042	1875	1562.5	12.5	0	19531.25	8.333	0	253.125	253.125	410.0625	68.34375	781.25
25-50	12.25	0	25	25	20.25	32.5	52.75	410.06	496.12	0	0	100.0416	0	1006.227	1875	1562.5	37.5	0	19531.25	33.333	812.5	406.25	1218.75	1056.25	176.0416	1562.5
50-100	32.5	0	50	50	32.5	65.0	97.5	1056.25	2112.5	0	0	704.166	0	3872.917	3750	3125	75	0	78,125	66.666	3250.0	1625.0	4875.0	4225	704.166	3125.0
100-150	33.5	1.5	50	100	65.0	100.0	165.0	4225.0	4355.0	97.5	50.25	748.166	0.75	9476.66	3750	3125	125	93.75	78,125	116.6	9925	2487.5	12,412.5	10,000	1666.6	4687.5
150-200	34.0	1.5	50	150	100.0	135.5	235.5	10,000	6800	150.0	51.0	770.66	0.75	17,772.417	3750	3125	175	93.75	78,125	116.6	20,212.5	3375.0	23,587.5	18,360.25	3060.416	6250.0
200-250	34.0	1.5	50	200	135.5	171.0	306.5	18,360.2	9214.0	203.25	51.0	770.66	0.75	28,600.0	3750	3125	225	93.75	78,125	216.6	34,050	4262.5	38,312.5	29,241	4873.5	7812.5
250-300	34.0	0	50	250	171.0	205.0	376.0	29,241	11,628	0	0	770.66	0	41,639.66	3750	3125	275	0	78,125	266.6	—	—	—	42,025	7004.16	9375.
300-350	34.0	0	50	300	205.0	239.0	444.0	42,025	13,940	0	0	770.66	0	56,735.66	3750	3125	325	0	78,125	316.6	—	—	—	57,121	9520.16	10,937.5
Col.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)

Segment Depth	$20.83(y_1+y_2)$	Weight of Segment	Weight of Segment Plus all those above	Moment of Weight about Toe of its Zone	Total Moment of this Segment+all above about Toe of Lowest Zone	Total Base Pressure in Terms of P_2 and P_1 (To Base of Segment)	Total Net Moment about Toe of Segment in Terms of P_2 and P_1	Reservoir Empty P_2 Lbs Per Sq.Ft.	P_2 Lbs. Sq.In.	P_1 Lbs Per Sq.Ft.	P_1 Lbs. Sq.In.	Water Press. on Upstr'm Batter of Segment	Total Water Pressure on Upstr. Batter to Crest	Moment of Vert. Water Press on Upstr. Batter Abt. Zone Toe	Moments of Vert. Batter Press's to Crest Abt. Toe of Lowest Zone	Total Projected Horizontal Water Pressures to Crest
0-25'	520.83	67,968.75	67,968.75	757,578	757,578	$10.125P_2+10.125P_1$	$68.34375P_2+136.6875P_1$	2341	16	4372	30	0	0	0	0	19,537.25
25-50	1041.666	98,906.25	166,875.0	1,886,676	3,476,871	$16.25P_2+16.25P_1$	$176.04167P_2+352.083P_1$	788	5	9481	66	0	0	0	0	78,125.0
50-100	2083.333	365,625	532,500	14,523,439	23,423,748	$32.5P_2+32.5P_1$	$704.166P_2+1408.333P_1$	-495	-3	16,880	117	0	0	0	0	312,500
100-150	3125.0	618,750	1,151,250	35,537,500	76,799,998	$50P_2+50P_1$	$1666.6P_2+3333.3P_1$	-30	0	23,055	160	11,718.75	11,718.75	1,163,672	1,163,672	703,125
150-200	4166.6	883,125	2,034,375	66,646,564	182,589,062	$67.75P_2+67.75P_1$	$3060.416P_2+6120.83P_1$	394	3	29,634	206	16,406.25	28,125.0	2,211,328	3,773,438	1,250,000
200-250	5208.3	1,149,375	3,183,750	107,250,000	359,007,812	$85.5P_2+85.5P_1$	$4873.5P_2+9747P_1$	808	6	36,428	253	21,093.75	49,218.75	3,591,797	8,321,485	1,953,125
250-300	6250.0	1,410,000	4,593,750	156,148,751	623,404,063	$102.5P_2+102.5P_1$	$7004.16P_2+14,008.33P_1$	629	4	44,188	307	0	49,218.75	0	9,994,923	2,812,500
300-350	7291.6	1,665,000	6,258,750	212,758,751	992,350,314	$119.5P_2+119.5P_1$	$9,520.16P_2+19,040.33P_1$	512	4	51,862	360	0	49,218.75	0	11,668,360	3,828,125
Col.	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)

Segment Depth	To. Moments of Horiz. Water Press. to Crest about Bottom of Segment	Slid'g factor 25% Uplift $\frac{\Sigma P}{\Sigma W-\text{Uplift}}$	Total Uplift on Base in Terms of C	Moment of Uplift Abt. Toe of Zone in Terms of C	Base Pressure Net-Reservoir Full Excluding Uplift	Net Moment of Base Pressure Abt. Toe of Lowest Zone Reservoir Full, no Uplift	Reservoir Full – No Uplift P_2 Lbs Per Sq.Ft.	P_2 Lbs. Sq.In.	P_1 Lbs Per Sq.Ft.	P_1 Lbs. Sq.In.	P_2 for $P_1=0$. Res. full with Uplift Lbs Per Sq.Ft.	Lbs. Sq.In.	Uplift Ratio C for $P_1=0$	Maximum Values Reservoir Full P_2 Lbs Per Sq.In.	$\csc^2\theta$	Cor. Stress Lbs/Sq.In.	Reservoir Empty P_1 Lbs. Per Sq.In.	$\csc^2\theta$	Cor. Stress Lbs/Sq.In.	Uplift at 25% Sliding Factor	Net Weight	Uplift
0-25'	162,760.4	0.305	15,820.3 c	213,574.2 c	67,968.75	594,818	4723	33	1990	14	4723	33	1.274	33			30			.305	64,014	3955
25-50	1,302,083	0.507	50,781.2 c	1,100,260 c	166,875	2,174,788	8185	57	2085	14	8185	57	0.667	57	1.4225	81	66	1	66	.615	154,180	12,695
50-100	10,416,667	0.649	203,125 c	8,802,083 c	532,500	13,007,081	14,298	99	2087	14	14,298	99	0.334	99	1.4225	141	117	1	117	.650	481,719	50,781
100-150	35,156,251	0.672	468,750 c	31,250,000 c	1,162,969	42,807,419	20,834	145	2425	17	20,834	145	0.259	145	1.4489	210	160	1	160	.673	1,045,781	117,188
150-200	83,333,333	0.675	846,875 c	76,501,042 c	2,062,500	103,029,167	27,221	189	3222	22	27,221	189	0.258	189	1.4624	276	206	1	206	.676	1,850,181	211,719
200-250	162,760,402	0.674	1,335,938 c	152,296,874 c	3,232,968	204,568,895	33,649	234	4163	29	33,649	234	0.266	234	"	342	253	1	253	.674	2,898,983	333,985
250-300	281,250,000	0.676	1,921,875 c	262,656,250 c	4,642,969	352,148,986	40,317	280	4980	35	40,317	280	0.266	280	"	409	307	1	307	.674	4,162,500	480,469
300-350	446,614,560	0.677	2,614,062 c	416,507,294 c	6,307,969	557,404,114	47,299	329	5487	38	47,299	329	0.263	329	"	481	360	1	360	.676	5,654,453	653,516
Col.	(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)	(61)	(62)	(63)	(64)	(65)

FIG. 20.—Computations for solid non-overflow gravity dam based on parallel-sided slice, 1 ft. thick.

Column 33 is derived from equation

$$W = \left(\frac{p_2 + p_1}{2}\right)T,$$

the coefficients of p_2 and p_1 being $T/2$.

Column 34 expresses the total moment of forces above the bottom of the segment about the toe of the segment in terms of p_2 and p_1. In other words, it is the moment about the toe of the base of the trapezoid of pressures upon the base, which is expressed by the equation

$$M = \frac{T^2}{6}(p_2 + 2p_1).$$

Thus the coefficient of p_2 is the value in Col. 26, and that of p_1 is twice this value.

Column 35 gives the value of p_2, the pressure at the toe of the segment, for the condition of reservoir empty, being the result in solving simultaneously the equations expressed in Cols. 33 and 34. This method of determining the values of p_1 and p_2 involves certain simplifications as indicated by the following example of solving for the values for segment 200 to 250 ft.:

(*a*)	From Cols. 33 and 30,	$85.5\, p_2 + 85.5\, p_1$	$= 3{,}183{,}750;$
(*b*)	From Cols. 34 and 32,	$4{,}873.5\, p_2 + 9{,}747\, p_1$	$= 359{,}007{,}812;$
(*c*)	$(a) \times \frac{T}{3}$,	$4{,}873.5\, p_2 + 4{,}873.5\, p_1$	$= 181{,}473{,}750;$
(*d*)	$(b) - (c)$,	$4{,}873.5\, p_1$	$= 177{,}534{,}062;$
		p_1	$= 36{,}428.$

Hence, the coefficient of p_1 in Eq. (*d*) for any segment is obtained by subtracting the coefficient of p_2 from that of p_1 in Col. 34, and the right side of Eq. (*d*) is obtained by subtracting from the value in Col. 32, the value in Col. 30 multiplied by $T/3$, or

$$p_1 = \frac{\text{Col. } 32 - \frac{T}{3}(\text{Col. } 30)}{\text{coef. } p_1 - \text{coef. } p_2 \text{ (in Col. 34)}}.$$

The value of p_2 may then be obtained in a manner similar to that used for finding the value of p_1.

Column 39 gives the vertical water pressure exerted upon the upstream batter of the segment, being equal to

$$mw(y_1 + \tfrac{1}{2}y_2),$$

or Col. 18 times Col. 19.

Column 40 gives the cumulative sums of Col. 39.

Column 41, the moment of the water pressure recorded in Col. 39, is equal to

$$mw\left(y_1 + \frac{1}{2}y_2\right)\left(T - \frac{m}{2}\right).$$

Column 42 summarizes the moments of the water pressures recorded in Col. 39 for all the segments above the base of the segment in question. This is accomplished by adding to the value given in Col. 41 for the segment, the value recorded in Col. 42 for the next segment above plus the value in Col. 40 for the next segment above times n of the segment in question. For example, to find value in Col. 42 for segment 200 to 250 ft., proceed as follows:

Value, Col. 41, seg. 200 to 250 ft.	=	3,591,797
Value, Col. 42, seg. 150 to 200 ft.	=	3,773,438
28,125 × 34	=	956,250
Value, Col. 42, seg. 200 to 250 ft.	=	8,321,485

Column 43 gives the total horizontal water pressure acting against the face of the dam above the base of the segment, being equal to

$$\frac{w}{2}(y_1 + y_2)^2,$$

or Col. 27 times $(y_1 + y_2)$.

Column 44 similarly records the moment of the horizontal water pressures about the base of the segment which is given by the familiar equation,

$$M = \frac{w(y_1 + y_2)^3}{6}.$$

This may be computed by taking Col. 43 times $\frac{y_1 + y_2}{3}$.

Column 45 records the sliding factor on the base of each segment assuming uplift at 25 per cent of the head at the upstream face, diminishing uniformly to zero at the downstream face. As stated in Eq. (143) above,

$$W_3' = \frac{cwT}{2}(y_1 + y_2),$$

or

$$\text{Uplift} = 0.25T \cdot \text{Col. } 27.$$

Hence, the values in Col. 45 are obtained by solving the fraction

$$\frac{\text{Col. } 43}{\text{Col. } 30 + \text{Col. } 40 - (0.25T \cdot \text{Col. } 27)}.$$

Column 46 presents the solution for each segment in terms of c (the fractional part of head effective as uplift), of the equation

$$W_3' = \frac{cwT}{2}(y_1 + y_2),$$

the numerical coefficients of c being $T \cdot$ Col. 27.

Column 47, similarly, presents the values of the moment of the uplift in terms of c, being the solution of Eq. (144) above,

$$M_3' = \frac{cwT^2}{3}(y_1 + y_2).$$

The numerical coefficients are equal to $T^2 \cdot$ Col. 82.

Column 48 is simply the sum of Col. 30 plus Col. 40.

Column 49 is the sum of Col. 32 + Col. 42 − Col. 44.

Column 50, the pressure at the toe of the segment, results from the solution of Eq. (52), *i.e.*,

$$p_2 = \frac{2W}{T}\left(2 - \frac{3u}{T}\right),$$

or since

$$u = \frac{M}{W},\ p_2 = \frac{4W}{T} - \frac{6M}{T^2}.$$

Thus

$$p_2 = \frac{4 \cdot \text{Col. } 48}{\text{Col. } 7} - \frac{6 \cdot \text{Col. } 49}{\text{Col. } 25}.$$

Column 52, the pressure at the heel of the segment, is similarly derived from Eq. (53), *i.e.*,

$$p_1 = \frac{2W}{T}\left(\frac{3u}{T} - 1\right) = \frac{6M}{T^2} - \frac{2W}{T}.$$

Column 54 gives the values of p_2 which will take place if the uplift under the dam is just enough to cause zero pressure at the heel, in other words, when

$$c = \frac{p_1}{w(y_1 + y_2)},$$

in which p_1 is the value from Col. 52. These values of p_2 will, of course, be the same as in Col. 50, since the uplift at the toe is zero.

Column 56 is the result of solving for the value of c as determined by the equation

$$c = \frac{p_1}{w(y_1 + y_2)},$$

this being the maximum value of uplift coefficient allowable if there is to be no tension at the heel.

Having determined the maximum values of p_2 and p_1, the vertical stresses in the downstream and upstream faces of the dam, respectively, it is next necessary to determine the maximum values of these stresses on inclined planes due to these vertical stresses. As shown in Eq. (74), the maximum value of a compressive stress at the face of the dam is

$$p \csc^2 \theta,$$

in which θ is the angle of the face with the horizontal. Columns 58 and 61 give the value of $\csc^2 \theta$ for the downstream and upstream faces of the dam at the bottom of each segment, and Cols. 59 and 62 give the results of multiplying the vertical stresses by $\csc^2 \theta$.

Column 63, the sliding factor with 25 per cent uplift, is the result of dividing the total horizontal water pressure as given in Col. 43 by the net weight as given in Col. 64. In other words, the quantity as given in Col. 64, the "net weight," is actually the effective pressure of the base

Segment Depth	y_2	Face Batter Down Strm n	Face Batter Up Strm m	t	T	r	R	$R-r$	$R+r$	$R+\frac{r}{2}$	$R+2r$	$2R+r$	$\frac{R^3-r^3}{R-r}$	$\frac{R^4-r^4}{R-r}$	r_o	R_o	R_o-r_o	$\frac{R_o^3-r_o^3}{R_o-r_o}$	$\frac{R_o^4-r_o^4}{R_o-r_o}$	$R^2-R_o^2$	$R^3-R_o^3$	$\frac{R^4-R_o^4}{1,000}$	$R-R_o\cos\frac{\theta}{2}$	$R_o\left(\frac{R^2-R_o^2}{2}\right)$	$R_o\left(\frac{R^3-R_o^3}{3}\right)$
0-25'	25'	4.25	0	16	20.25	1200	1200	0	2400	1800	3600	3600	4,320,000	6,912,000,000	1184	1179.75	-4.25	4,190,480	6,603,504,000	48,190	86,012,035	136,464,600	20.25	28,426,076	
25-50	25	12.25	0	20.25	32.5	1200	1200	0	2400	1800	3600	3600	"	"	1179.75	1157.5	-12.25	4,132,233	6,466,379,000	76,944	136,631,900	215,677,800	32.5	44,916,060	
50-100	50	32.5	0	32.5	65.0	1200	1200	0	2400	1800	3600	3600	"	"	1167.5	1135	-32.5	3,976,392	6,104,571,000	151,775	265,864,600	414,076,349	65.0	86,132,880	
100-150	50	33.5	1.5	65.0	100.0	1200	1201.5	1.5	2401.5	1801.5	3601.5	3603	4,325,362	6,924,875,000	1135	1101.5	-33.5	3,751,732	5,594,669,300	230,300	398,035,670	611,885,084	100.0	126,837,725	146,145,433,840
150-200	50	34.0	1.5	100.0	135.5	1201.5	1203	1.5	2404.5	1803.75	3606	3607.5	4,336,256	6,951,052,000	1101.5	1067.5	-34.0	3,528,710	5,103,349,900	307,653	524,516,183	795,825,557	135.5	164,209,730	186,640,333,600
200-250	50	34.0	1.5	135.5	171.0	1203	1204.5	1.5	2407.5	1806	3610.5	3612.0	4,347,003	6,976,909,000	1067.5	1033.5	-34.0	3,310,940	4,638,332,300	382,698	643,608,637	963,994,219	171.0	197,759,190	221,723,173,000
250-300	50	34.0	0	171.0	205.0	1204.5	1204.5	0	2409	1806.75	3613.5	3613.5	4,352,461	6,990,041,700	1033.5	999.5	-34.0	3,100,104	4,202,458,400	451,820	749,012,181	1,106,877,803	205.0	225,797,040	249,545,894,600
300-350	50	34.0	0	205.0	239.0	1204.5	1204.5	0	2409	1806.75	3613.5	3613.5	"	"	999.5	965.5	-34.0	2,896,208	3,794,789,200	518,630	847,483,245	1,235,900,637	239.0	250,368,630	272,748,352,900
Col.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)

Segment Depth	Weight of Segment	Weight of Segment Plus All Those Above	Moment of Weight About "Center Axis"	Moment of Segment Plus Those Above About "Center Axis"	Total Base Pressure in Terms of P_2 And P_1 (To Base of Segment)	Total Net Moment About Center Axis in Terms of P_2 And P_1	Reservoir Empty P_2 Lbs Per Sq. Ft.	Reservoir Empty P_2 Lbs. Sq. In.	Reservoir Empty P_1 Lbs Per Sq. Ft.	Reservoir Empty P_1 Lbs Sq. In.	Pressure on Upstrm Batter of Segment	Total Pressure on Upstr. Batter to Crest	Moment of Pressure on Upstr. Batter of Segment	Moment of Pressure on Upstr. Batter To Crest	Projected Horiz. Water Pressure, on Segment	Projected Horiz. Water Pressure, to Crest.	Moment Horiz. Pressure on Segment Abt. Bot. of Segment.
0-25'	67,458	67,458	80,337,501	80,337,501	$10.0132P_2+10.0660P_1$	$11,879.153P_2+12,013.079P_1$	2207	15	4506	31	0	0	0	0		19,531.3	
25-50	97,795	165,254	116,047,134	196,384,635	$15.9598P_2+16.1002P_1$	$18,802.826P_2+19,150.480P_1$	910	6	9362	65	0	0	0	0		78,125.1	
50-100	357,925	523,179	420,535,950	616,920,585	$31.3265P_2+31.9131P_1$	$36,240.447P_2+37,610.831P_1$	-492	-3	16,876	117	0	0	0	0		312,500	
100-150	597,531	1,120,710	692,815,464	1,309,736,050	$47.2847P_2+48.6736P_1$	$53,683.47P_2+56,882.00P_1$	-18	0	23,042	160	11,726.6	11,726.6	14,081,153	14,081,153	390,885	703,385	9,118,653
150-200	841,194	1,961,904	962,344,838	2,272,080,890	$62.8201P_2+65.3686P_1$	$69,954.41P_2+75,744.53P_1$	435	3	29,595	205	16,437.5	28,164.1	19,762,732	33,843,885	547,917	1,251,302	13,042,807
200-250	1,079,232	3,041,136	1,218,008,690	3,490,089,580	$77.6981P_2+81.7594P_1$	$84,845.52P_2+93,934.65P_1$	886	6	36,354	253	21,160.2	49,324.3	25,471,922	59,315,807	705,339	1,956,641	16,976,726
250-300	1,304,539	4,345,675	1,451,866,293	4,941,955,870	$91.2108P_2+97.0475P_1$	$97,597.20P_2+110,461.67P_1$	545	4	44,267	307	0	49,324.3	0	59,315,807	862,598	2,819,239	20,911,459
300-350	1,516,930	5,862,605	1,664,194,000	6,606,149,870	$104.0812P_2+112.0146P_1$	$109,098.17P_2+126,313.84P_1$	586	4	51,793	360	0	49,324.3	0	59,315,807	1,019,434	3,838,673	24,832,358
Col.	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)

Segment Depth	Total Moment Horiz Pressure To Crest About Bottom of Segmt.	$\frac{R^2-R_o^2}{2}$	Total Uplift on Base in Terms of C	Moment of Uplift About Center Axis in Terms of C	Net Base Pressure Reservoir Full, Exclud. Uplift	Net Moment of Base Press. Abt Center Axis. Res. Full. No Uplift	Reservoir Full-No Uplift P_2 Lbs Per Sq. Ft.	Reservoir Full-No Uplift P_2 Lbs. Sq. In.	Reservoir Full-No Uplift P_1 Lbs Per Sq. Ft.	Reservoir Full-No Uplift P_1 Lbs. Sq. In.	P_2 For $P_1=0$ Reserv. Full with Uplift Lbs Per Sq. Ft.	P_2 For $P_1=0$ Reserv. Full with Uplift Lbs. Sq. In.	Uplift Ratio C for $P_1=0$	Maximum Values Reservoir Full P_2 Lbs. Pr. Sq. In	Maximum Values Reservoir Full $1+\theta_D^2$	Maximum Values Reservoir Full Compr. Str. Lbs. Sq. In.	Maximum Values Res. Empty P_1 Lbs Pr. Sq. In	Maximum Values Res. Empty Compr. Str. Lbs. Sq. In	Sliding Factor 25% Upl'ft $\frac{\Sigma Hor. Pr.}{\Sigma Wt.-Upl.}$	Required Properties of θ' = .000833 Radians	Slid'g Fact'r with Max. Allow Upl'ft Max. C=100%	$\frac{R^4-R_o^4}{4000}$
0-25'	162,760.5		15,727c	18,772,893c	67,458	80,187,692	4502	31	2223	15	4502	31	1.422	31			31	31		$\sin\frac{1}{2}\theta'=.0004166$	0.377	
25-50	1,302,019		50,313c	59,845,206c	165,254	195,095,580	8117	56	2218	15	8117	56	0.709	56			65	65		$\cos\frac{1}{2}\theta'=.99999$	0.603	
50-100	10,416,667		199,457c	235,068,000c	523,179	606,503,918	14,848	103	1818	13	14,848	103	0.291	103	1.4725	147	117	117	0.660	$\frac{2}{3}\sin\frac{1}{2}\theta'=.000277$	0.671	
100-150	35,160,320	115,150	456,315c	533,268,438c	1,132,437	1,288,656,883	22,061	153	1834	13	22,061	153	0.196	153	1.4489	222	160	160	0.675	$\theta'\cos\frac{1}{2}\theta'=.000833$	0.675	152,971,270
150-200	83,372,377	153,826.5	817,108c	946,806,290c	1,990,068	2,222,552,398	29,396	204	2194	15	29,396	204	0.176	204	1.4624	298	205	205	0.678	$\sin\theta'=.000833$	0.678	198,956,390
200-250	162,914,203	191,349	1,277,490c	1,467,728,200c	3,090,460	3,386,491,184	37,121	258	2522	18	37,121	258	0.161	258	"	377	253	253	0.679	$\frac{1}{\theta'}(\theta'+\sin\theta')=\frac{.000-}{.20833}$	0.679	240,998,555
250-300	281,657,712	225,910	1,819,640c	2,071,155,810c	4,395,000	4,719,613,965	45,455	316	2566	18	45,455	316	0.136	316	"	462	307	307	0.680	$62.5\theta'=.0520833$	0.680	276,719,450
300-350	477,452,020	259,315	2,450,319c	2,763,115,300c	5,911,929	6,218,013,657	54,248	377	2372	16	54,248	377	0.108	377	"	551	360	360	0.680	$125\sin\frac{\theta'}{2}=\frac{.052-}{.0833}$	0.680	308,975,160
Col.	(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)	(61)	(62)	(63)	(64)	(65)

FIG. 21.—Computations for solid non-overflow gravity dam based on slice bounded by radial planes.

of the segment on the surface below, with 25 per cent uplift. The values are obtained by subtracting Col. 65 from Col. 48. Column 65 is the result of substituting the value of $c = 0.25$ in Col. 46.

8-25. Method of Analysis by the Radial-plane Theory.—The complete computations for the stress analysis of a dam, based on the radial-plane theory, are presented in Fig. 21. In this analysis the element of the dam to be considered is bounded by two radial planes at right angles with the curved axis of the dam, and spaced 1 ft. apart at the upstream face of the dam.

In order to indicate the differences in the resulting stresses by the two theories of analysis, the section of the dam is taken to be the same as that shown in Fig. 19, thus making the values of n, m, t, and T for the various segments into which the dam is divided by horizontal planes, the same as in the former illustration. The radii r, R, r_0, and R_0 are as tabulated in Cols. 7, 8, 16, and 17, r (the upstream radius) being 1,200 ft. at the top of the dam and increasing to 1,204.5 ft. at the bottom.

The computation sheet presents the solution of the following equations which are derived in Chap. VII, as Eqs. (81) to (99), inclusive:

$$W = \frac{w'y_2\theta'}{6}\left[\left(\frac{R^3 - r^3}{R - r}\right) - \left(\frac{R_0^3 - r_0^3}{R_0 - r_0}\right)\right]; \tag{145}$$

$$M = \frac{w'y_2 \sin \theta'/2}{6}\left[\left(\frac{R^4 - r^4}{R - r}\right) - \left(\frac{R_0^4 - r_0^4}{R_0 - r_0}\right)\right]; \tag{146}$$

$$W_1 = \frac{w}{6}\theta'(R - r)[3y_1(R + r) + y_2(2R + r)]; \tag{147}$$

$$M_1 = \frac{2w}{3} \sin \frac{\theta'}{2}(R - r)\left[y_1\left(\frac{R^3 - r^3}{R - r}\right) + \frac{1}{4}y_2(3R^2 + 2Rr + r^2)\right]; \tag{148}$$

$$W_2 = 2wy_2 \sin \frac{\theta'}{2}\left[\frac{y_1}{2}(R + r) + \frac{y_2}{3}\left(R + \frac{1}{2}r\right)\right]; \tag{149}$$

$$M_2 = \frac{w}{6} y_2^2 \sin \frac{\theta'}{2} [2y_1(R + 2r) + y_2(R + r)]; \tag{150}$$

$$W_3 = c\left[\frac{w(y_1 + y_2)\theta'}{(R - R_0)}\right]\left[\frac{1}{3}(R^3 - R_0^3) - \frac{R_0}{2}(R^2 - R_0^2)\right]; \tag{151}$$

$$M_3 = 2wc \sin \frac{\theta'}{2}(y_1 + y_2)\left[\frac{1}{4}\left(\frac{R^4 - R_0^4}{R - R_0}\right) - \frac{R_0}{3}\left(\frac{R^3 - R_0^3}{R - R_0}\right)\right]; \tag{152}$$

$$W_4 = p_2\theta'\left(\frac{R^2 - R_0^2}{2}\right) - (p_2 - p_1)$$

$$\left[\frac{2 \sin \frac{\theta'}{2}\left(\frac{R^3 - R_0^3}{3}\right) - \theta' R_0 \cos \frac{\theta'}{2}\left(\frac{R^2 - R_0^2}{2}\right)}{R - R_0 \cos \theta'/2}\right]; \tag{153}$$

$$M_4 = \frac{2p_2}{3} \sin \frac{\theta'}{2}(R^3 - R_0^3) - (p_2 - p_1)$$

$$\left[\frac{\frac{1}{8}(\theta' + \sin \theta')(R^4 - R_0^4) - \frac{1}{3}R_0(\sin \theta')(R^3 - R_0^3)}{R - R_0 \cos \theta'/2}\right]. \tag{154}$$

The characteristics of the section and curvature of the dam and derived properties required in solving the above equations, are tabulated in Cols. 2 to 26. The values given in the ensuing columns are obtained as follows:

Column 27, the weight of the segment may be expressed as

$$W = \frac{w' \cdot (\text{Col. } 2) \cdot \theta'}{6}(\text{Col. } 14 - \text{Col. } 19),$$

in which θ' is to be expressed in radians. (Throughout this analysis $\theta' = 0.000833$ radians as determined by the properties of the dam and the thickness of the element under consideration. Various properties of θ' are tabulated in Col. 63.)

Column 28 is the cumulative summation of Col. 27.

Column 29 is the result of solving Eq. (146), or

$$M = w'(\text{Col. } 2)\left(\sin \frac{\theta'}{2}\right)(\text{Col. } 14 - \text{Col. } 20).$$

Column 30 is the cumulative summation of Col. 29.

Column 31 is the result of substituting the values of θ', R, and R_0 in Eq. (153) and gathering terms. Since the value of θ' is constant, this equation may be greatly simplified for a particular analysis. Thus for $\theta' = 0.000833$ radians as in this case, Eq. (153) becomes:

$$W_4 = 0.000416(\text{Col. }21)p_2 - \left[\frac{0.00027\dot{7}(\text{Col. }22) - 0.000833(\text{Col. }25)}{(\text{Col. }24)}\right](p_2 - p_1).$$

Column 32 is similarly the result of substituting the values of θ', R, and R_0 in Eq. (154), which, for the analysis under discussion, may be simplified by the substitution of the functions of θ' therein to the following form:

$$M_4 = 0.00027\dot{7}(\text{Col. }22)p_2 - \left[\frac{0.20833(\text{Col. }23) - 0.00027\dot{7}(\text{Col. }17)(\text{Col. }22)}{(\text{Col. }24)}\right](p_2 - p_1).$$

It is next necessary to equate the values in Col. 31 to those in Col. 28, and those in Col. 32 to those in Col. 30, having thus established two equations with only the two unknown quantities p_2 and p_1 for each segment. The numerical values of these quantities may next be obtained by solving the equations simultaneously. The results are tabulated in Cols. 50 and 52, expressed in pounds per square foot, and in Cols. 51 and 53 expressed in pounds per square inch. These values of p_2 and p_1 are the foundation pressures at the downstream and upstream edges of the segment respectively, due to the weight of the dam only, the reservoir being empty, and hence no uplift being effective.

Column 37 tabulates the pressures W_1 on the upstream batter of each segment, being the solution of Eq. (147). As is obvious from the form of the equation,

$$W_1 = \frac{w\theta'}{6}(\text{Col. }9)[3y_1(\text{Col. }10) + y_2(\text{Col. }13)],$$

in which $w\theta'/6$ is constant, and y_1 is the depth of the water above the top of the segment, which in this case, is the height of the dam above the top of the segment.

Column 38 gives the cumulative totals of the quantities in Col. 37.

Column 39 tabulates the moment M_1 of the pressures on the upstream batter of each segment about the center axis, being the solution of Eq. (148).

Column 40 shows the cumulative totals of the quantities in Col. 39.

In like manner, the values tabulated in Cols. 41 and 43 are the solutions for W_2 and M_2, by means of Eqs. (149) and (150), and the values in Cols. 42 and 44 are the cumulative totals of these quantities.

Column 46 expresses the uplift W_3 under each segment in terms of c, the percentage of uplift effective at the upstream face, and is the solution of Eq. (151). Similarly, Col. 47 tabulates the values of M_3, the moment of this uplift about the center axis, being the solution of Eq. (152).

Column 48 gives the net base pressures with reservoir full, but without the uplift being taken into account. The quantities are the result of adding the values in Col. 28 to those in Col. 38.

Similarly, Col. 49 presents the net moments of the base pressures about the center axis, these being equal to Col. 30 plus Col. 40 minus Col. 44.

The pressures p_2 and p_1 at the downstream and upstream faces, respectively, of each segment may next be computed for the condition of reservoir full but with no uplift, by establishing two equations involving these quantities and solving them simultaneously. These equations may be established in the same way as for the condition of reservoir empty, *i.e.*, by equating Col. 31 to Col. 48, and by equating Col. 32 to Col. 49. The results of these solutions are tabulated in Cols. 50, 51, 52, and 53.

If now the uplift at the heel is assumed to be just equal to the value of p_1 in Col. 52, the resulting heel pressure will be zero, the limiting value if no tension is allowed. Since the uplift decreases to zero at the toe of the segment, the introduction of this force under the base does not affect pressure p_2 and, hence, the values in Cols. 54 and 55 are the same as those in Cols. 50 and 51.

The value of the uplift coefficient c, for the condition of zero pressure at the heel, is computed by the solution of the equation

$$c = \frac{p_1 \text{ for no uplift}}{wy_2}$$

or

$$c = \frac{(\text{Col. } 52)}{62.5(y_1 + y_2)}.$$

Column 57 restates the maximum values of p_2, the vertical stress at the toe, this being from Col. 55; and Col. 60 lists the maximum values of p_1, the vertical stress at the heel from Col. 36. To obtain the maximum inclined stresses it is now necessary to multiply these values by $\csc^2 \theta'$, which is equivalent to $1 + B^2$, B being the batter of the face of the dam. These maximum inclined stresses are as given in Cols. 59 and 61.

The sliding factor, with c = 25 per cent, is next computed and tabulated in Col. 62. The value of this sliding factor is obtained by solving the equation,

$$f = \frac{\Sigma \text{ horizontal pressures}}{\Sigma \text{ weight} - \Sigma \text{ uplift}} = \frac{(\text{Col. } 42)}{(\text{Col. } 48) - (\text{Col. } 46)}.$$

Similarly, the sliding factor with $p_1 = 0$ is found. The value of c is taken from Col. 56, but cannot, of course, be greater than 100 per cent. The resulting values of the sliding factor are tabulated in Col. 64.

DESCRIPTION OF ELEPHANT BUTTE DAM

8-26. General Statement.—An interesting large non-overflow gravity dam, straight in plan, is the Elephant Butte Dam situated on the Rio Grande River about 120 miles northwest of El Paso, Tex. This dam was built by the United States Reclamation Service during 1911 to 1916 for the purpose of furnishing water for irrigation use on lands in the Rio Grande Valley in the states of New Mexico and Texas, and in Mexico as a result of a treaty entered into between the government of Mexico and the government of the United States, May 21, 1906. This reservoir is at the present time the largest artificially

constructed reservoir in the world and contains when full 2,638,000 acre-ft. of water, of which 60,000 acre-ft. are to be turned over to Mexico for irrigation uses and the balance is to be used in the United States. The maximum section of this dam is shown in Fig. 22, and a photograph of the completed dam is shown in Fig. 23.

8-27. Dimensions of Dam.—The Elephant Butte Dam is 306 ft. high above the lowest point in the foundation,

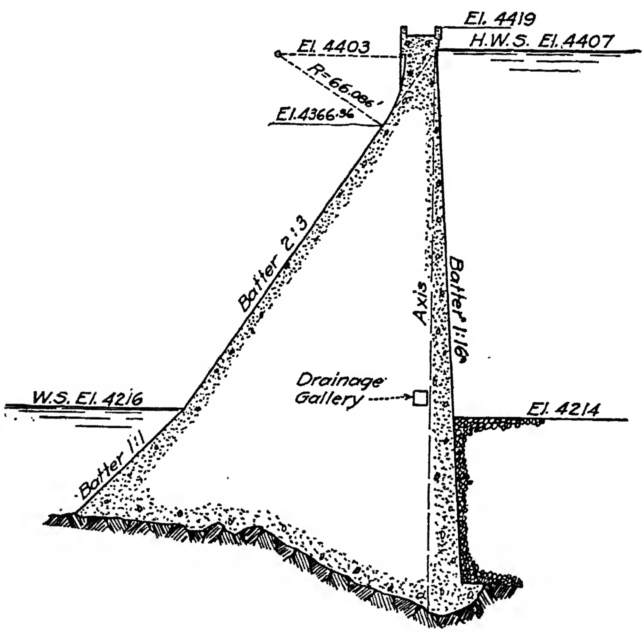


FIG. 22.—Section of Elephant Butte Dam. (*U. S. Reclamation Bureau.*)

its base width is 154 ft. at stream-bed level, its top width is 20 ft. exclusive of parapet walls, and its crest length is 1,274 ft. not including the spillway. The top of the dam is rectangular for the first 14⅓ ft. below the roadway. From the base of the rectangle the upstream face of the dam has a batter of 1:16. The rectangle at the top is joined to the downstream slope with a circular curve of 66.086 ft. radius. This curve becomes tangent to the downstream batter of 2:3 of the elementary triangle with

its apex at 4 ft. below the crest. Below the river-bed level at about 195 ft. below the crest, the downstream batter becomes 1:1. The dam itself contains 550,000 cu. yd. of masonry. The foundation was excavated to a maximum of about 100 ft. below the river-bed level, and a large volume of the concrete lies below this level.

8-28. Drainage and Inspection Provisions.—A drainage and inspection gallery, 7 by 8 ft. in cross-section, was provided across the base of the dam and up the sides of the canyon walls. The elevation of the floor of the gallery

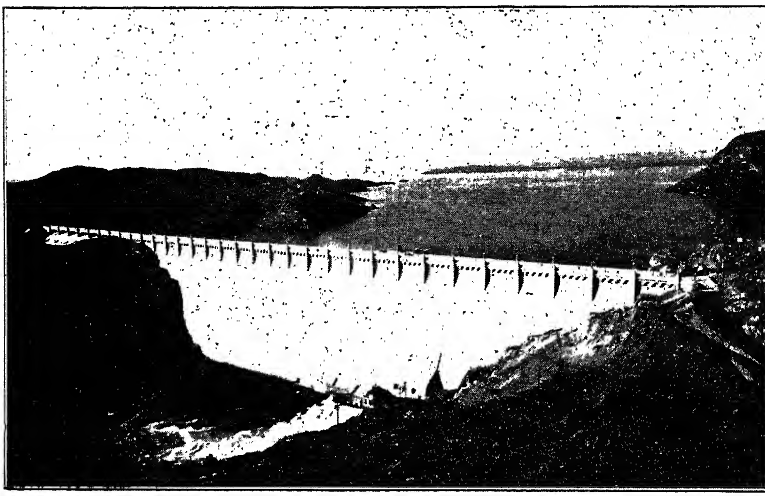


FIG. 23.—Elephant Butte Dam. (*U. S. Reclamation Bureau.*)

is 195 ft. below the crest of the dam and about 4 ft. above the original river-bed level. It runs the entire length of the base of the dam and is carried up the abutment slopes. It is connected at the low point with the downstream face of the dam for drainage purposes.

An upstream row of drainage wells, 6 in. in diameter and on 8-ft. centers, was drilled down to a depth of 45 ft. below the foundation for the purpose of intercepting seepage water coming under the toe of the dam and delivering it into 12-in. drainage wells leading into the drainage gallery at its upstream edge. Five feet further downstream, an

additional row of drainage wells 12 in. in diameter are brought into the gallery directly from the foundation. These wells discharge into drainage gutters at the sides of the drainage gallery which is connected with the downstream face of the dam. Similar drainage wells are also contained from the roof of the gallery to the top of the dam to intercept and carry off any water that enters the dam.

8-29. Expansion Joints and Grouting Provisions.—Expansion joints are placed in the dam 100 ft. apart from the base up to within 100 ft. of the top and every 50 ft. apart from this level to the crest.

Grout holes were drilled at intervals of 10 ft. for 45 ft. into the foundation of the dam in the cutoff trench at the upstream toe. Wherever cracks were found in the foundation, additional drilling and grouting were done.

8-30. Quality of Concrete.—The portions of the dam upstream from the upper row of drainage wells were made of a mix of 1:1.8:4.2. Downstream from this point the mix generally used was 1:2.9:6.1. There was no sharply defined line between the two types of concrete, but they were generally blended together in a distance from 1 to 3 ft. The aggregates consisted of a mixture of river sand and crusher run of sandstone, the crusher run below ¼ in. in diameter being classed as fine aggregate, and the balance as coarse aggregate. The cement used in the mixture of concrete was sand cement which was made on the work by blending sand, made from the sandstone from the rock quarries, with Portland cement. The blend used consisted of about equal parts of cement and sand. To still further decrease seepage through the dam, the upstream face was coated with gunite, consisting of 1 part cement to 2 parts sand comprising a layer about 1 in. thick.

8-31. Conduits through Dam.—There are two 4-ft. by 7½-in. sluiceway conduits through the dam each provided with two 47- by 60-in. sluice gates in series and lined with iron castings for a distance of 47 ft. from the entrance. Six penstock entrances are also provided in the dam for hydroelectric development, each being closed by a sliding

gate at the entrance immediately below which is a transition casting from a 47- by 60-in. rectangular section to a 60-in. circular section. There are also four 60-in. balanced valves installed in two separate wells extending vertically through the dam from top to bottom. There are two valves in each well at different elevations, one set at 60 ft. above the other. Water is admitted at the bottom of the wells through sliding surface gates which can be closed for inspection of the balanced valves.

DESCRIPTION OF PARDEE DAM

8-32. General Statement.—A description of a large, non-overflow, solid-gravity dam, curved in plan, will be presented at this point. As a part of the Mokelumne River Project for a present mountain-water supply of 60,000,000 gal. daily and an ultimate supply of 200,000,000 gal. daily, the East Bay Municipal Utility District has constructed the Pardee Reservoir with a capacity of 222,000 acre-ft. on the Mokelumne River in the foothills of the Sierra Nevada mountains near Valley Springs, Calif. The principal controlling structure of this reservoir is the Pardee Dam. Figure 24 shows a plan, elevation, and a cross-section of this dam.

8-33. Dimensions of Dam.—This dam is curved in plan to an upstream radius of 1,200 ft., 16 ft. thick at the top, 241 ft. thick at the base, 358 ft. high, 1,337 ft. long on the crest, and contains 617,700 cu. yd. of concrete. The downstream face of a transverse section of the dam is curved to a radius of 103 ft. 4 in. in the first 50 ft. downward from the top; then has a slope of 0.65 to 1 for the next 50 ft.; 0.67 to 1 for the next 50 ft., and 0.68 to 1 from there to the base. The upstream face is vertical for the first 100 ft. downward from the crest, has a batter of 3 to 100 in the next 150 ft. and is vertical from that elevation to the base. For a dam of its height, the stresses are not high. For reservoir empty, the computed vertical pressure on a horizontal section 350 ft. below roadway crest is 360 lb. per square inch at the heel and 4 lb. per square inch at the

toe; and for reservoir full, it is zero at the heel and 329 lb. at the toe, with an uplift ratio at this level of 26.3 per cent and a sliding factor of 0.68. The maximum inclined

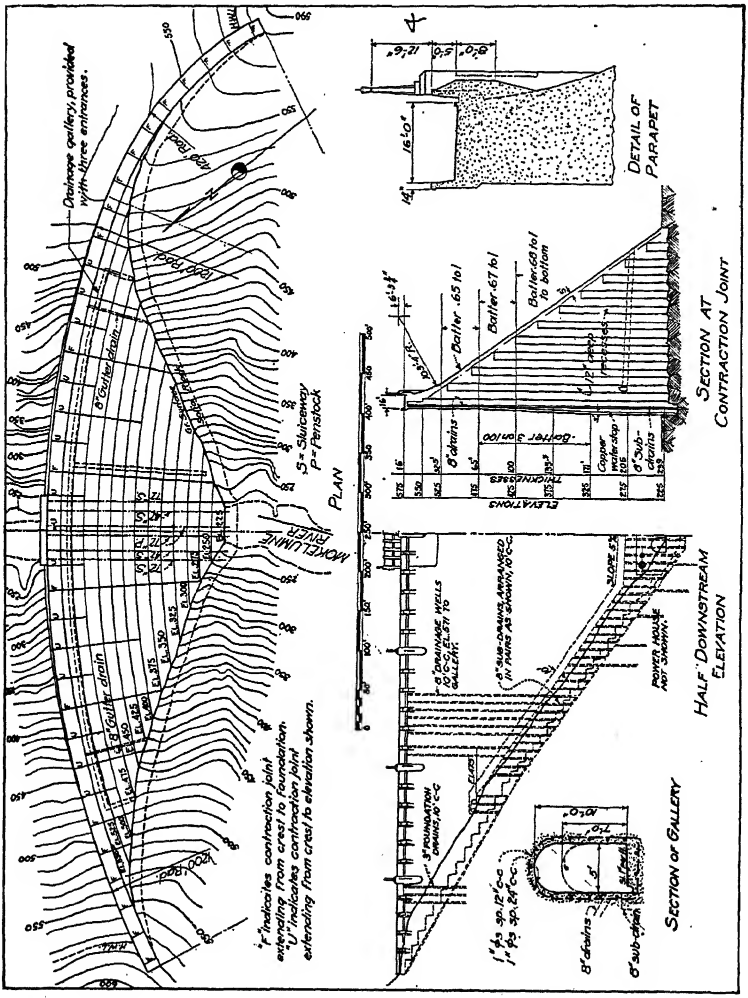


FIG. 24.—Details of Pardee Dam. (*East Bay Municipal Utility District.*)

pressure at the toe is about 493 lb. per square inch.

8-34. Foundation of Dam.—The dam is founded on an igneous ridge of diabase intruded into the upturned sedimentary rocks of the locality. This diabase is a hard,

non-erosive material and forms an excellent foundation for the dam. The formation, both upstream and downstream from the igneous ridge, consists of comparatively soft sedimentary rocks.

8-35. Inspection and Drainage Galleries.—The dam contains a 5- by 10-ft. inspection gallery running parallel to the base of the dam across the bottom of the canyon and at an inclination up each of the abutments paralleling and approximately 25 ft. from the upstream face to a height of 100 ft. below the top. The inspection gallery is connected with the downstream face of the dam with 5- by 7-ft. entrance tunnels, one at the bottom and another at the top of each of the inclined galleries. Drainage wells 8 in. in diameter, 10 ft. apart, and 7½ ft. from the upstream face of the dam are connected with the cutoff trench at the upstream toe and extend to within 25 ft. of the top of the dam. These drains are joined together in pairs and are arranged to discharge into a gutter located at the upstream side of the main inspection gallery and extend to the downstream face of the dam through the lower entrance tunnel.

8-36. Contraction Joints.—Careful provisions are made in this dam for taking care of the shrinkage due to the setting of the concrete and to temperature changes through the use of contraction joints as shown in Fig. 24. Up to a height of 179 ft. above the base, contraction joints are spaced at 150-ft. intervals; from here to a height of 279 ft. above the bottom, a 75-ft. spacing is used, while from this level to the top, the spacing is 37½ ft. Since this is an arched gravity dam, the spacings as given are measured along the arc of the downstream edge and the joints are located along radial planes. For the purpose of making a more tortuous passage for water through the dam at the joints and providing ample shearing area, each contraction joint is recessed to the depth of 1 ft. and a width of 10 ft. at intervals of 10 ft. across the face of the contraction joint. Figure 25 is a photograph of these recesses. It will be noted that the recesses stop short of the downstream face

and top of the dam, so that visible contraction seams on the downstream face of the dam appear as straight lines.

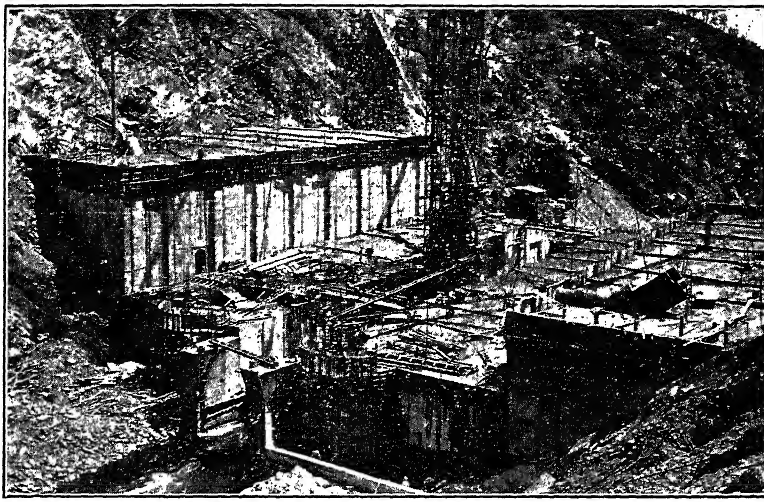


FIG. 25.—Pardee Dam, recesses at contraction joints. (*East Bay Municipal Utility District.*)

8-37. Water Stops and Grout Stops.—There is a copper water stop in this dam across the contraction joint, 18 in. in from the upstream face of the dam, to prevent, so far as possible, the entrance of water. The detail of these water stops is shown in Fig. 26. Also a galvanized-iron grout stop is placed just upstream from the drainage wells near the upstream face, and another is placed near the downstream face of the dam for the purpose of retaining grout under pressure within the contraction joints. A similar grout stop passes around the inspection gallery. Pipes with spigots in them are placed at the downstream

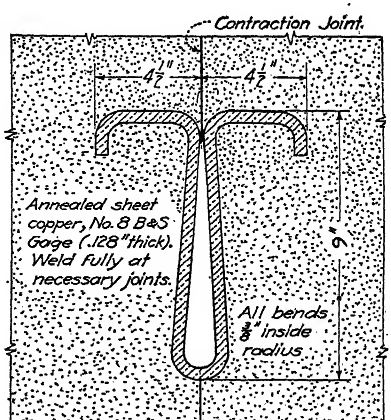


FIG. 26—Detail of water stop for Pardee Dam. (*East Bay Municipal Utility District.*)

face of the dam in the contraction joints at their bottoms passing through the galvanized-iron grout stop. These pipes were placed after the dam was grouted. Their purpose is to permit their being opened for the purpose of draining out any water that may gather behind the grout stop.

Practically in the center of each recess and of each intervening section of the normal face of the contraction joint, there is a grout hole extending up from the base of the dam to the downstream face of the dam. Within the confining

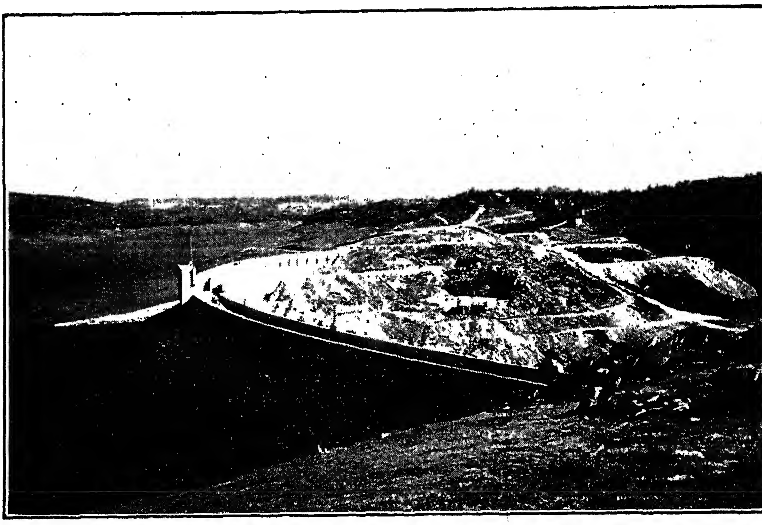


FIG. 27.—Pardee Dam, south spillway in background. (*East Bay Municipal Utility District.*)

limits of the grout plates, the grout holes consist of a combined triangular and circular segment, and outside of the downstream grout plate the hole is a 2-in. pipe extending to the face of the dam. Tightness between the pipe and grout plate is secured by means of a sleeve on each side of the plate. The outer end of the pipe is threaded to provide for attaching the grouting machine.

8-38. Reservoir Outlets.—There are four large cast-iron pipe sluiceways through the dam, two of them of 72-in. diameter and two of 42-in. diameter, each controlled at the lower end by means of a throttle valve and a butterfly

valve, and at the upper end by means of a roller gate. There is a power plant at the base of the dam which has a capacity of 15,000 kw. and this receives its water through two 72-in. cast-iron pipes through the dam, each controlled at the lower face of the dam with a 72-in. butterfly valve, and at the upper end by means of a roller gate. An ample spillway is provided at a low place in the rim of the reservoir a short distance from the dam.

8-39. Quality of Concrete.—The aggregates used in the construction of the dam were separated into sand con-

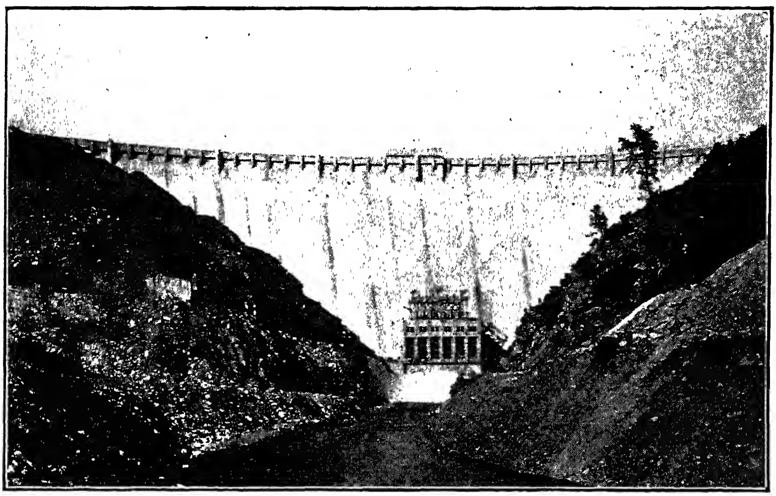


FIG. 28.—Pardee Dam, downstream view. (*East Bay Municipal Utility District.*)

sisting of particles of $\frac{7}{16}$-in. diameter and under; fine gravel consisting of particles of $1\frac{7}{8}$-in. diameter and under; and coarse gravel consisting of particles 8 in. in diameter and under. The concrete used in the dam has an average of one barrel of Portland cement to the cubic yard, an average true mix of about 1:6.6; a fineness modulus of 6.8, a water-cement ratio of about 1, and a slump of from 5 to 6 in. The average compressive strength in pounds per square inch of all samples tested during construction of the dam was 2,732 for the 28-day tests, 3,280 for the 3-month tests,

3,415 for the 6-month tests, and 3,550 for the 1-year tests.

8-40. Time of Construction.—The construction of this dam was begun in July, 1927, and was completed in the same month during 1929. There were 153,000 cu. yd. of rock and overburden removed and 617,700 cu. yd. of concrete placed. Figure 27 shows a photograph of the dam with the spillway in the background; Fig. 28 shows a downstream view of the dam, and Fig. 29 shows

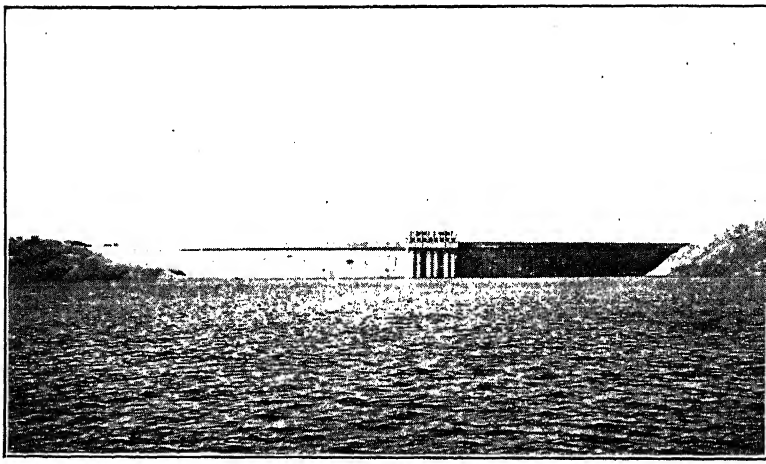


FIG. 29.—Pardee Dam, upstream view. (*East Bay Municipal Utility District.*)

an upstream view with the reservoir partially full of water. A view of the main spillway of the reservoir is shown in the chapter on Accessories for Dams.

8-41. List of Important Non-overflow Solid-gravity Dams.—In Table XI there are given some of the important, non-overflow solid-gravity dams now in existence. This list is not intended to be exhaustive as to number, but is selected more for the purpose of illustrating well-designed successful structures, particularly in America. The engineer who is contemplating the designing of a dam of this general type can profitably give careful consideration to the cross-sectional data given in this list of dams.

TABLE XI.—IMPORTANT, NON-OVERFLOW, SOLID-GRAVITY DAMS IN THE UNITED STATES

Name of dam	Location	River	Year completed	Height above foundation, ft.	Height above stream bed, ft.	Crest length, ft.	Type	Thickness at top, ft.	Thickness at bottom, ft.	Upstream radius at top, ft.	Volume of dam, cu. yd.
Boulder	Arizona-Nevada	Colorado	1936	730	...	1,180	C.G.	45	650	500	3,410,000
Owyhee	Oregon	Owyhee	1932	405	317	835	C.G.	30	255	500	540,000
Pardee	California	Mokelumne	1929	358	346	1,337	C.G.	16	241	1,200	617,700
Arrowrock	Idaho	Boise	1915	349	251	1,100	C.G.	15.5	223	672.5	585,130
O'Shaughnessy	California	Tuolumne	1938	430	313	605	C.G.	15	298	600	650,000
Exchequer	California	Merced	1926	333	325	950	C.G.	16	220	674	440,000
Kensico	New York	Bronx	1916	307	177	1,843	S.G.	28	235	....	913,000
Elephant Butte	New Mexico	Rio Grande	1916	306	206	1,274	S.G.	20	154	....	550,000
Don Pedro	California	Tuolumne	1923	285	279	1,040	C.G.	16	176	675	281,552
Roosevelt	Arizona	Salt	1911	280	262	1,125	C.G.	16	158	418	342,325
Ashokan	New York	Esopus	1911	252	150	1,000	S.G.	26	190	....	390,000
New Croton	New York	Croton	1907	238	166	1,600	S.G.	18	206	....	855,000
Wachusett	Massachusetts	Nashua	1906	228	114	1,476	S.G.	22.5	187	....	274,439
Lake Cheeseman	Colorado	South Platte	1904	227	217	710	C.G.	18	176	400	103,000
Barrett	California	Cottonwood	1922	215	...	750	C.G.	15	167	400	
Bull Run	Oregon	Bull Run	1928	200	170	1,000	C.G.	20	145	620	
Gibson	Montana	Sun	1928	195	87	900	C.G.	15	87	405	160,000
Medina	Texas	Medina	1913	180	164	1,580	S.G.	25	128	....	205,000
Lower Otay	California	Otay	1919	175	125	750	C.G.	15	150	325	
Croton Falls	New York	Croton	1911	173	113	1,100	S.G.	23	128	....	290,540
Barker	Colorado	Boulder	1909	172	145	625	S.G.	16	124	625	
Cross River	New York	Croton	1909	170	135	986	S.G.	23	116	....	156,467

NOTE.—C.G. = curved gravity; S.G. = straight gravity.

CHAPTER IX

DESIGN OF OVERFLOW SOLID-GRAVITY DAMS AND SPILLWAYS

INTRODUCTION

9-1. General Statement.—The design of overflow solid-gravity dams differs slightly from that of the non-overflow type. This difference lies in the fact that, (*a*) the overflow type generally is somewhat effected by tail-water pressure on the downstream face, (*b*) the top of the dam carries vertical water pressure, (*c*) the dam is subjected to flotation when not built on an impervious foundation, (*d*) the downstream face is often modified to care for the overflowing sheet of water, and (*e*) provision is made against undermining of the toe of the dam by the overflowing sheet of water. When proper consideration is given to these differences, the procedure in the analysis is the same as for the non-overflow type as described in the preceding chapter, to which the reader is referred for the method of computing and recording stresses.

9-2. Requirements for Stability.—The general requirements for the stability of overflow solid-gravity dams include those of non-overflow solid-gravity dams, together with some additional requirements, and may for this type of dam, therefore, be stated as follows:

1. There shall be no tension in the masonry.
2. The maximum pressure on any plane shall not exceed a certain, prescribed, safe working stress.
3. There shall be no sliding on horizontal sections.
4. The dam near the top shall be able to resist shocks from waves, floating bodies, and ice pressure.
5. The downstream face of the dam shall be protected against the tendency toward formation of a vacuum.
6. The foundation at the downstream toe of the dam shall be protected against erosion.

9-3. Symbols Utilized.—In the discussion of overflow gravity dams, the principal symbols used will have the following significance:

b = base length of the section of the dam.
h_1 = the height of the maximum water surface above the horizontal section under consideration.
h_0 = the depth of the water over the top of the dam far enough upstream from the crest to avoid the effect of drop in the water surface.
t = the effective top width of the dam.
s = the specific gravity of the masonry.

DESIGN OF AN OVERFLOW GRAVITY DAM FOR STABILITY AGAINST OVERTURNING FORCES

9-4. Theoretical Section of an Overflow Gravity Dam.—The theoretical section of an overflow gravity dam is a trapezoid, either with the upstream face vertical or battered slightly. The crest is often modified so as to give increased discharge and adherence of the discharge sheet to the body of the dam at all stages of overflow.

9-5. Forces Acting on an Overflow Dam.—In addition to the forces acting on a non-overflow dam, the following forces act on an overflow dam: (*a*) hydraulic pressure on the top, (*b*) forces resulting from impact of overflowing logs, etc., and (*c*) the pressure of the overflowing sheet on the downstream slope which pressure is small and may be neglected. In fact, if the downstream slope is too steep, there will be no pressure, and a partial vacuum may result in negative pressure unless provision is made to prevent it. The erosive effect of the overflowing sheet of water is overcome by providing a "bucket" at the toe of the dam to deflect the water into a horizontal direction, a non-erosive apron immediately below the dam, or a deep pool of water below it to dissipate the overflow energy.

9-6. Section of a Low Overflow Gravity Dam on a Pervious Foundation.—When a dam is built on a foundation such that practically full uplift pressure exists under its base, it is said to be subject to "flotation." In such a dam of height 30 ft. or under, the greatest instability occurs

when the depth of the tail-water is about four-fifths the height of the dam, with very little change from that level to full crest level, the stability improving rapidly for depths greater than this. The maximum stability of such an overflow dam will be found by trial methods to result when the crest width is determined approximately by the equation,

$$t = \sqrt{h_1}, \tag{155}$$

and the base width is determined approximately by the equation,

$$b = \frac{h_1}{\sqrt{s}}. \tag{156}$$

The base and top widths of the dam having been approximately determined by Eqs. (155) and (156), these widths should be checked and revised by a consideration of the forces acting on the dam.

9-7. Section of a Low Overflow Gravity Dam on an Impervious Foundation.—Overflow dams founded on impervious foundations, and consequently not subject to uplift due to flotation, evidently have the advantage of the full weight of the masonry in them, which permits of narrowing the base under that required for those subject to uplift. The top width of such a dam of height 30 ft. or under may be found by Eq. (155), and the approximate base width by the following equation:

$$b = \frac{h_1 - 0.4\,h_0}{\sqrt{s}}. \tag{157}$$

As a general rule the maximum stress on such an overflow dam, not subject to uplift, occurs when the backwater level is at about half the height of the dam above its base. Having determined the approximate dimensions of the dam by Eqs. (155) and (157), they should be checked and revised by considering the forces acting on the dam.

9-8. Section of a High Overflow Gravity Dam.—For higher dams than above discussed, the base width and slopes will be approximately as given in Chap. VIII for

non-overflow dams, except that the top of the triangle is removed and the crest modified to care for impact and to provide desirable hydraulic conditions. Computations including all the forces acting on the dam may lead to slight changes in the assumed section.

MODIFICATIONS OF AN OVERFLOW GRAVITY DAM FOR HYDRAULIC CONSIDERATIONS

9-9. Forms of Crest for Overflow Gravity Dams.—An overflow solid-gravity dam is a weir with its top and downstream face so formed as to permit the water to pass over it without damage to the dam. There are two prevalent types of overflow dams in use: (*a*) the plain trapezoidal type with unmodified crest and downstream face so constructed as to allow a free fall to the water; and (*b*) the modified trapezoidal type with crest and downstream face formed to control the direction of the overflowing water. The former type is used largely in India and the latter almost exclusively in the United States.

9-10. Overflow Dams with Unmodified Crests.—The Indian type of design is applicable to relatively low dams not exceeding 30 ft. in height. The height is limited by the inability to secure an economical section that will permit of free fall of the water, and by the erosive action of the falling water on the stream bed at the toe of the dam. This erosive action is overcome in dams of 100 ft. or less in height with moderate depths of overflow by providing an apron or a pool at the toe of the dam through the construction of a subsidiary weir below the main structure. The depth of this pool should be about one-fifth the height of the dam. In this type of dam, ample provision must be made for admittance of air, back of the falling water sheet, to avoid a partial vacuum which may produce serious vibrations and excessive unbalanced air pressure on the dam. This unbalanced air pressure may in some cases amount to 10 lb. or more per square inch.

9-11. Desirability of Modified Crests for Overflow Gravity Dams.—Although for greatest economy of masonry,

it would be desirable to provide a flat top on an overflow dam, considerations of a high discharge capacity as a weir, smooth flow, and freedom from vacuum forces indicate that the flat top should be somewhat modified.

In Chap. XXIII experiments by the United States Bureau of Reclamation on various-shaped weirs are discussed. It is shown that the weir having an upstream slope of 45 deg. to the crest and curved below the crest until tangent with the downstream slope of the weir, possesses the highest discharge coefficient of any of the types used in the experiments. It is also shown that rounding the upstream face of the crest slightly, increases the discharge. It is desirable, therefore, to adhere to this shape for the crest of an overflow dam, as the discharge capacity is usually a determining factor in both the cost and the safety of the structure.

9-12. Shape of the Modified Crest.—It has been previously shown, that for low overflow dams the proper width of the top of the dam, as determined by stability considerations, is approximately equal to the square root of the height of the dam, and the width at the bottom is equal to approximately

$$\frac{h_1}{\sqrt{s}} \text{ or } \frac{h_1 - 0.4\,h_0}{\sqrt{s}}.$$

Thus the theoretical section for stability is completely determined, and it is important that changes in the crest shape do not cause a great departure from this section. For higher dams, the top and base width are determined as described in paragraph 9–8. It is not possible, therefore, to fix the shape of the crest and the downstream face by considering the head of water over the dam alone. Each dam requires special study in order to provide the most perfect weir action in conjunction with the greatest economy of masonry. It will seldom be found that hydraulic considerations determine the crest width.

For hydraulic considerations it is recommended that the 45-deg. slope upstream from the crest be at least 3 ft. wide. From this point the section should be either a single circular

curve or a series of tangent circular curves to a point of tangency with the downstream slope as shown in Fig. 30. These curves will be determined by the necessity for maintaining the concrete surface slightly above the curve that the sheet of water would assume if falling freely in air. The curve of the lower surface of the water sheet may be plotted from the equation derived in the ensuing paragraphs, and the concrete section may then be easily drawn to encroach slightly into this curve. The stability of this section will depart in but negligible degree from the stability of the trapezoidal section, the weight of the masonry removed to acquire the proper weir shape being partly or entirely replaced by the weight of the overflowing water.

9-13. Theoretical Curve of Water Falling Freely in Air.— In order to determine whether the curve of the concrete derived as above actually encroaches into the ideal curve of the falling water, it is necessary to construct this ideal curve. Reference is made to Fig. 30. It may be assumed that the sheet is moving in a horizontal direction at the crest if the width of the sloping upstream face is greater than $h_0/4$, h_0 being the head of water over the crest of the weir, measured far enough back to avoid the drop-down curve of the water surface. Hence the curve of the filament, having the mean velocity of the sheet, will be determined by the fact that the velocity in the horizontal direction is constant while the vertical velocity is accelerated according to the laws of a freely falling body.

The Francis formula for a weir 1 ft. long and shaped as shown in Fig. 30 is,

$$Q = M\, h_0^{\frac{3}{2}}, \tag{158}$$

in which $M = (3.78 - 0.16\, S)h_0^{1/20}$, S being the batter of the face upstream from the crest.

Experiments by Bazin have shown the thickness of the water sheet directly over the crest of the weir to be $d = 0.69\ h_0$ for the type here under consideration. Thus, the mean horizontal velocity v_m at this point will be,

$$v_m = \frac{Q}{d} = \frac{M\,h_0^{3/2}}{0.69\,h_0} = \frac{M}{0.69}\sqrt{h_0}. \tag{159}$$

At any time t,

$$x = v_m t \text{ and } y = \frac{gt^2}{2}.$$

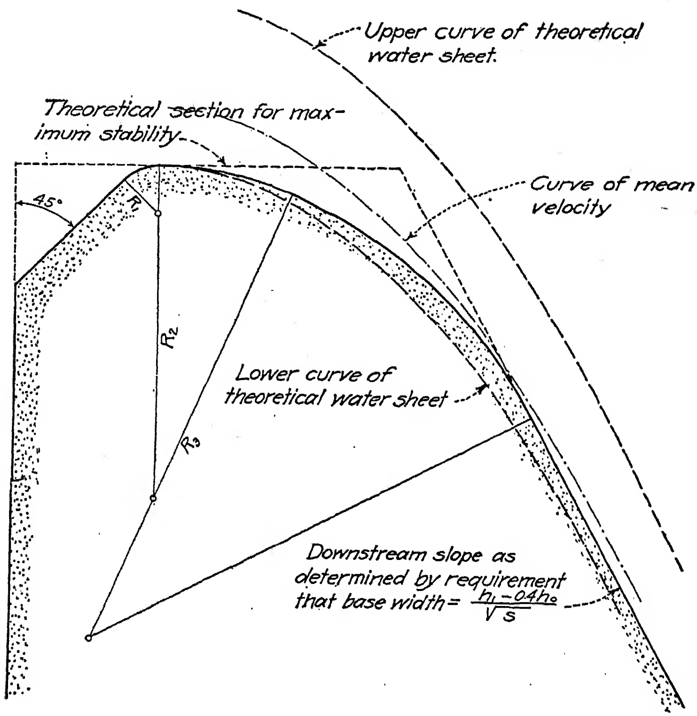


FIG. 30.—Illustration of proper curve for crest of overflow gravity dam.

Eliminating t from these equations gives

$$y = \frac{gx^2}{2v_m^2},$$

which is the equation of a parabola. Substituting the value of v_m derived above gives,

$$y = \frac{gx^2}{2M^2h_0/(0.69)^2} = \frac{gx^2}{4.2M^2h_0},$$

or

$$x^2 = \frac{M^2h_0y}{7.67}. \tag{161}$$

This is the equation of the line of mean velocity. It may be easily shown that the mean velocity of the sheet flowing over the weir occurs at two-thirds of the depth below the surface. It will be necessary to measure ⅓ d below and normal to the curve at any point to determine the lower nappe, and ⅔ d above and normal to the curve for the upper nappe. To obtain the value of d at any point, it is only necessary to divide the constant Q by the resultant velocity, or

$$d = \frac{Q}{\sqrt{v_x^2 + v_y^2}} = \frac{M h_0^{\frac{3}{2}}}{\sqrt{\frac{M^2 h_0}{(0.69)^2} + 2gy}} = \frac{M h_0^{\frac{3}{2}}}{\sqrt{2.1 M^2 h_0 + 2gy}}, \qquad (162)$$

v_x and v_y being the mean velocities in the horizontal and vertical directions, respectively.

In the above equations, no account has been taken of the velocity of approach. If such velocity exists, as it usually does to some degree, it may be allowed for by determining the equivalent velocity head and adding the same directly to the value of h_0 in all of these equations.

SIPHON SPILLWAYS

9-14. General Statement.—Siphon-spillway crests are sometimes used on overflow dams where it is desirable to secure a large discharge of water with a small fluctuation of water level in the reservoir. There are two general types of siphons used for this purpose, the water-sealing type and the self-sealing type. The water-sealing type has the outlet end of the siphon submerged in water; the self-sealing type is so constructed that the water flowing through the siphon will seal the discharge end by the flowing water. Figure 31 shows a cross-section of a siphon of the water-sealing type and also of the self-sealing type.

9-15. Parts of a Siphon Spillway.—The spillway usually consists of several individual siphons; and each siphon consists essentially of an inlet, upper leg, throat, lower leg, outlet section, and air vent. The upper leg is made of sufficient length to place the inlet below the upstream water surface, and the lower leg in the water-sealing type is made of sufficient length to place the outlet below the downstream water surface. The inlet is usually made considerably larger than the throat and is rounded to minimize the entrance-head loss. The upper leg gradually contracts to the throat at the crest of the siphon. The throat is also rounded with curves of the largest practicable radii to prevent loss of head from curvature. From the throat

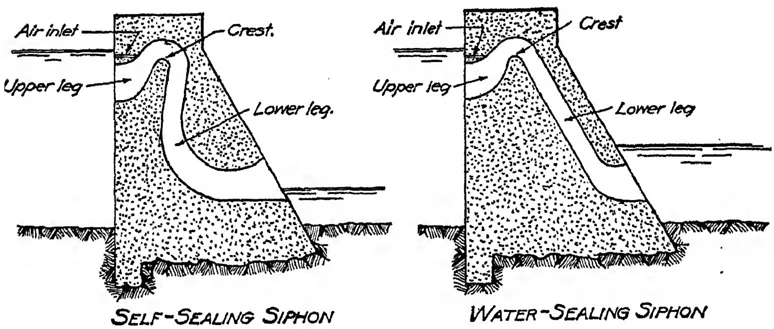


FIG. 31.—Types of siphon spillways.

the siphon area usually remains constant for siphons with full suction lengths and usually gradually expands through the lower leg into the outlet and through the outlet into the water below for siphons of less than full suction length. The lower leg is made as long as practicable up to the siphon limit. The air vent is usually placed at the siphon crest level, but may be placed at any convenient elevation, or may be adjusted to break the siphon at any desired level by bringing the opening out to the upstream face of the dam and attaching to it an adjustable metal air inlet.

9-16. Operation of a Siphon Spillway.—In action, the water in the reservoir spills over the crest, and entrains and removes the air in the throat, thus priming the siphon.

After priming, the effective head h, causing flow through the throat of the siphon, is due to the pressure on the upstream side of the throat at its center from water and air minus the pressure on the downstream side of the throat at its center from water and air. This pressure on the downstream side of the throat is due to air pressure on the tail-water for the water-sealing type, or on the discharging column of water in the self-sealing type, reduced by the weight of the column of water between the siphon-throat center level and the tail-water level in the water-sealing type, or the issuing column of discharge in the self-sealing type. On account of the fact that it is not possible to secure a perfect vacuum in the siphon, this difference in air pressure cannot exceed about 28 ft., whereas theoretically it could be about 34 ft. at sea-level. If the siphon capacity is just equal to the amount of water flowing into the reservoir, it will control the water level at its priming elevation; if the reservoir inflow is greater than the priming capacity of the siphon, the water will rise and increase the siphon capacity slightly; if the capacity is greater than the amount of water coming into the reservoir, the water surface will be lowered and the priming of the siphon will partially break and diminish its efficiency sufficiently to correspond to the inflow. Too much air drawn into the siphon will cause it to cease operating as a siphon and to operate as a mere overflow spillway.

9-17. Discharge Formula for a Siphon Spillway.—If airtight construction is secured, the siphon when in action has the hydraulic properties of a submerged tube and the flow through the orifice may, therefore, be computed by the following formula:

$$Q = CA\sqrt{2gh}, \tag{163}$$

in which Q is the discharge in second-feet, C the coefficient depending on the characteristics of the siphon, A the area of the siphon throat in square feet, g the acceleration of gravity, and h the head causing the flow through the siphon. The coefficient C varies from about 0.60 to 0.80.

9-18. Considerations in the Design of a Siphon Spillway.—In Eq. (163),

$$h = h_v + h_e + h_c + h_f + h_b,$$

in which h_v is the velocity head, h_e the entry head, h_c the conversion head, h_f the friction head, and h_b the bend head. Since h is the sum of these various heads and since h_v is the effective discharge-producing head, efficiency of design calls for making all of the other terms on the right-hand side of the equations as small as possible. Consequently, the entrance to the siphon should be well rounded, the changes in section should be gradual, the interior surface should be smooth, and the bends should have long radii. In the degree of proper control of these factors lies the selection of the value of the coefficient C. The losses of head from these several sources may be computed from the general principles of hydraulics.

In designing the section of a dam with a siphon-spillway crest, the principles involved are similar to those involved in the design of the section of other dams, as already set forth. The material removed to produce the siphon throat, etc., should be noted and the effect on the stability of the dam should be considered in the design.

MOVABLE CRESTS

9-19. Purpose and Types of Movable Crests.—Many overflow dams are provided with crests that can be taken down, raised or lowered at flood stages of the stream and then put into place again when the high stages have passed. In this way the discharge capacity of the dam for a given elevation of water in the reservoir is increased during freshets and the storage capacity is increased during low stages.

The types of movable crests consist principally of ordinary flashboards, sliding gates, Taintor gates, Stoney gates, or roller gates. These different types are discussed elsewhere in this work.

9-20. Effect of Movable Crests.—The general effect of placing movable crests on the top of a dam may be to

increase the top width of the structure, although as a general rule the top width as given in this chapter will be applicable. Moreover, the installation of movable crests will often affect the shape of the crest of the structure in order to fit it to a type of movable crest. It may happen also that the maximum stresses in the dam will occur when the movable crests are in place. Consequently, it is desirable in the designing of a dam with a movable crest to investigate the usual stresses of sliding, overturning, and crushing both for flood stages with the movable crest removed and for low-water stages with the movable crest in position.

METHOD OF PROTECTION AGAINST EROSION

9-21. General Statement.—Cutoff walls are generally placed under overflow dams near the heel and are usually carried down to impervious material if possible. Where it is not practicable to reach impervious material, the cutoff wall is carried down sufficiently far to reduce the percolation velocity to a safe rate of flow, that is, a rate that will cause no disturbance of the foundation material or cause too much loss of water. In hard foundation material the cutoff wall very generally consists of concrete, but in soft foundation material the cutoff wall usually consists of either wooden or steel-sheet piling. Where the foundation material is very pervious, a second row of sheet piling is often placed under the toe of the dam. Another cutoff wall should always be provided at the downstream end of the apron. In hard foundations this wall should consist of concrete material, and in soft foundations of sheet piling. The proper depth and disposition of cutoff walls is exceedingly important and should be carefully considered. In this connection the problem of percolation should be carefully considered, as discussed in Chap. IV.

In all classes of earth foundations and in practically all classes of rock foundations, the river bed below an overflow dam must be protected from erosion caused by the over-

flowing water, and in some cases against piping and erosion from underflowing water.

9-22. Design of Aprons.—Where the foundation material is relatively impervious, the main problem is protection against erosion from the overflowing water sheet. This protection is usually secured by covering the river bed below the dam with an apron of concrete or other non-erosive material, extending far enough downstream to secure a non-scouring velocity at the downstream end of the apron. The length of the apron is made to depend on the height of the dam, the depth of the overflowing sheet, and the hydraulic conditions of the channel below the dam, and hence, specific study must be given to each individual design.

Inasmuch as the height of the pressure line above the bottom of the masonry of the apron is greatest next to the dam and gradually decreases toward the downstream end of the apron, it is sometimes desirable to make the apron thicker adjacent to the dam than it is near the downstream end. In some cases the upstream part of the apron is made thicker for a given distance and a row of sheet piling is placed at the downstream end of the thickened portion. This sheet-piling cutoff drops the hydraulic gradient and permits the reduction of the thickness of the apron below it, thus securing better gravitational and frictional stability with a thinner apron.

A portion of the uplift on the apron may be reduced by providing weep holes throughout it at sufficient intervals to relieve the hydrostatic pressure. These weep holes may be founded on gravel to reduce the possibility of carrying silt or sand through them by the escaping water. In some cases the entire apron is underlain with crushed rock or gravel to provide a general passage for escaping water which is carried out from under the apron by means of drains through the cutoff wall at the downstream end of the apron.

9-23. Use of the Hydraulic Jump.—The hydraulic jump forms an important method for dissipating the dynamic

energy of the water flowing over the apron below a dam, which energy would cause damage through erosion of the natural bed beyond the apron. Theoretically, there is a standing wave where conditions are favorable for producing the hydraulic jump which may take place at the toe of the dam or farther downstream. By depressing the apron at the toe of the dam and giving it a gradual rise towards the downstream end of the apron, or by obstructing the free progress of the water with a lower dam or baffle piers, the standing wave may be brought very close to the toe of the dam.

In this connection the reader is referred to the treatment of the hydraulic jump given in Chap. XXII.

DESCRIPTION OF WILLWOOD DAM

9-24. General Statement.—The Willwood Dam is located on the Shoshone River about 10 miles southwest of Powell, Wyo., and was constructed by the United States Reclamation Service for the purpose of diverting water from Shoshone River to an irrigable area of about 17,000 acres. Work was begun on the dam Aug. 17, 1922, and was completed June 26, 1923. The dam is an overflow solid gravity structure constructed of concrete with a mix of 1:2.5:6.3. It is 271 ft. long, 55 ft. high, and is surmounted by a highway bridge. Figure 32 shows the maximum section of the dam. It is designed for a maximum depth of water 9 ft. over the crest, allowances being made for uplift pressure over half the area of the base, varying from full reservoir pressure at the heel to full tail-water pressure at the toe. The structure is founded in a shale and sandstone gorge in the Shoshone River, and for this reason a solid-gravity type of structure was chosen in preference to the deck type. The formation is stratified horizontally and is in a seamy condition.

9-25. Methods for Preventing Erosion.—The Willwood Dam is unusual in the means adopted for preventing erosion of the stream bed below the toe. This is accomplished by depressing the surface of the apron below river-bed level

at the toe of the dam, then raising it on a gradual slope to an elevation of 5 ft. higher at the downstream edge of the apron. Observation of the structure in flood stages shows that the hydraulic jump occurs at the toe of the dam and that the water movement is reduced to a non-scouring velocity at the downstream edge of the apron.

Cutoff walls are provided at the heel of the dam and at the downstream edge of the apron. Transverse contraction joints fitted with keyways 24 in. wide by 12 in. deep, and approximately 6 ft. apart, are provided in the overflow section at intervals of about 30 ft. along the dam. The apron is reinforced in both directions with ⅝-in. square bars spaced 16 in. apart and was built as continuous construction without joints.

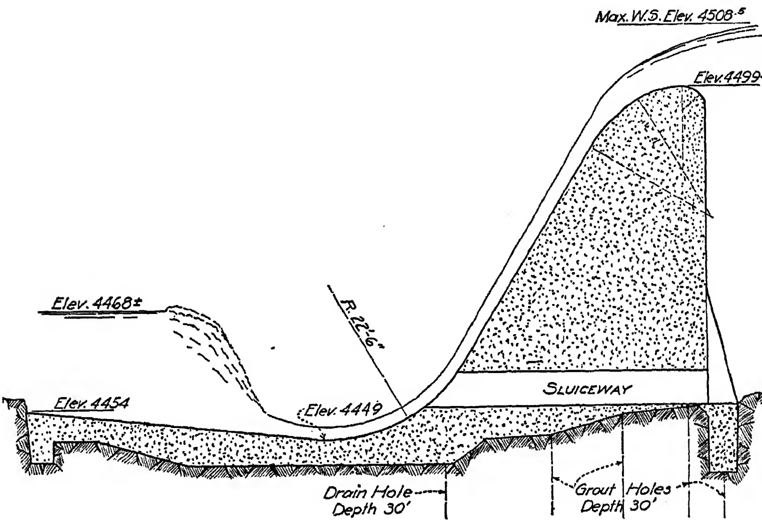


Fig. 32.—Willwood Dam, maximum section. (*U. S. Reclamation Bureau.*)

9-26. Methods for Preventing Uplift.—The foundation and abutments were thoroughly grouted under pressure of 100 lb. per square inch, grouting being applied in holes drilled approximately 30 ft. into the rock. Grout holes are spaced 10 ft. apart longitudinally and are arranged in four staggered lines across the stream bed. The tightness of

TABLE XII.—IMPORTANT OVERFLOW SOLID-GRAVITY DAMS IN THE UNITED STATES

Name of dam	Location	River	Year completed	Height above foundation, ft.	Height above stream bed, ft.	Crest length, ft.	Type	Thickness at top, ft.	Thickness at bottom, ft.	Upstream radius at top, ft.	Volume of dam, cu. yd.
Cheoah	North Carolina	Little Tennessee	1919	200	180	700	C.G.	..	...	400	
Yadkin	North Carolina	Yadkin	1919	217	169	1,400	C.G.	..	...	1,678	525,000
Shaver Lake	California	Stevenson	1927	...	185	2,105	S.G.	..	121	.....	275,000
Black Canyon	Idaho	Payette	1924	184	.....	218*	S.G.	..	130	.....	79,844
La Grange	California	Tuolumne	1894	129	125	310	C.G.	24	90	300	39,500
Houser Lake	Montana	Missouri	1912	132	110	600	S.G.	..	96	.....	83,000
Pit River No. 3	California	Pit	1925	112	.....	500	C.G.	12	107		
Folsom	California	American	1891	98	69.5	650	S.G.	24	87	.....	48,590
Wilson (Muscle Shoal No. 2)	Alabama	Tennessee	1927	140	98	3,098	S.G.	..	100	.....	1,400,000

NOTE.—C.G. = curved gravity; S.G. = straight gravity.
* Overflow section only.

the rock formations was tested under an 80-ft. hydrostatic head before grouting. The shale was found to be comparatively tight but some of the sandstone took considerable water. Measurements of uplift pressure on the base of the dam after construction showed that the total resulting uplift did not exceed the value assumed in the design.

One line of drain holes was located under the downstream edge of the dam, the holes being 10 ft. apart and drilled to depths of approximately 30 ft. These holes were connected to horizontal drain pipes leading to the downstream face of the dam at the lowest practicable elevation and were also piped vertically through the concrete in the face of the dam to facilitate cleaning.

9-27. List of Important Overflow Solid-gravity Dams.— In the preceding list, Table XII, there are given some of the important overflow solid-gravity dams. This list is selected more for the purpose of illustrating well-designed successful structures than to be exhaustive as to number. The purpose of the table is to serve the engineer by furnishing information that will be of use in designing this general type of dam.

CHAPTER X

DECK DAMS

INTRODUCTION

10-1. General Statement.—The term *deck dam* is here used to designate a hollow type of gravity dam consisting of a series of concrete buttresses, usually of trapezoidal shape, with slab faces resting on them. Such dams are variously called deck dams, hollow dams, or Ambursen dams, the later designation being applied because of patents held by the Ambursen Dam Company on some of these types of dams. In the non-overflow type, the face slab either connects directly with or rests on the upstream face of the buttresses, while in the overflow type the downstream face is also usually so covered. Until recently the buttresses of deck dams were designed by assuming a layout conforming to certain general principles, testing it by an analysis of the stresses, and then correcting the design to prevent overstressed parts. Recently Schorer[1] has presented a method of buttress design that largely eliminates the cut-and-try procedure formerly employed. The old or former method may properly be called the trial method of buttress design, and the newer or latter method the direct method of buttress design.

10-2. Forces Acting on Deck Dams.—The forces acting on a deck dam are the same as those acting on a solid-gravity dam and they may, therefore, be determined in the same manner as described in preceding chapters.

10-3. Requirements for Stability.—The general requirements for the stability of a deck dam are as follows:

a. The dam near the top shall be able to resist shocks from waves, floating bodies, and ice pressure.

[1] *Proc. Am. Soc. Civil Eng.*, Vol. 56, pp. 1947–1968, November, 1930.

b. The deck shall be capable of transmitting the water load to the buttresses without exceeding the specified working stresses.

c. The maximum compression, tension, or shear on any horizontal or inclined plane in the buttresses shall not exceed certain, prescribed, safe working stresses.

d. There shall be no sliding on horizontal sections. Overturning is not generally involved in the design of a deck dam because the upstream face is given sufficient slope to bring the resultant well within the middle third of all horizontal sections of the buttresses.

DESIGN OF DECK

10-4. General Statement.—The design of the deck of a deck dam is generally an ordinary problem in the design of reinforced concrete beams. Care should be taken not to place reinforcing steel too near the water face of the deck because of the danger of the spalling of the concrete and the rusting of the steel. The deck usually consists of the fixed beam, simple beam, cantilever beam, or cylinder type.

10-5. Fixed-beam Type.—In the fixed-beam type, the deck of the dam is fixed integrally with the buttresses with expansion joints at suitable intervals. The beams may be fixed at one end and free at the other so as to form expansion joints at alternate buttresses, or they may be fixed at both ends with expansion joints provided at suitable intervals, usually at the buttresses. This type of deck is no longer extensively used on account of the requirements of placing reinforcing steel in the concrete near the upstream face in order to take care of the bending moments in the beam. Difficulties are also experienced due to difference in shrinkage of the concrete in the buttresses and in the deck, to unequal settlement of the foundation of the deck and the buttresses, and to the difference of temperature variations between these two parts of the dam, the deck being thin and subjected to the cooling effect of the water in the reservoir, whereas the buttresses are not so affected. For

these reasons this type is used at the present time only for low dams. An example of this construction is shown in Fig. 33.

10-6. Simple-beam Type.—The simple-beam type of deck is used extensively not only for low dams but also for high dams. The simple-beam type has the advantage of eliminating the influences of the difference in temperature,

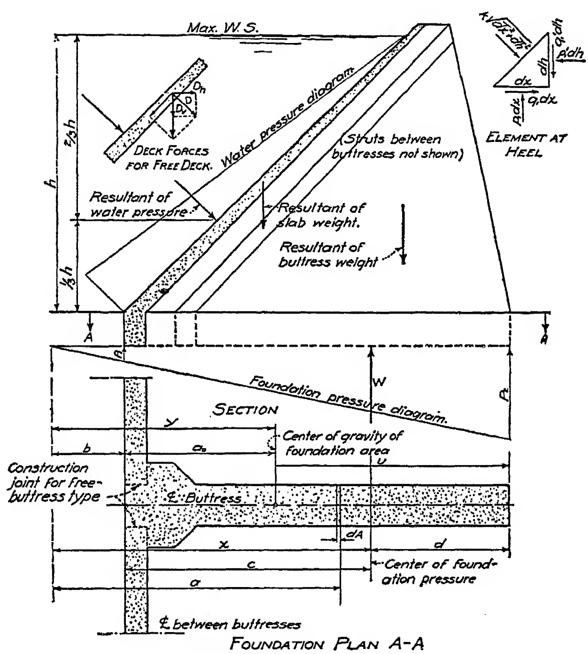


FIG. 33.—Fixed-deck type of deck dam.

shrinkage, and foundation settlement, and also of placing the reinforcing steel, for taking up the bending moment, near the downstream face of the deck. This location of the reinforcing steel removes it from the proximity of the water surface in the reservoir and, therefore, frees it from the great danger of rust of the steel and places it where it can be re-covered with concrete in case of spalling. The manner of connecting this type of deck with the buttresses

is shown in Fig. 33*A* which is taken from "Recent Advances in Buttress Type of Dams" by Davis.[1]

10-7. Cantilever Type.—In the cantilever type of deck the water load is carried to each buttress by means of cantilever slabs extending from the buttress to the center of the adjacent spans. This type requires the monolithic consideration of the buttresses and deck. Reinforcing

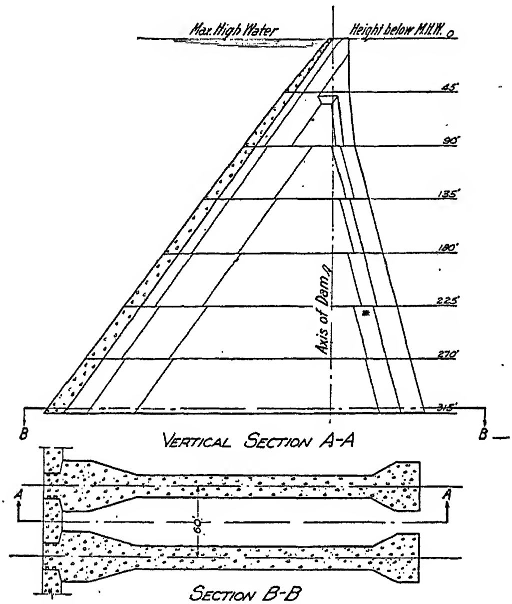


Fig. 33*A*.—Free-deck type of deck dam.

steel and roughened concrete are used to secure unity and continuity of action between the deck and the buttresses. The reinforcing steel, to take up the bending moment in the cantilever type, must, however, of necessity be placed on the upstream face of the cantilevers in order to take up the bending moment in them, this being a disadvantage that does not apply to the simple-beam type. Other objectionable features inherent in this type are those already noted in connection with the monolithic con-

[1] "Civil Engineering," *Am. Soc. Civil Eng.*, Vol. 1, pp. 387–391, February, 1931.

struction of the buttresses and deck, thus being subject to shrinkage strains, settlement strains, and temperature strains. The method of connecting the cantilevers to the buttresses is shown in Fig. 33*B*, which is taken from "The Buttressed Dam of Uniform Strength" by Schorer.[1]

10-8. Cylinder Type.—The cylinder type of deck sometimes called the *round-head buttress type,* appears to be an

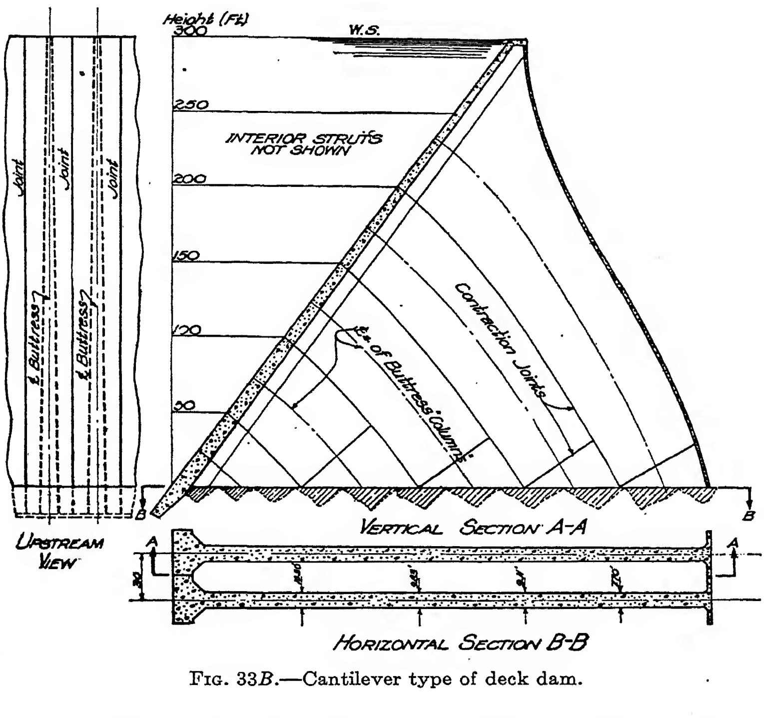


FIG. 33*B*.—Cantilever type of deck dam.

improvement over all of the other types of deck. In this type of construction the upstream end of the buttress is expanded to a cylindrical surface that extends laterally to meet the similar surfaces on the adjacent buttresses. The cylinder type of deck has recently been developed by Noetzli.[2] This type of deck and buttress combined has the advantage that all of the water load is transmitted

[1] *Proc. Am. Soc. Civil Eng.*, Vol. 56, pp. 1947–1969, November, 1930.
[2] *Proc. Am. Soc. Civil Eng.*, Vol. 57, pp. 176–180, January, 1931.

from the deck to the buttress through the cylindrical deck. Since the water pressure acts radial to the water-bearing surface, all of the pressures are normal, and both bending and diagonal tensions in the upstream part of the dam are eliminated. The shape of the deck gives an ideal distribution of the principal stresses and shear throughout the entire dam and all of the members are massive, thus eliminating thin sections and reinforcing steel, both of which

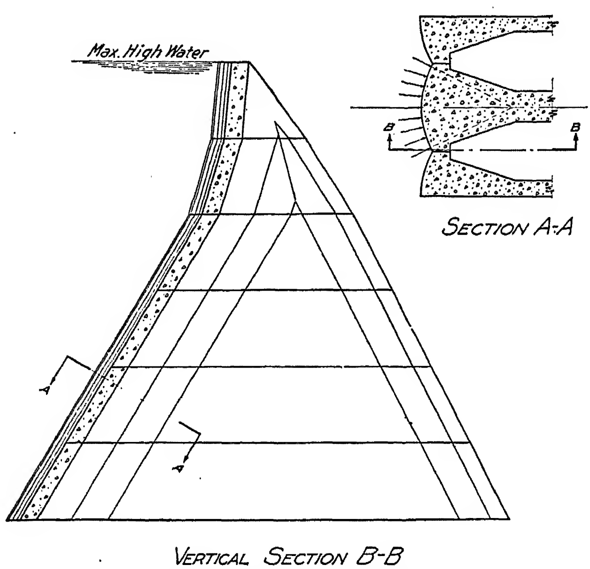


Fig. 33*C*.—Cylinder type of deck dam.

are undesirable in a dam that is to be built for long life. Reference is also made to this type of buttress by Davis in his article on the "Recent Advances in Buttress Type Dams,"[1] from which source Fig. 33*C* is taken.

10-9. Form of Crest for Overflow Deck Dams.—The required shape of the crest, the downstream slope, and the bucket of an overflow deck dam are identical with those of an overflow solid-gravity dam. In the non-overflow type, the top width and the superelevation above the water

[1] "Civil Engineering," *Am. Soc. Civil Eng.*, Vol. 1, pp. 387–391, February, 1931.

surface to prevent overflow are fixed by the same general principles as used for the solid-gravity type. The deck at the crest and on the downstream face are usually connected with the buttresses in a manner similar to that used for the deck on the upstream face, but care has to be taken to see that the deck slab and the downstream face slab are sufficiently anchored to the buttresses to prevent their being torn loose by ice, drift, and impact of the water. This may be accomplished by tying the reinforcing steel in the slabs to projecting steel on the faces of the buttresses.

BUTTRESS DESIGN

10-10. General Statement.—In figuring the load on the buttresses, it is customary to consider that the load on the face slabs between the buttresses is carried to the adjoining buttresses and combined with their loads. Consequently the buttresses are designed for resisting overturning, sliding, and crushing on horizontal or inclined sections. It is usually an easy matter to keep the horizontal forces near the center of the base of the buttress by giving suitable inclination to the upstream face slab of the dam. Owing to the large concentration of the water load on a comparatively small foundation area of the buttresses, unusual care must be given to compressive stresses. In any deck dam special attention must be given to the principal stresses at the faces of the buttresses, as tension is likely to exist at the upstream faces and the inclined stress at the toe may be excessive. In high deck dams it is desirable to compute the principal stresses in horizontal planes for 10- to 20-ft. intervals so that the trajectories of equal stresses can be traced out on a vertical cross-section of each buttress. Generally the principal stresses can be determined from the shearing, and tensional or compressive stresses, in the horizontal planes. Consequently, it is necessary to determine these stresses first in the horizontal plane of the buttresses. The trajectories of equal principal compressive stresses will be found to approach catenary curves, the tangents of which at the upstream face of the

dam are perpendicular to that face, and the tangents of the curves at the foundation parallel to the downstream face of the buttresses. Sections of the buttress of a deck dam, taken perpendicular to the length of the dam, are usually nearly triangular or trapezoidal in shape. The upstream face slopes with the horizontal from 35 to 45 deg. and the downstream face slopes in the non-overflow type from 0 to 75 deg., depending on the requirements for centering the resultant pressure on the base to give suitable foundation pressure. Sometimes the buttresses are steepened in their upper parts, particularly in high structures. In the overflow type, the downstream slope is made so that the deck will coincide with the overflowing water sheet in a manner similar to the downstream slope of an overflow solid-gravity dam. It can be shown from concrete examples that a slope of from 35 to 45 deg. for the upstream face of the buttress is required to secure proper sliding resistance in a deck dam. Too low an upstream slope gives too high inclined stresses at the heel, and too steep slopes increase the sliding tendency; so the design must be made to fit in between these two considerations. The buttresses are usually made with plane surfaces, although more modern theories indicate that economy may be secured by tapering them toward the upstream and downstream faces, thus securing equal stresses throughout.

Buttresses are spaced at economical intervals which are found to vary from about 10 to 25 ft. in the thin-buttress type and up to 50 or 60 ft. for the massive-buttress type. The buttresses are given sufficient width to safely carry the load on them to the foundation. Where the foundation is soft the buttresses are provided with footings of sufficient width to spread the load, to meet the requirements of the bearing capacity of the foundation. In some cases the footings extend the full width between the buttress, thus forming a reinforced floor. Weep holes should be placed in such footings to prevent hydrostatic uplift. Economy requires high buttresses to be braced laterally with struts at suitable intervals, particularly near the downstream

faces. These struts should be of ample size to prevent the buckling tendency of the buttresses and should be spaced both vertically and horizontally so as to divide the buttresses into lengths not greater than twelve times their thickness. Holes should be provided through the buttresses to allow the passage of men and materials. Except in very low dams of the overflow type, an inspection gallery should be built into the structure.

10-11. Methods of Computing Buttress Stresses.—Considerable discussion has arisen as to whether the stresses in a deck dam should be computed on sections perpendicular to the altitude, median or upstream face of the deck, but results for low dams will generally be sufficiently accurate when computed on horizontal planes. In important structures it may be well to compute the stresses on all three planes as a check. The question has also been raised as to whether the buttresses should carry the entire load to the foundation or whether the deck and buttress should act together as a monolith in carrying this load to the foundation. If the buttresses are separated from the deck by a frictionless joint, the method of computation is called the free-buttress theory. If the buttresses and deck are constructed monolithically, the method is called the deck-buttress monolith theory. In important dams it is well to compute the stresses by both theories and so to design as to make the dam safe under either of them. In the deck-buttress monolith theory the deck and buttress should be considered as rigidly joined together with reinforcing steel and stepped facing, but in the free-buttress theory the jointure may be considered as disconnected. Regardless of which of these theories is used, computations of the stresses in the buttresses may be made either by the trial method or by the direct method of buttress design already referred to.

TRIAL METHOD OF BUTTRESS DESIGN

10-12. Monolithic Deck and Buttress.—Let Fig. 33 represent a section and a foundation plan of a non-overflow

deck dam to be analyzed by the deck-buttress monolith theory, the section and plan being included between two vertical planes at the middles of the adjacent spans. Assume the whole structure to act monolithically and to be supported by the entire foundation shown in the figure.

The structure shown in the figure may be looked upon as a column under eccentric loading, and the general column formula made applicable to the determination of the foundation pressures. This formula is,

$$p = \frac{W}{A} + \frac{Mc}{I}, \tag{164}$$

in which p is the pressure on the extreme fibers, W the resultant of the forces normal to the section on which stresses are to be determined, A the area of this section, M the moment of W due to its eccentricity from the center of gravity of the section, c the distance from the center of gravity to the extreme fibers, and I the moment of inertia of the section about a line through its center of gravity. If e is this eccentricity measured from the center of gravity of the section, then

$$p = \frac{W}{A} \pm \frac{Wec}{I}. \tag{165}$$

The values of A, c, and I can be found by the well-known principles of geometry and mechanics, and W and e from the forces acting on the dam in a manner similar to that used for solid gravity dams. Then p_2 and p_1 may be computed by substitution of these values in Eq. (165), having a proper regard for the algebraic signs. p_2 and p_1 having been found, it is next necessary to find the governing principal stresses.

10-13. Principal Stresses—Monolithic Deck and Buttress.—The maximum stresses at the toe of the buttress may now be found by the use of Eqs. (74) and (76) of Chap. VI, except where backwater pressure exists, in which case the method as developed below for the heel of the buttress is applicable also for the toe.

The vertical shear q_1' and horizontal shear q_1 at the heel are equal and of opposite signs (see Eq. (71), Chap. VI) and, by referring to Figs. 12 and 33, and noting that the sum of the vertical forces on the element must equal zero, it will be seen that

$$p_1 dx - q_1' \, dh - k \, dx = 0,$$

and therefore that, since

$$\frac{dh}{dx} = \tan \theta',$$

$$q_1' = (p_1 - k) \cot \theta', \tag{166}$$

in which k equals the unit water pressure on the deck at the heel. Also, since the sum of the horizontal forces must be equal to zero,

$$q_1 dx - p_1' \, dh + k \, dh = 0,$$

or

$$-p_1' + q_1 \frac{dx}{dh} + k = 0,$$

or

$$-p_1' + q_1 \cot \theta' + k = 0,$$

p_1' being the horizontal force against dh. By substituting the value of q_1' from Eq. (166) and solving for p_1',

$$p_1' = (p_1 - k) \cot^2 \theta' + k. \tag{167}$$

At the heel the water pressure k is one of the principal normal stresses and the other is parallel to the deck. This stress may now be readily found from Eq. (68) of Chap. VI by substituting in it the values of p_1, p_1', and q_1'. If this stress is found to be negative, tension will exist in the buttress and the design should be revised to cure this defect. This may be done by steepening the upstream face of the dam a little. The maximum shearing stress may be found by applying Eq. (70). Stresses on higher horizontal planes may be found in the same manner.

The solution of the principal stresses by the method just presented will be sufficient for low dams, but for high dams the principal stresses should be found for several points in each horizontal plane. This may be accomplished by use of Eqs. (68) and (70).

10-14. Free Buttress.—Let Fig. 33 represent the section and foundation plan of a non-overflow deck dam to be analyzed by the free-buttress theory, the plan being included laterally between two vertical planes in the centers of the two adjacent spans. The water pressure on the deck is perpendicular to its face with its center at two-thirds of the depth h of the water. This pressure may be resolved into a horizontal force H and a vertical force V, where

$$H = \frac{wlh^2}{2}$$

and

$$V = \frac{wlh^2 \cot \theta'}{2}.$$

The weight of the deck acts vertically at its center of gravity, which will generally be at an elevation above the center of pressure of water on the deck, and should be resolved into a force parallel to the face of the deck and another force perpendicular thereto. The former force is opposed by the foundation of the deck and does not enter into the analysis of the buttress stresses. The second force acts on the buttress and should be resolved into horizontal and vertical components. If D is taken as the normal component of the weight, D_h as its horizontal component, and D_v as its vertical component, then $D_h = D \sin \theta'$, and $D_v = D \cos \theta'$. The weight of the buttress will act vertically through its center of gravity. The position and the amount of the resultant of these three vertical forces can be found from the well-known principle of mechanics applying to parallel forces. The horizontal water pressure will meet this resultant within the buttress, and the resultant of these two forces can be found and its point of incidence on the base located. This point of incidence should be well within the middle third of the base to avoid tension in the upstream face. Pressures at the heel and toe may be found as in solid-gravity dams by applying Eqs. (54) and (55) of Chap. VI.

10-15. Principal Stresses—Free Buttress.—The principal stresses at the toe of the buttress are found in the same manner as just given in the deck-buttress monolith method. One of the principal stresses at the heel of the buttress is assumed to be perpendicular and the other parallel to the face of the buttress. The former can be computed in the same manner as just developed for the heel stresses in the deck-buttress monolith method, except that the normal pressure on the buttress is that due to the water pressure plus the normal component of the deck at that point. As stated in paragraph 10-13, it will be sufficient to determine the principal stresses at the heel and toe for low dams, but for high dams these stresses should be found for several points in each horizontal plane.

DIRECT METHOD OF BUTTRESS DESIGN

10-16. Monolithic Deck and Buttress Design.—Schorer[1] has devised an ingenious method for the direct design of the buttresses of a deck or multiple-arch dam, thus obviating the necessity of trial designs. This method permits of the direct determination of the shape and thickness of the buttress to obtain one of the principal stresses at a constant predetermined magnitude while the other principal stress is zero. The result is a saving in both the cost of the design and the quantity of concrete. By Schorer's method the slope of the upstream face is first determined from equations or curves based upon the allowable sliding factor and the unit weight of the masonry. In the monolithic-deck and buttress type of dam, the deck is considered as divided into independent units by horizontal joints, and each unit is supported, together with its water load, by a portion of the buttress forming an independent column. It has been shown that the axis of this column will conform to the curve known as the "catenary of uniform strength," the curved shape resulting, of course, from consideration of the weight of the column itself. By laying out the axes of a number of these elementary curved columns and

[1] *Proc. Am. Soc. Civil Eng.*, Vol. 56, pp. 1947–1968, November, 1930.

assuming that the limits of adjacent columns lie half-way between their axes, the width and, therefore, the necessary thickness are determined. The buttress may actually be poured as a single monolithic mass, or it may be divided by convenient curved construction joints, either parallel to or at right angles to the theoretical column axes. The deck dam shown in Fig. 33*B* is a uniform-strength monolithic type, designed by Schorer, using this method. The theoretical thickness of the buttress would require curved lateral faces giving zero thickness at the downstream edge, and a maximum thickness between the upstream and downstream faces. In practice some variation from this shape will be necessary.

10-17. Free-buttress Design.—If the deck and the buttress are considered to be in contact but not attached to one another, then the buttress will support only the component of the deck weight which is normal to the face of the deck, the tangential component being carried directly to the foundation by the deck itself. The buttress columns will thus be loaded by (1) the water load, (2) the normal component of the deck weight, and (3) the weight of the column itself. At the face of the dam, the curved column axes will be normal to the deck. From this point they will curve downward on a slope somewhat less steep and longer than those of the dam having the monolithic deck and buttress. For this reason, a saving in concrete will result from the use of the latter type of dam, while, for reasons covered in paragraph 10-6, the free-buttress-type dam involving the simple-beam slab has other important advantages.

10-18. Contraction Joints.—Vertical contraction joints should be placed in the deck of the dam at each buttress or at the midpoints between them. It is also desirable to provide horizontal contraction joints in the deck, so as to avoid shrinkage cracks. These joints will overcome the possibility of an overturning force caused by the buttresses shrinking more than the deck during setting, or due to unequal temperature or water-soaking effects.

Contraction joints may be made at double buttresses with copper or other suitable water stops between them. These double buttresses should be designed for the loads carried by them, thus permitting a reduction in thickness.

10-19. Cutoff Walls and Aprons.—Cutoff walls should be provided in connection with all deck dams for the same purpose as for solid-gravity dams, and aprons should be provided for overflow deck dams. In a non-overflow deck dam, only one cutoff wall is required and that should be at the connection of the deck with the foundation; but in overflow deck dams, cutoff walls should also be provided at the foot of the downstream deck under the bucket and at the downstream end of the apron. The design of the apron for a deck dam does not differ in any respect from that for the overflow solid-gravity dam, as described in Chap. IX.

10-20. Working Stresses.—The working stresses applicable to deck dams for foundation materials should be the same as those given in Chap. IV. For the reinforced concrete in the deck and piers, the 1924 Standard Specifications of the American Society of Civil Engineers should be used, a summary of which follows.

ALLOWABLE STRESSES FOR REINFORCED CONCRETE

Kind of Stress	Allowable Stress, Lb. per Sq. In.
Concrete	
Tension	$0.00 fc$*
Axial compression on piers and columns for $L < 40R$	$0.25 fc$
Bending compression	$0.35 fc$
Bending compression at supports for continuous beams	$0.375 fc$
Shear in beams without web reinforcement	$0.020 fc$
Shear in beams with complete web reinforcement	$0.060 fc$
Punching shear	$0.060 fc$
Bond for plain bars	$0.040 fc$
Bond for deformed bars	$0.050 fc$
Reinforcing Steel	
Tension and compression in dry locations	16,000
Tension and compression in wet locations	15,000

DESCRIPTION OF STONY GORGE DAM

10-21. General Statement.—The Stony Gorge Dam was constructed by the United States Bureau of Reclamation on Stony Creek, a tributary of the Sacramento River in California, during 1927 to 1928. The dam is located about 25 miles northwest of Willows, Calif. The dam has for its purpose the storing of about 50,000 acre-ft. of water for irrigation use in connection with the Orland Project of the bureau.

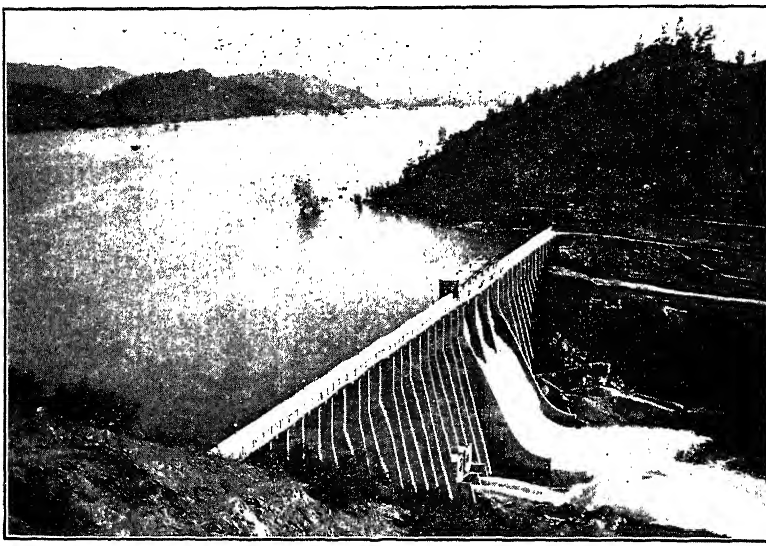


FIG. 34.—Stony Gorge Dam. (*U. S. Reclamation Bureau.*)

The Stony Gorge Dam is of the deck type, 142½ ft. high above the lowest point in the foundation, and 125 ft. above stream-bed level. It is 868 ft. long on top including 310 ft. in the left abutment section, 108 ft. in the spillway section, and 450 ft. in the right abutment section, topped with a concrete walkway. A photograph of the dam with reservoir full is shown in Fig. 34.

10-22. Spillway and Outlet Features.—The spillway section has a downstream ogee face with a 50-ft. concrete apron and is controlled by three 30- by 30-ft. steel gates.

A gate house is provided over the spillway for housing the hoisting machinery and supporting walls, for a traveling crane for handling the gates. Water is let out of the reservoir through two 50-in. riveted steel pipes controlled by two 42-in. balanced needle valves at the downstream ends and by two 3½- by 3½-ft. high-pressure gates at the upstream ends.

10-23. Foundation of Dam.—The foundation of the dam consists of sandstone with a small amount of shale content, pebble and boulder conglomerate with some sandstone and interbedded shales, shaly sandstone, and sandstone. The rock formations at the dam site are comparatively impervious. There is an inactive fault along the gorge at which a horizontal movement of about 150 ft. has taken place. The foundation was thoroughly grouted along the line of the cutoff trench to a depth between 18 to 40 ft. under a pressure of about 100 lb. per square inch.

10-24. Details of Deck.—The deck tapers uniformly from 15 in. in thickness at the top to 50 in. at a depth of 120 ft., the taper averaging 3½ in. to 12 ft. vertically. The deck is not tied into the buttresses but is corbelled adjacent to them to prevent lateral movement. The deck is reinforced with 1-in. steel bars so spaced that the resulting stresses range between 12,500 and 17,000 lb. per square inch. The corresponding stresses in the concrete vary from 380 to 624 lb. per square inch. The maximum shear in the deck at the edge of the buttress corbel is 78 lb. per square inch. Every third bar in the face slab is bent up, near the top of the support, to secure shearing resistance.

10-25. Details of Buttresses.—A study was made of the spacing of the buttresses with a result that a spacing of 18 ft. was found most economical for this structure. The upstream faces of the buttresses make an angle of 45 deg. with the horizontal. The downstream faces of the buttresses are vertical for the first 24 ft. below the top and then slope one horizontal to four vertical to the bottom.

In the design of the dam the limiting value of 0.65 was assumed for the sliding factor and 100 lb. per square inch

for horizontal shear in the buttresses. The buttresses are reinforced with ⅝-in. round bars 18 in. on centers, spaced diagonally and parallel to the deck in both sides of the buttresses, and with ⅝-in. round bars 3 ft. on centers, placed vertically in both sides of the buttress. Owing to the appearance of vertical shrinkage cracks in some of the earlier constructed buttresses, horizontal reinforcing steel amounting to about 0.3 per cent of the vertical cross-sectional area of the concrete was also placed in all buttresses up to within 30 ft. of the tops on both faces. Reinforced-concrete struts, 18 by 24 in. in cross-section and spaced 24 ft. on centers both ways, are placed between all buttresses. Some of these struts were precast and others were of monolithic construction. The ends of the struts were either grouted or poured into pockets in the sides of the buttresses.

10-26. Contraction Joints.—Contraction joints were placed between the deck and the buttresses. The seats of the deck on the buttresses were coated with about ⅛-in. thickness of plastic asphaltic putty prior to pouring the deck for the purpose of waterproofing and prevention of bonding between the deck and buttresses.

10-27. List of Important Deck Dams.—In the following list, Table XIII, are given some of the important deck dams. The list is not exhaustive as to number but is representative of successful structures, particularly in America.

TABLE XIII.—IMPORTANT DECK DAMS IN THE UNITED STATES AND CANADA

Name of dam	Location	River	Year completed	Height above foundation, ft.	Height above stream bed, ft.	Crest length, ft.	Volume of dam, cu. yd.
Stony Gorge..	California	Stony Creek	1928	142.5	125	868	
La Prêle	Wyoming	....	1909	.....	135	250	15,000
Jordan River.	British Columbia	Jordan	1912	126.0	...	756	21,200
Remmel......	Arkansas	Onachita	1924	75.0	...	900	30,000
Ellsworth	Maine	Union	1908	72.0	72	500	8,000
Patapsco.....	Maryland	Patapsco	....	.....	30	200	2,200

CHAPTER XI

GENERAL THEORY OF ARCH DAMS

INTRODUCTION

11-1. General Statement.—Arch dams differ from gravity masonry dams principally in that they transmit the water load to the sides rather than to the bottom of the canyon. The stability of arch dams depends, therefore, on the crushing strength of the material, rather than on the weight as in gravity dams. A suitable site for an arch dam then must have canyon walls capable of resisting the arch thrust and sufficiently close together to insure arch action.

11-2. Increased Use of Arch Dams.—Until recent years arch dams have been used but little, probably because stresses in them were not well understood. At the present time, stresses in arch dams are fully as well understood as those in gravity dams, and their relative number is being rapidly increased. The earlier arch dams were largely of the constant-radius type, and stresses in them were computed by the thin-cylinder formula. Since arch dams are not thin and are generally fixed at the abutments, this formula gives erroneous results, and should be relied upon only for the purpose of roughly determining the general dimensions of the arches of the dam, to be revised by more accurate methods. The thin-cylinder formula may be written as follows:

$$p = \frac{q'r}{t}, \tag{168}$$

in which p equals the compressive stress in pounds per square foot, q' the load on the arch in pounds per square foot, r the upstream radius in feet, and t the thickness of the arch in feet.

11-3. Economic Types of Arch Dams.—Neither a constant-radius arch nor a constant ring-thickness arch at a given level uses the construction material with the best economy. The constant radius causes the central angle to diminish from the top to the bottom of the dam in the ordinary canyon, while economy of material requires this angle to be practically constant[1] at 133.5 deg. Practically, however, the central angle should range from about 100 to 120 deg., on account of extra cost of excavation and forms for the rounder arch. Since the rings of the arch dam are fixed at the abutments, the moment at the crown and that at the abutments are maximum quantities of opposite signs with points of zero moment between them. Ordinarily the magnitude of the moment at the abutments is about twice that at the crown.[2] Economy of material, therefore, requires the thickening of the arch rings towards the abutments. Since the eccentricity of the thrust causing the moment at the crown is on the extradosal side and that at the abutments is on the intradosal side of the neutral axis, the excess of the abutment moments may be reduced somewhat by compounding the extradosal and introdosal curves through the use of smaller radii of curvature in the portions near the abutments.

11-4. Necessity for Trial Designs.—Considerable difficulty is generally experienced in getting the most economical design for any site. In the first place, the canyon walls must be sound and capable of resisting the thrust of the arch rings; second, the span must not be too great to produce arch action with reasonable thicknesses of the rings; third, the shape of the ring must be such as to meet the canyon walls without being too flat for economy of material; and fourth, the ratio of the length of any arch ring to its thickness must not be too great to conform to column requirements. All of these requirements necessitate numerous trial designs. In these trials, the cylinder formula and formulas for circular rings of constant thickness

[1] *Trans., Am. Soc. Civil Eng.*, Vol. 78, p. 685.
[2] *Western Construction News*, p. 336, May 25, 1928.

are of great aid. A tentative design, or several tentative designs, having been made, more elaborate shapes and formulas must be applied.

11-5. General Symbols Used.—The units of measure will be taken as pounds, feet, pounds per square foot, and foot-pounds. The following designations will be given to the symbols used in the formulas as far as they are applicable:

A = area of the radial cross-section through the point g.
C = distribution coefficient for shear.
$\frac{CE}{E'}$ = 3, or more exactly 2.75, for rectangular cross-section.
ds = selected elementary length of neutral axis or arc parallel thereto.
dl = selected elementary length of extrados.
E = elastic modulus for tension and compression.
E' = elastic modulus for shear.
$j = M \div N$ = eccentricity of thrust.
f = stress per square foot of arch ring at the point g.
g = any stress point on the neutral axis.
I = moment of inertia of the radial cross-section.
N, M, and V = thrust, moment, and shear on the section through the point g in the arch ring, due to all of the acting forces under consideration.
N_0, M_0, and V_0 = thrust, moment, and shear on the section through the point g in the arch ring, due to water pressure between free end of the assumed cantilever and point g, only.
N_c, M_c, and V_c = thrust, moment, and shear at crown of symmetrical arch ring.
p = unit water pressure on upstream face of any arch ring.
p' = unit water pressure on neutral axis of any arch ring.
R = radius of extrados.
r = radius of neutral axis.
t = thickness of arch ring radially to the neutral axis at point g.
X = redundant force acting along x-axis.
Y = redundant force acting along y-axis.
Z = redundant rotary couple acting at the elastic center.
k = distance between crown center and elastic center.
ϕ = angle between axis of symmetry and radius of extrados at any point of semiarch ring to the right of point g.
ϕ_a = one-half central angle of extrados for loaded arch ring.
ϕ_1 = angle between the crown radius and the radius of extrados through the point g.

θ_1 = angle between tangent to neutral axis at point g and chord through that point.
Δ = linear deflection of any point in base system.
Δ' = linear deflection at elastic center along x-axis under water load.
Δ_1 = linear deflection at elastic center along x-axis under dummy unit load.
a' = angular deflection at elastic center under water load.
a_1 = angular deflection at elastic center under dummy unit load.
T = change in temperature in degrees Farhenheit.
c = coefficient of expansion per degree Fahrenheit.
$\int$ is used to indicate either integration or summation.

HANNA METHOD OF ARCH-DAM ANALYSIS

11-6. General Statement.—In the analysis, used in this treatise, of stresses in an arch dam, the dam is assumed to be divided by horizontal planes into arch rings each 1 ft. high, and these rings are then considered to carry the entire load on the dam as independent arches. Resistance due to gravitational stability against overturning of a portion of the dam bounded by vertical slices distributes the load among the arches, but this resistance cannot be correctly determined by means of the radial deflections of the vertical slices because these deflections are not radial. Torsional and tangential stresses differing for the various arch rings are set up and resisted by the dam as a whole. This resistance can best be approximated by analyzing the stresses transferred by an arch ring to the adjacent arch rings through consideration of beam shear of the circular beam formed by the arch ring.[1] Such an analysis shows that considerable assistance is given to the arch rings from this shear. It is thus indicated that it is on the side of safety to assume all of the load on the dam to be carried by the arch rings.

The arch rings are assumed to be either of variable or uniform thickness, symmetrically or unsymmetrically water loaded, and fixed at the ends. Formulas for arch rings, both of variable and of uniform thickness, are here developed, the former because in rings under considerable

[1] *Trans. Am. Soc. Civil Eng.*, Vol. 93, pp. 1248–1250.

water pressure, economy of material requires their thickening towards the abutments to care for the increased moment and stresses in these parts; and the latter because in rings under low water pressure, the thickness is fixed by simplicity of design and minimum thickness of material rather than by stress conditions. In the rings of variable thickness and in those of uniform thickness with variable radii, the rings are divided into voussoirs and the sum of the stresses is obtained by summations instead of by integrations. From the assumed conditions, it is obvious that the arch rings are statically indeterminate, but the analysis of stresses in them may be approached through the *principle of least work* as employed by Castigliano, or by the *principle of deflections*. In this treatise the deflection method as employed by Maxwell and Mohr will be used. This involves the use of the "dummy unit load," as explained below.

11-7. Dummy Unit Loading Method.—In the dummy unit loading method, the deflection of any point in an arch ring under water load or of any point in a rigid bracket attached thereto is obtained through its relation to the distortion of any desired part of the arch ring through the application of a unit load at the deflection point. The unit load consists of a unit force or unit couple applied at the deflected point acting in the direction of the desired deflection or rotation. In general, the principal forces acting on any elementary section of the arch ring are the perpendicular thrust on the section, the moment of thrust about the center, and the shear parallel to the face of the section.

11-8. General Deflection Formula.—In order to develop the general deflection formula for use in arch-dam analysis, let AB represent any portion of an arch ring, as shown in Fig. 35, and let ds represent an elementary length of the ring. Assume the entire arch ring to be acted upon by a specified loading, thus producing an axial stress N, a bending stress M, and a shearing stress V, on any cross-section of the ring. These stresses will cause linear,

angular, and radial deformations of the arch ring. If the specified loading is now removed and a dummy unit load is applied at the point q in the arch ring whose deflection is in question, and acting in the direction in which the deflection is desired as shown in Fig. 35, it will produce, at any cross-section of the ring, an axial stress n, a moment stress m, and a shearing stress v.

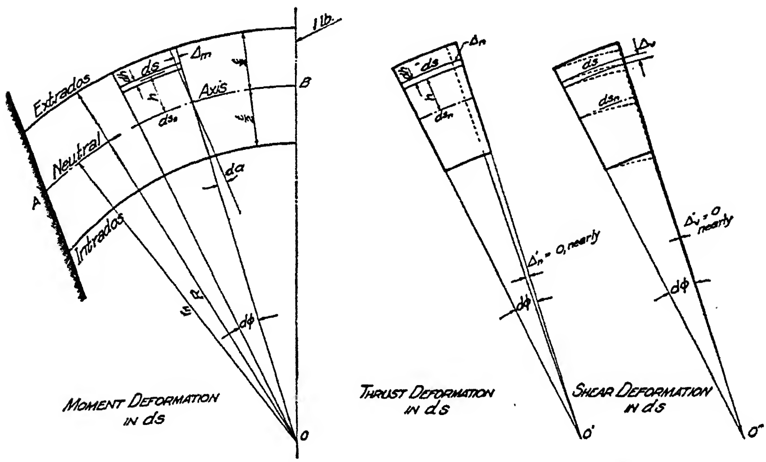


FIG. 35.—Diagrams illustrating deformations in an arch ring under water loading.

Since the external work of the forces producing deformation in the length ds is equal to the internal work in this length, it follows that, for a unit load,

$$\frac{1 \cdot \delta_q}{2} = \frac{n\Delta_{ds}}{2} + \frac{m\Delta_{d\phi}}{2} + \frac{v\Delta_{dr}}{2},$$

or the deflection of q, due to the deformations in the length ds, obviously equals

$$\delta_q = n\Delta_{ds} + m\Delta_{d\phi} + v\Delta_{dr}.$$

Now from Hooke's law the deformations in the length ds, due to the specified loading, are

$$\Delta_{ds} = \frac{N\,ds}{AE}, \quad \Delta_{d\phi} = \frac{M\,ds}{EI},$$

and

$$\Delta_{dr} = \frac{CV\,ds}{AE'},$$

the distribution coefficient C being applied to the shearing term because of lack of uniform distribution and inexact radial deformation. Substituting these values in the preceding equation, and integrating between the limits s and 0, there results the equation for the deflection of the point q, due to the specified loading,

$$\Delta_q = \int_0^s \frac{Nn\,ds}{AE} + \int_0^s \frac{Mm\,ds}{EI} + C\int_0^s \frac{Vv\,ds}{AE'}.$$

In addition to the deformation due to applied forces, there are deformations due to changes in volume caused by temperature variations, curing shrinkage, and water soaking. The effects of these forces are all similar and tend to alter the elementary length ds of the arch ring in the same manner as an axial thrust. All of these forces may, therefore, be considered as equivalent to a temperature change. It is obvious that a uniform temperature change of $T°$ will deform the length ds by $cT\,ds$, where c is the coefficient of thermal expansion. It is also obvious that the deformation due to axial thrust is the only such temperature deformation, since there is no net internal work due directly to moment and shear. The deflection in the direction of the unit load will then be $ncT\,ds$; and, for the length s, the total deflection will be

$$\int_0^s ncT\,ds.$$

The total deflection of the point q resulting from deformations between the limits s and 0, due to the specified loading and temperature, will then be

$$\Delta_q = \int_0^s \frac{Nn\,ds}{AE} + \int_0^s \frac{Mm\,ds}{EI} + C\int_0^s \frac{Vv\,ds}{AE'} + \int_0^s ncT\,ds. \tag{169}$$

This is the general formula for the deflection of any point q in any specified direction produced by the deformation of the arch ring between the limits s and 0. There is an additional angular deflection shown in Fig. 35 as Δ_n' and a consequent linear deformation of

$$\frac{\Delta_n'}{r},$$

due to the effect of thrust N in the arch ring, but since Δ_n' is small as compared with r, this deformation is generally omitted. Similarly the additional angular deflection Δ_v' due to shear is generally omitted. Equation (169) has very broad and useful application in the solution of statically indeterminate stresses in arch rings of arch dams and many other engineering structures.

11-9. Types of Arch Dams Considered.—In the following chapters there will be considered symmetrically water-loaded symmetrical arch rings of variable thickness, single-centered arch rings of uniform thickness, three-centered arch rings of uniform thickness, multicentered arch rings of uniform thickness, and unsymmetrical arch rings of variable thickness. The stress formulas will, in the main, be based on the general deflection equation (Eq. [169]), as given in this chapter.

STRESS DISTRIBUTION IN ARCH RINGS

11-10. General Assumptions.—Although the arch ring considered is taken to be 1 ft. high, it is represented by a plane section midway between its top and bottom faces for the field of action of the forces, all sections parallel to the faces being assumed equal. Let Figs. 36 and 37 represent such plane sections of a thin and a thick arch ring, respectively. In this treatise radial cross-sections of both thin and thick arch rings which are plane before loading will be considered plane after loading, *i.e.*, the strain or deformation distribution will be taken as planar. The *stress* distribution in thin rings and that in thick rings will, however, not be the same, since in the former the stress

distribution will be assumed to be planar, while in the latter it will be assumed to be curved.

11-11. Distinction between "Thin" and "Thick" Arch Rings.—An arch ring will be considered thin if the thickness is less than about one-fifth of the length of the radius of curvature. In such an arch there is relatively little difference in the lengths of the extradosal and introdosal fibers, and consequently the arch ring may, without serious error, be considered to be straight, the stress distribution planer,

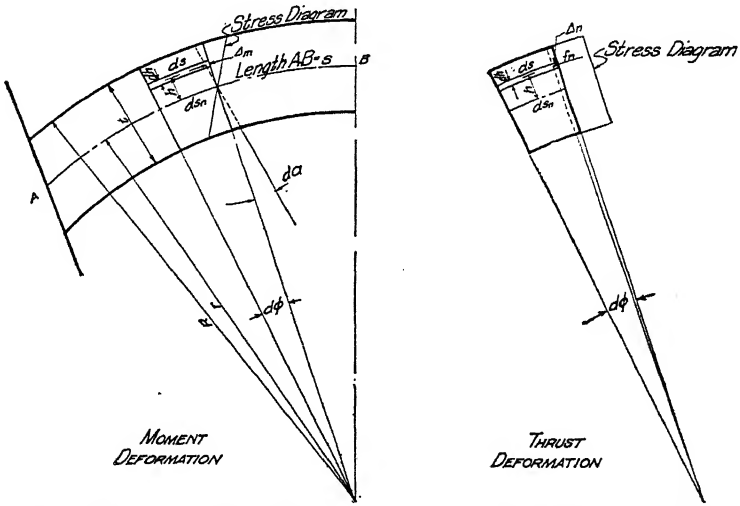


FIG. 36.—Diagrams illustrating deformations in a thin arch ring under water loading.

and the neutral axis coincident with the gravity axis of the radial cross-section. Such assumptions simplify the stress formulas for arch rings considerably, and the error involved amounts to less than 2 per cent where the radius is five times the thickness of the arch ring for small central angles, and considerably less than this for large central angles. The error involved in the assumption of planar-stress distribution for planar deformation increases, however, as the ratio of thickness to radius of curvature of the arch ring increases, due to the fact that the fiber lengths between radial planes are no longer closely equal, being shorter on the intradosal

side of the neutral axis at a given distance from it than on the extradosal side at the same distance, as illustrated in Fig. 37. In thick arches, if planar *deformation* distribution is assumed, then the *stress* distribution cannot be planar. The inequality of the fiber lengths between radial planes disturbs the equilibrium which is assumed to be obtained on the *central* axis for thin arch rings, and makes it neces-

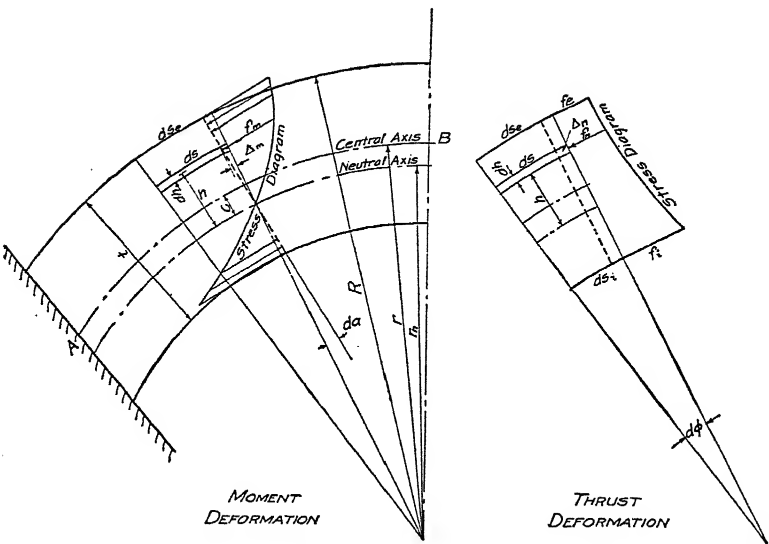


FIG. 37.—Diagrams illustrating deformations in a thick arch ring under water loading.

sary to obtain that equilibrium on the *neutral* axis, which is shifted towards the intradosal part of the arch ring as indicated in Fig. 37. In Fig. 36, since the faces of the voussoir are assumed to be parallel, it is clear from Hooke's law that

$$\Delta_m = \frac{f_m dh\ ds}{E\ dh} = \frac{f_m ds}{E}.$$

But, by assumption, E and ds are constant throughout the cross-section, and it follows, therefore, that Δ_m varies directly as f_m, and f_m varies directly as Δ_m, or as h, since $\Delta_m = h\ da$, where da is constant. This relation evidently

is rectilinear. In Fig. 37, where curvature is taken into consideration, it is also clear that

$$\Delta_m = \frac{f_m dh\, ds}{E\, dh} = \frac{f_m ds}{E}.$$

As before, E is constant, but ds varies according to the distance from the neutral axis. From Fig. 37,

$$\frac{ds}{ds_e} = \frac{(r_n + h)d\phi}{R\, d\phi} = \frac{r_n + h}{R},$$

whence

$$ds = \frac{(r_n + h)ds_e}{R};$$

therefore,

$$\Delta_m = \frac{f_m(r_n + h)ds_e}{ER}.$$

In this expression, r_n, ds_e, E, and R are constant and h is a variable, whence Δ_m varies as $f_m(r_n + h)$ and f_m varies as

$$\frac{\Delta_m}{r_n + h}.$$

Obviously, this relation is not rectilinear. Now, $\Delta_m = h\, da$, and, therefore,

$$hda = \frac{f_m(r_n + h)ds_e}{ER};$$

whence

$$f_m = \frac{ERh\, da}{(r_n + h)ds_e} = \left(\frac{ER\, da}{ds_e}\right) \cdot \frac{h}{r_n + h}$$

for a given loading on the cross-section of the arch ring. This relation between f_m and h is hyperbolic and, hence, the stress distribution diagram is a hyperbola.

11-12. Position of Neutral Axis of Thick Arch Rings.—To find the radius of the neutral axis, let $1 \cdot dh = dh$ in Fig. 37, be an elementary area distant h from the neutral axis. Let f_m be the intensity of the bending stress on dh, then the total bending stress on dh will be $f_m dh$, and from Hooke's law,

$$\Delta_m = h\, da = \frac{f_m dh\, ds}{E\, dh}.$$

Solving for the stress on dh from this equation,

$$f_m dh = \frac{E\,da\,h\,dh}{ds}.$$

But $ds = d\phi(r_n + h)$ and, therefore,

$$f_m dh = \frac{E\,da\,h\,dh}{d\phi(r_n + h)}.$$

Now, the sum S of the bending stresses on the entire cross-section of the arch ring must equal zero, whence

$$S = \int_{-\left(\frac{t}{2}-c\right)}^{\frac{t}{2}+c} f_m dh = \int_{-\left(\frac{t}{2}-c\right)}^{\frac{t}{2}+c} \frac{E\,da\,h\,dh}{d\phi(r_n + h)} = 0. \tag{170}$$

But E, $d\phi$, and da are constants for the condition of material and loading assumed, whence

$$\int_{-\left(\frac{t}{2}-c\right)}^{\frac{t}{2}+c} \frac{h\,dh}{r_n + h} = 0. \tag{171}$$

Performing the indicated integration and solving for r_n, it is found that

$$r_n = \frac{t}{\log_e\left(\frac{R}{R - t}\right)}. \tag{172}$$

Also, from Fig. 37,

$$c = r - r_n. \tag{173}$$

11-13. Unit Compressive-stress Formulas for Thin Arch Rings.—The thrust generally meets the cross-section of a thin arch ring eccentrically with reference to the central axis. This thrust can be resolved into a thrust N, normal to the section, and a shearing stress V, parallel to the section. If two equal opposite forces of the value of and parallel to N are placed at the central axis, one of these forces will form a moment couple with the normal thrust N about the central axis, and the other a uniform compressive stress over the cross-section. As in a straight column

eccentrically loaded on the end with a load acting parallel to its axis, the unit compressive stress

$$f_n = \frac{N}{1 \cdot t} = \frac{N}{t}$$

and the unit bending stress

$$f_m = \pm\frac{Mc}{I} = \pm\frac{M\frac{t}{2}}{t^3/12} = \pm\frac{6M}{t^2}.$$

Then the combined unit stress $f = f_n + f_m$ and, therefore, the general compressive-stress formula becomes,

$$f = \frac{N}{t} \pm \frac{6M}{t^2}. \tag{174}$$

11-14. Unit Shearing-stress Formulas for Thin Arch Rings.—To find the distribution of the shearing stress

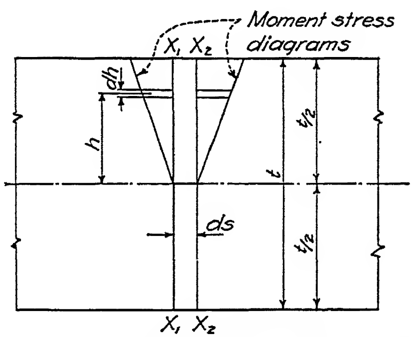


FIG. 38.—Diagram illustrating shearing stress in a thin arch ring.

over the cross-section of the arch ring, let X_1X_1 and X_2X_2 of Fig. 38 represent top views of two cross-sections of the arch ring an elementary distance ds apart and each perpendicular to the neutral axis of the ring. Also, let the plane of the paper represent, in Fig. 38, the horizontal plane through the neutral axis of the arch ring. Now the vertical shear f_v at any point in either of the cross-sections is equal in intensity to the horizontal shear f_h. The normal stress f_m on the elementary area $1 \cdot dh = dh$, is

$$f_m = \frac{Mh\,dh}{I},$$

where M is the moment and I is the moment of inertia of any cross-section of the arch ring such as X_1X_1, and the total normal moment stress N on the area above the elementary area dh is

$$M\int_h^{\frac{t}{2}}\frac{h\,dh}{I},$$

that is,

$$N = M\int_h^{\frac{t}{2}}\frac{h\,dh}{I},$$

Now the total change in N due to the change in the moment M in the length ds is

$$dN = dM\int_h^{\frac{t}{2}}\frac{h\,dh}{I},$$

and the unit change is

$$\frac{dN}{ds} = \frac{dM}{ds}\int_h^{\frac{t}{2}}\frac{h\,dh}{I},$$

This unit change dN/ds must be equal to and taken up by the unit stress f_h in the horizontal shear; also dM/ds equals V from the well-known principle of mechanics of beams. Therefore, by substitution of these values in, and the integration of, the preceding equation, it is found that

$$f_h = \frac{V}{2I}\left(\frac{t^2}{4} - h^2\right). \tag{175}$$

Now $f_h = f_v$ and $I = t^3/12$, therefore, by substitution of these values in Eq. (175),

$$f_v = \frac{6V}{t^3}\left(\frac{t^2}{4} - h^2\right). \tag{176}$$

The relation shown in Eq. (176) between f_v and h is that of a parabolic curve. In this equation, when $h = t/2$, $f_v = 0$, and when $h = 0$,

$$f_v = \frac{3V}{2t}. \tag{177}$$

This shows that the intensity of shear at the central axis of the arch ring is $1\frac{1}{2}$ times the intensity for shear if assumed distributed evenly over the section. Inspection of Eq. (176) also shows that f_v is a maximum for $h = 0$.

11-15. Normal Compressive-stress Formulas for Thick Arch Rings.—The thrust in a thick arch ring generally meets the cross-section eccentrically with reference to the neutral axis. This thrust can here, as in the case of thin arch rings, be resolved into a normal component N, and a parallel component V on the cross-section. As in the thin arch ring, two equal opposite forces of the value of and parallel to N may be placed at the neutral axis. One of these forces will form a compressive stress on the cross-section and the other will form with the thrust, a moment M around the neutral axis. Under planar deformation distribution, neither of these stresses will conform with planar distribution. In order to develop compressive stress formulas to cover this case, consider an elementary area $1 \cdot dh = dh$, and consider the intensity of stress f_n due to the compressive force acting at the neutral axis and the intensity of stress f_m due to the moment on this area. Then,

$$N = \int f_n dh$$

and

$$M = \int f_m dh.$$

If Δ_n be taken as the deflection of the elementary prism with end area dh,

$$\Delta_n = \frac{f_n\, dh\, ds}{E\, dh},$$

whence

$$f_n\, dh = \frac{E\, \Delta_n\, dh}{ds},$$

and therefore

$$N = \int f_n dh = \int \frac{E\, \Delta_n\, dh}{ds}.$$

From Fig. 37,

$$ds = (r_n + h)d\phi.$$

Therefore,

$$N = \frac{E\Delta_n}{d\phi}\int_{-\left(\frac{t}{2}-c\right)}^{\frac{t}{2}+c} \frac{dh}{r_n + h} = \frac{E\Delta_n}{d\phi}\log_e \frac{R}{R-t}. \tag{178}$$

E, Δ_n, and $d\phi$ are all constant in this equation, and it follows that

$$\frac{E\Delta_n}{d\phi} = \frac{N}{\log_e \frac{R}{R-t}}.$$

Utilizing Hooke's law, it will be seen from Fig. 37 that

$$f_n = \frac{E\Delta_n}{ds} = \frac{E\Delta_n}{(r_n + h)d\phi},$$

whence

$$\frac{E\Delta_n}{d\phi} = (r_n + h)f_n.$$

Substituting this value of $\frac{E\Delta_n}{d\phi}$ in Eq. (178), and solving for f_n, it is found that

$$f_n = \frac{N}{(r_n + h)\log_e \frac{R}{R-t}}. \tag{179}$$

If $h = \frac{t}{2} + c$, $r_n + h = R$, and $f_n = f_e$, the stress at the extrados, then from Eqs. (179) and (172),

$$f_e = \frac{N}{R\log_e \frac{R}{R-t}} = \frac{Nr_n}{Rt}. \tag{180}$$

Also, if $h = -\left(\frac{t}{2} - c\right)$, $r_n + h = R - t$, and $f_n = f_i$, the stress at the intrados, then from Eqs. (179) and (172),

$$f_i = \frac{N}{(R-t)\log_e \frac{R}{R-t}} = \frac{Nr_n}{(R-t)t}. \tag{181}$$

11-16. Moment Compressive-stress Formulas for Thick Arch Rings.—In paragraph 11–12, it was shown that the bending stress on the elementary area dh in Fig. 37 is

$$f_m dh = \frac{E\,da\,h\,dh}{d\phi(r_n + h)}. \tag{182}$$

Since

$$M = \int f_m h\,dh,$$

$$M = \frac{E\,da}{d\phi\,r_n}\int_{-\left(\frac{t}{2}-c\right)}^{\frac{t}{2}+c} \frac{r_n h^2 dh}{r_n + h}. \tag{183}$$

The integral expression is approximately the moment of inertia I_n of the cross-section. Also since the neutral axis is not far from the central axis in practical dam design, it follows that I_n may be taken equal to I with an error of not more than 1 per cent for all ratios of thickness to mean radius of arch rings that will be encountered in practice. Therefore,

$$M = \frac{E\,da\,I}{d\phi\,r_n}. \tag{184}$$

Changing the form of Eq. (184),

$$\frac{E\,da}{d\phi} = \frac{M\,r_n}{I}.$$

Also, from Eq. (182),

$$\frac{E\,da}{d\phi} = \frac{f_m(r_n + h)}{h}.$$

Therefore,

$$\frac{f_m(r_n + h)}{h} = \frac{Mr_n}{I},$$

and

$$f_m = \frac{M\,r_n h}{(r_n + h)I}. \tag{185}$$

When

$$h = \frac{t}{2} + c,\ r_n + h = R,$$

and the stress at the extrados is

$$f_m = \frac{Mr_n\left(\frac{t}{2} + c\right)}{RI}. \tag{186}$$

When

$$h = -\left(\frac{t}{2} - c\right),\ r_n + h = R - t,$$

and the stress at the intrados is

$$f_m = -\frac{Mr_n\left(\frac{t}{2} - c\right)}{(R - t)I}. \tag{187}$$

11-17. Combined Compressive-stress Formulas for Thick Arch Rings.—By combining Eq. (180) with (186), the total compressive stress at the extrados is found to be

$$f_e = \frac{Nr_n}{Rt} + \frac{Mr_n\left(\frac{t}{2} + c\right)}{RI}. \tag{188}$$

Also combining Eq. (181) with (187), the total compressive stress at the intrados is found to be,

$$f_i = \frac{Nr_n}{(R - t)I} - \frac{Mr_n\left(\frac{t}{2} - c\right)}{(R - t)I}. \tag{189}$$

11-18. Unit Shearing-stress Formulas for Thick Arch Rings.—The intensity of shear at any point in the cross-section of a thick arch ring can be found by the same method as used in paragraph 11-14 for thin arch rings. As in that case the intensity of stress is greatest at the neutral axis (which there coincided with the central axis) and decreases to zero at the edges. The development of the formula for thick arch rings will not be given here, as shear in arch rings is of relatively minor importance and the shearing stresses can be found with sufficient accuracy by use of the formula for thin arch rings.

11-19. Thrust, Moment, and Shear in Arch Rings.—The stress formulas given in this chapter involve the thrust, moment, and shear in arch rings of the various types

considered in this work. Each of these types will be discussed in the following chapters and the general formulas for computing thrust, moment, and shear in them will there be developed. Having thus computed these values, the stresses developed in the arch ring can be obtained from the formulas given in this chapter.

TRIAL-LOAD METHOD OF ARCH-DAM ANALYSIS

11-20. Explanation of Trial-load Method.—The United States Bureau of Reclamation has developed a method of arch-dam analysis known as the *trial-load method* that is worthy of note here. This method is based on the assumption that an arch dam is composed of a series of vertical cantilever beams and horizontal arches, which are assumed to carry the water and temperature loads distributed between them on the basis of equal radial deflections at common points. In the computations, proportionate parts of the load are first assumed to be carried by the beams and the arches, and the corresponding deflections at the common points are found. Discrepancies in the deflections are noted, estimated changes in the load distribution are made, and the deflections are again computed. This process is repeated a sufficient number of times to obtain reasonable agreement in the computed deflections. The resulting stresses are then computed from this final load distribution. The earlier development of the trial-load method was presented by Howell and Jaquith in a paper entitled "Analysis of Arch Dams by the Trial Load Method,"[1] to which article the reader is referred. The method presented in that article did not take into consideration the torsional and shearing effects, thus neglecting important factors in the analysis, as was brought out in the discussions of the article.[2]

In a more recent development of this method, torsional stresses in the beams and arches, shearing stresses in horizontal sections and corresponding shearing stresses in the vertical radial sections, and the effect of Poisson's

[1] *Trans. Am. Soc. Civil Eng.*, Vol. 93, pp. 1191–1225.
[2] *Trans. Am. Soc. Civil Eng.*, Vol. 93, pp. 1246–1250.

ratio are considered. In addition to the load adjustments to secure equality of deflections, therefore, the recent development requires adjustments of loads for torsion, shear, and Poisson's ratio. This new development has recently been published by Westergaard.[1]

11-21. Stresses in Arch Ring under Variable Water Load.—The procedure of arch-dam analysis used in this work is based on an assumed uniform water load on each horizontal arch ring. If, however, part of the load is assumed to be carried by vertical cantilevers as required by the trial-load method, the water load can no longer be considered uniformly distributed over the arch ring, and consequently the values of N_0, M_0, and V_0 in Chaps. XII, XIV, and XVI, must be found by summation instead of by integration. In this summation the water load is to be taken as constant on the water face of any voussoir, but is to be varied from voussoir to voussoir in the arch ring to suit the requirements of the assumed load distribution between the cantilevers and the arch ring. With this change made the procedure for the arch stresses is otherwise unaltered.

The stresses in the cantilevers are determined by utilizing the part of the load assumed to be carried by them, in addition to the weight of the concrete, the vertical water load, and the negative load produced by the resistance of the arches at the top of the dam to the downstream deflections of the cantilevers. Adjustments and recomputations are then made, if necessary, as discussed in the above paragraph, until the deflections of the arches and cantilevers are found to be approximately equal at all points.

Although the trial-load method of arch-dam analysis as now developed has many meritorious features, yet, on account of the uncertainty of some of the underlying assumptions and of the lack of mathematical precision, it is not given further treatment in this work.

USE OF MODELS IN ARCH-DAM DESIGN

11-22. Value of Models.—There is a growing use of models in the solution of hydraulic problems and as an aid in the design of dams. The chief value of model tests in

[1] *Eng. News-Record*, Vol. 106, pp. 141–143, Jan. 22, 1931.

dams is for the checking of stresses and deflections after the design based on theoretical considerations is complete. This is particularly important in the case of dams wherein the water load is carried partly by arch action and partly by gravity action. The United States Bureau of Reclamation has made extensive use of models in the design of the Boulder, Norris, and Grand Coulee dams.

11-23. Model Theory.—As set forth by E. D. Smith,[1] the general requirements for dam models and the relationships between model and prototype may be stated as follows:

1. One of the important requirements of a structural model of a dam is that, upon application of the loads, the resulting strains and deformations must be of magnitudes susceptible of measurement with available laboratory equipment. Since the model must necessarily be constructed to a fairly small scale, it will ordinarily require a higher specific gravity of loading or much less stiffness than the prototype's in order to comply with this requirement. Often it is necessary to combine both these properties.

2. The model must be a true scalar representation of the prototype.

3. The loading of the model must be proportional to the loading of the prototype.

4. If the model represents a massive structure where the stress distribution is influenced by the volume changes occurring under strain, Poisson's ratio must be the same for model and prototype. However, if only a cross-section of the structure is being investigated, giving a model under two-dimensioned stress, the stress distribution is independent of Poisson's ratio.

5. If the effect of both live load and gravity forces is to be investigated, the ratio of specific gravities of the dam and loading medium must be the same for both model and prototype. If the effect of external forces only is to be investigated, the results are not affected by the specific gravity of the model, provided that the action of the structure is elastic.

[1] "Dams and Control Works," 2d ed., U. S. Department of Interior, Bureau of Reclamation, February, 1938.

6. The model material must be homogeneous and isotropic and must obey Hooke's law.

7. Foundations and abutments for the model must be sufficiently extensive to allow it freedom to deform in a manner similar to the prototype.

If these requirements are fulfilled, the ratios between stresses and deformations in the dam and in the model will be as follows:

With a scale ratio between model and prototype equal to $1:n$ and a specific gravity of loading for the model equal to G times that of water, the unit liquid pressure at the base of the dam will be n/G times that for the model. This ratio will also apply to stress conditions. If the modulus of elasticity of the material in the dam is equal to E_d and in the model is equal to E_m, the ratio between the unit strain in the model and in the dam will be the ratio of the unit stresses multiplied by the ratio of the two moduli of elasticity, or nE_m/GE_d. These strains are acting over lengths for the dam n times the corresponding lengths for the model. The total deformation for the dam expressed as the deflection at a certain point will therefore be n times the ratios of the unit strains compared with the total deformation for the model. The ratio between the deflections of the dam and the deflection of the model will be n^2E_m/GE_d.

11-24. Materials Used in Models.—In order to satisfy the requirements as set forth in the preceding paragraph, it is necessary to choose carefully the material of which the model is to be constructed. If the model is relatively massive, the use of a hard material such as concrete will result in such small deflections as to be difficult of measurement and subject to relatively large errors. Thin models, if to proper scale, can be successfully constructed of concrete, with the advantage of similarity of material in model and prototype. Other materials used are rubber, celluloid, and plaster mixed with "Celite." For loading the model, water may be used; or if higher pressures are desired, mercury may be substituted.

CHAPTER XII

SYMMETRICAL THIN ARCH RINGS OF VARIABLE THICKNESS

INTRODUCTION

12-1. General Statement.—As stated in Chap. IX, economy of material in an arch dam requires the thickening of the arch rings towards the abutments. Consequently, an analysis of the stresses in rings of variable thickness is of first importance in the design of arch dams. This type of ring also lends itself admirably to thickening by compounding the introdosal curve through the use of shorter radii for the arcs adjacent to the abutments. A three-centered intradosal curve is generally sufficient for the purpose, but multicentering may readily be applied. Through shifting of the crown section of the arch rings, adjustment of central angles, and other manipulations, the arch ring of a dam can generally be made symmetrical, and, as a result, the computations of the stresses in the rings are greatly simplified. Formulas will, therefore, first be developed for symmetrical rings of the type under consideration in this chapter.

12-2. Coordinates, Signs, and Symbols.—Let Fig. 39 represent the left half, 1 ft. high, of a symmetrical and symmetrically water-loaded thin arch ring with a circular extrados, a compound circular intrados, and a compound circular line drawn between them as a neutral axis. Assume the origin of rectangular coordinates and redundant forces to be placed cn the axis of symmetry of the arch at the elastic center, a distance, k, from the center of the crown, with the y-axis coincident with the axis of symmetry. Let the values of x and of all angles be taken positive towards the left, and the values of y positive on the crown side of the x-axis and negative on the opposite side. Thrusts are considered positive when they act toward the section under considera-

tion, that is, tend to cause compression in the arch ring, moments positive when they tend to cause counterclockwise motion, and shears positive when acting toward the extrados. The redundant forces will be considered positive when acting as shown in Fig. 39. Except as hereinafter designated, the symbols used will be those defined in Chap. XI.

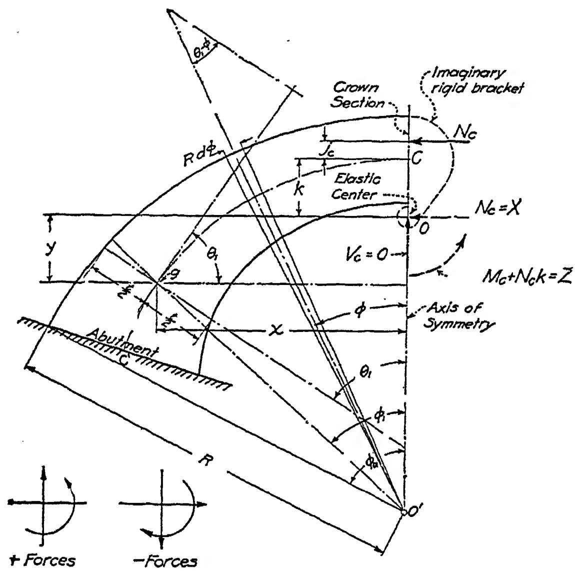


FIG. 39.—Diagram for analysis of compound circular arch ring.

DETERMINATIONS OF REDUNDANT FORCES

12-3. Removal of Redundant Member.—The whole arch ring, of which Fig. 39 represents the left half, forms a statically indeterminate structure with the redundant member consisting of the right half of the arch ring. First, consider this redundant member removed and equilibrium maintained by a single force N_c, acting normally to the crown section at the point P, distant j_c above the center line CC' of the ring. By placing two oppositely acting forces, each equal to and parallel to N_c at the point C, the effect of the single force N_c becomes equivalent to an equal horizontal force N_c, acting at C, and a moment force

M_c equal to $N_c j_c$ tending to produce counterclockwise motion about C.

12-4. Substitution of Bracket Support.—Second, let the crown of the semiarch be connected to the elastic center of the entire arch ring by means of an infinitely rigid bracket, and let the support N_c and M_c of the arch at the crown be removed and replaced by the force X and the moment Z at the elastic center. Now $X = N_c$ and $Z = M_c + N_c k$. This can be seen by considering two forces each equal to and parallel to N_c, applied at O and acting in opposite directions. One of these forces will give a force acting along the x-axis at O equal to N_c and the other force will form with N_c at the crown, a couple acting counter-clockwise, whose arm is k and the numerical value of which is $N_c k$. Therefore, the moment of Z about the point O will be equal to the moment M_c at the crown, plus the moment of this new couple. The arch will still be in equilibrium with the application of these two new forces in place of the forces at the crown.

12-5. Use of Residual Cantilever.—Third, let the redundant forces X and Z be removed and the left half of the arch ring be loaded with the water pressure and swung as a cantilever from the left abutment. Then the bracket attached to the crown section will be deflected a distance, Δ', towards the right in opposition to the action of the former force X and rotated clockwise through an angle a' in opposition to the action of the former moment Z and it will require forces equivalent to X and Z to bring the bracket back to its original position.

12-6. Application of Unit Dummy Loads.—Fourth, consider the water load removed and a dummy unit load and a dummy unit couple applied at the point of application of and acting in the direction, respectively, of X and Z, which will deflect the bracket a linear distance Δ_1, and an angular distance of a_1. Now the sum of the deflections due to the water load and the deflections due to the dummy unit loads and redundants must equal zero, that is, $\Delta' + X\Delta_1 = 0$ and $a' + Za_1 = 0$.

Therefore,

$$X = -\frac{\Delta'}{\Delta_1}, \; and \; Z = \frac{-a'}{a_1}. \tag{190}$$

Also, since the elastic center is at the center of gravity of the arch ring,

$$k = \frac{\int \frac{y\,ds}{I}}{\int \frac{ds}{I}}. \tag{191}$$

12-7. Deflections for Horizontal Redundant.—Now the linear deflections Δ' and Δ_1 may be computed from Eq. (169) and their values substituted in the right member of Eq. (190). It is best to omit the temperature term in Eq. (169) at this point, and later to consider it separately. In Eq. (169), it should be observed that for Δ', $N = N_0$, $M = M_0$, and $V = V_0$, the respective thrust, moment, and shear due to water load acting on the base system, and $n = n_x = \cos\ \theta_1$, $m = m_x = -y$, and $v = v_x = \sin\ \theta_1$ for that system. Again in Eq. (169), for Δ_1, $N = n_x = \cos\ \theta_1$, $M = m_x = -y$, and $V = v_x = \sin\ \theta_1$, the respective thrust, moment, and shear due to the dummy unit load acting on the base system, and as before $n = n_x = \cos\ \theta_1$, $m = m_x = -y$, and $v = v_x = \sin\ \theta_1$ as with Δ_1, since the dummy unit load acts in the direction of the redundant force. Substituting the values of Δ' and Δ_1 in the expression for X in Eq. (190), noting that $CE/E' = 3$, and canceling common terms, the following value of the redundant crown thrust is found,

$$X = \frac{-\int_0^s N_0 \cos\ \theta_1 \frac{ds}{A} + \int_0^s M_0 y \frac{ds}{I} - 3\int_0^s V_0 \sin\ \theta_1 \frac{ds}{A}}{\int_0^s \cos^2\ \theta_1 \frac{ds}{A} + \int_0^s y^2 \frac{ds}{I} + 3\int_0^s \sin^2\ \theta_1 \frac{ds}{A}}. \tag{192}$$

12-8. Deflections for Angular Redundant.—The angular deflections a' and a_1 may also be computed from Eq. (169) and their values substituted in the right member of the

expression for Z in Eq. (190). In applying Eq. (169) to angular movements, it should be observed that temperature change is temporarily neglected and that the thrust and shear terms in the equation are reduced to zero because a unit couple cannot produce any net thrust or net radial shear. In the moment term of the formula it should be observed that for a', $M = M_0$, the moment due to water load acting on the base system, and $m = m_z = 1$, $n = 0$, and $v = 0$ for that system. Again in Eq. (169), for a_1, $M = m_z = 1$, the moment due to the dummy unit load acting on the base system; and $m = m_z = 1$ in both cases, since the dummy unit load acts in the direction of the redundant force. Substituting the values of a' and a_1 thus determined in Eq. (190), the following value of the elastic center moment is found,

$$Z = -\frac{\int_0^s M_0 \frac{ds}{I}}{\int_0^s \frac{ds}{I}}. \tag{193}$$

Also since neither semiarch can transmit net shear to the other, because both the arch shape and loading are symmetrical about the crown section, therefore,

$$Y = V_c = 0. \tag{194}$$

EQUATIONS FOR THRUST, MOMENT, AND SHEAR

12-9. Stresses in Cantilever Due to Water Load.—Considering the cantilever under water load again unsupported by the redundant forces, the thrust, moment, and radial shear at any point g due to the water load to the right thereof may readily be determined from Fig. 39 to be as follows:

$$N_0 = \int_0^{\phi_1} pR\, d\phi \cdot \sin(\theta_1 - \phi) = pR[\cos(\theta_1 - \phi_1) - \cos\theta_1]; \tag{195}$$

$$M_0 = \int_0^{\phi_1} pR\ d\phi \cdot \left(R - \frac{t}{2}\right) \sin\ (\phi_1 - \phi) = -pR\left(R - \frac{t}{2}\right) [1 - \cos \phi_1]; \quad (196)$$

$$V_0 = \int_0^{\phi_1} -pR\ d\phi \cdot \cos\ (\theta_1 - \phi) = pR[\sin\ (\theta_1 - \phi_1) - \sin \theta_1]. \quad (197)$$

12-10. Stresses on Arch Ring Due to Water Load and Redundants.—Now the thrust, moment, and shear at any point g of the arch ring are those due to the water load and the redundant forces at the elastic center, resolved onto the radial section through the point g. Whence $N = N_0 + Xn_x + Yn_y + Z n_z$, $M = M_0 + Xm_x + Ym_y + Zm_z$, and $V = V_0 + Xv_x + Yv_y + Zv_z$; and since $n_x = \cos \theta_1$, $n_y = \sin \theta_1$, $n_z = 0$, $m_x = -y$, $m_y = x$, $m_z = 1$, $v_x = \sin \theta_1$, $v_y = \cos \theta_1$, and $v_z = 0$; and since $Y = 0$ due to the symmetry of the arch ring and its loading, it follows that,

$$N = N_0 + \mathrm{X} \cos \theta_1; \quad (198)$$

$$M = M_0 - \mathrm{Xy} + Z; \quad (199)$$

$$V = V_0 + \mathrm{X} \sin \theta_1. \quad (200)$$

In applying these equations, it should be noted that N_0 is positive and that M_0 and V_0 are negative. If the arch ring has a variable thickness, the integral signs in Eqs. (192) and (193) should be changed to summation signs, and the semiarch ring should be divided into a convenient number of voussoirs by radial lines through the neutral axis. The voussoirs may be taken of any convenient lengths consistent with accuracy of summation, or they may be made such that $ds \div I$ or $12\ ds \div t^3$ is constant. The elastic center is then located on the axis of symmetry of the arch ring so that, respectively,

$$\int \frac{y\ ds}{I} \text{ or } \int y = 0,$$

the summation being taken for the values of y measured to the voussoir centers on the neutral axis. All centers, angles, and coordinates of the divisions are drawn to scale, the angles and coordinates scaled or computed, and the redundant and stress forces computed. N, M, and V having been determined for the various voussoirs, the value of f_n, the unit compressive stress, can be obtained by means of Eq. (174), and f_v, the unit shear, can be obtained from Eq. (177).

12-11. Stresses Due to Temperature Changes.—Formulas for stresses due to uniform temperature changes for arch rings of the character here discussed are found in a manner similar to those due to water load. The stresses may be determined by application of Eq. (190), substituting therein the value of Δ' and Δ_1 computed from Eq. (169) as in the case for water load. In computing Δ', the redundant forces are removed and the cantilever load is taken as that due to temperature only; and, therefore, the first three terms of the right-hand member of Eq. (169) become zero, whence

$$\Delta' = \int_0^s ncT\, ds.$$

A rise of temperature will elongate the cantilever and deflect the elastic center O in Fig. 39 an amount Δ' towards the right. To find this deflection apply a dummy unit load at O acting along XX towards the right. The deflection of O due to the deformation in the elementary length ds will be equal to $n_x cT\, ds$; but, since the unit force acts toward the right, it will produce tension or negative compression in any cross-section of the length ds, whence, for the point g,

$$n_x = -1 \cdot \cos \theta_1 = -\cos \theta_1.$$

Therefore,

$$\Delta' = -\int_0^s \cos \theta_1\, cT\, ds.$$

To restore the point O to its original position, the redundant force X must be employed. Consider the temperature load

removed and apply a dummy unit load at O acting to the left along XX, then in Eq. (169) the last term becomes zero, and since $N = n = n_x = \cos \theta_1$, $M = m = m_x = -y$, and $V = v = v_x = \sin \theta_1$, the first three terms thereof will become equal to

$$\int_0^s \frac{\cos^2 \theta_1 ds}{AE} + \int_0^s \frac{y^2 ds}{EI} + 3\int_0^s \frac{\sin^2 \theta_1 ds}{AE}.$$

Substituting these values of Δ' and Δ_1 in Eq. (190), the value for $N_c = X$ becomes,

$$X = \frac{-\int_0^s \cos \theta_1 \, cT \, ds}{\int_0^s \frac{\cos^2 \theta_1 ds}{AE} + \int_0^s \frac{y^2 ds}{EI} + 3\int_0^s \frac{\sin^2 \theta_1 ds}{AE}} \tag{201}$$

In this equation, T is considered positive for a temperature rise and negative for a temperature fall.

Then by applying Eqs. (198), (199), and (200), the thrust, moment, and shear for temperature changes may be computed by noting that N_0, M_0, V_0, and Z are equal to zero. Thus,

$$N = X \cos \theta_1; \tag{202}$$

$$M = -Xy; \tag{203}$$

$$V = X \sin \theta_1. \tag{204}$$

12-12. Eccentricity of Thrust.—The thrust N, as found at any point of the arch ring from Eq. (198) or (202) meets the cross-section of the arch ring at a radial distance j from the center of the ring and, therefore, $Nj = M$, or $j = M \div N$. If j, as found, is less than $t/6$, there will be no tension in the arch ring at the section due to the forces producing N and M at that section. The critical points will generally be at the crown and the abutments, the eccentricity usually being a maximum at these three points. The divergence of the thrust is above the center at the crown and crosses the central axis between the crown and the abutments.

12-13. Deflection at Crown.—Equation (169) can be used to find the deflection at the crown due to the loads on the semiarch ring. Assume the arch ring to be loaded with the water pressure and redundants X and Z as shown in Fig. 39. If these loads are removed the crown of the arch will rise a definite amount and if a dummy unit load is applied at O acting downward along YY, it will cause at O a deflection due to the thrust n_y, moment m_y, and shear v_y on the section through any point g. It will be observed that $n_y = \sin\ \theta_1$, $m_y = -x$, and $v_y = -\cos\ \theta_1$. Substituting these values in Eq. (169), the following equation results,

$$\Delta = \int_0^s \frac{N \sin \theta_1 ds}{AE} - \int_0^s \frac{Mx\ ds}{EI} - 3\int_0^s \frac{V \cos \theta_1 ds}{AE}. \quad (205)$$

By substituting the values of N, M, and V from Eqs. (198), (199), and (200), and integrating over the semiarch, the deflection due to the water load is found. To determine the deflection due to temperature changes or equivalent shrinkage or swelling, use the values of N, M, and V from Eqs. (202), (203), and (204).

12-14. Another View of the Dummy Unit Load Method. Assume all conditions as described in paragraphs 12–3 and 12–4 and as shown in Fig. 39. Assume a unit load applied at O acting along XX to the left and a unit couple acting counterclockwise. Now the unit load along XX will tend to deflect the point O to the left along XX against the action of the water load an amount Δ_1 and the redundant X times Δ_1 will prevent a potential deflection Δ' in the opposite direction due to water load. Therefore,

$$\Delta' + X\Delta_1 = 0.$$

Similarly, the unit couple will tend to deflect the point O angularly counterclockwise against the action of the water load an amount a_1 and the redundant Z times a_1 will prevent the potential deflection a' in the opposite direction due to the water load, therefore,

$$a' + Za_1 = 0.$$

Now the thrust, moment, and shear on any section of the arch ring such as that through the point g in Fig. 39 due to the unit load along XX are, respectively, $n_x = N = \cos\theta_1$, $m_x = M = -y$, and $v_x = V = \sin\theta_1$, and the potential deflections of O for an elementary length ds of the arch ring is, from Eq. (169),

$$\Delta_1 = \int_0^s \frac{\cos^2\theta_1 ds}{AE} + \int_0^s \frac{y^2\,ds}{EI} + \int_0^s \frac{\sin^2\theta_1 ds}{AE}.$$

Also the thrust, moment, and shear on the section through g due to the water load on the arch ring to the right of g are, respectively, N_0, M_0, and V_0, and the potential deflections of O due to them for the elementary length ds of the arch ring are those due to the unit load and N_0, M_0, and V_0, with proper consideration of signs. When the potential deflections due to the unit load and forces act in the same direction, the total potential deflections are positive and when in opposite directions, these deflections are negative. Therefore, Eq. (169) gives

$$\Delta' = \int_0^s \frac{N_0 \cos\theta_1 ds}{AE} - \int_0^s \frac{M_0 y\,ds}{EI} + 3\int_0^s \frac{V_0 \sin\theta_1 ds}{AE}.$$

Substitution of the values Δ' and Δ_1 just computed in Eq. (190) gives Eq. (192).

In considering the thrust, moment, and shear on the section through g due to the unit couple acting at O, it will be observed that $n_z = 0$ and $v_z = 0$, since the unit couple can produce no thrust or shear at any point in the arch ring and that it will produce a moment $m_z = 1$ in a counterclockwise direction. The potential deflection produced at O by the unit couple for an elementary length ds of the arch ring is, therefore, from Eq. (169),

$$a_1 = \int_0^s \frac{ds}{EI}.$$

Also the potential deflection due to the moment M_0 on the section through g due to the water load is

$$-\int_0^s M_0 \cdot \frac{ds}{EI},$$

Voussoir	t	ds	$\frac{ds}{I}$	a	$a\frac{ds}{I}$	y	$y\frac{ds}{I}$	$y^2\frac{ds}{I}$	$\frac{ds}{A}$	θ_1	ϕ_1	$\theta_1-\phi_1$	$\cos\theta_1$	$\sin\theta_1$	$\cos(\theta_1-\phi_1)$	$\sin(\theta_1-\phi_1)$	$\cos\phi_1$	Voussoir
1	46.0	0	0	+43.2	0	+31.15	0	0	0	0	0	0	1.00000	0	1.00000	0	1.00000	1
2	46.1	25.0	.00306210	+43.1	+.131977	+31.05	+.095078	2.95217	.54230	3°-06'	2°-48'	0°-18'	.99854	.05408	.99999	.00524	.99881	2
3	46.8	34.4	.00402720	+39.8	+.160283	+27.75	+.111755	3.10120	.73504	10°-20'	9°-27'	0°-53'	.98378	.17937	.99988	.01542	.98643	3
4	48.1	"	.00370939	+31.0	+.114991	+18.95	+.070293	1.33205	.71518	18°-40'	17°-07'	1°-33'	.94740	.32006	.99963	.02705	.95571	4
5	50.5	"	.00320527	+17.9	+.057374	+5.85	+.018751	0.10969	.68119	27°-02'	24°-51'	2°-11'	.89074	.45451	.99927	.03810	.90741	5
6	53.5	"	.00269575	0	0	-12.05	-.032484	0.39143	.64299	35°-24'	32°-36'	2°-48'	.81513	.57928	.99881	.04885	.84245	6
7	58.3	"	.00208322	-22.0	-.045831	-34.05	-.070934	2.41530	.59005	43°-47'	40°-27'	3°-20'	.72196	.69193	.99831	.05814	.76097	7
8	61.7	34.1	.00174213	-47.5	-.082751	-59.55	-.103744	6.17796	.55267	52°-11'	48°-19'	3°-52'	.61314	.78998	.99772	.06743	.66501	8
9	65.0	25.1	.00105725	-71.9	-.076016	-83.95	-.088756	7.45107	.38146	59°-25'	55°-11'	4°-14'	.50879	.86089	.99727	.07382	.51095	9
10	68.5	0	0	-83.0	0	-95.05	0	0	0	62°-34'	58°-02'	4°-32'	.46072	.88755	.99687	.07904	.52943	10
TOTALS			.02158231	+12.05	+.260027		-.000041	+23.93087	Water depth 150 ft. p=9,375 lbs./sq.ft. R=279 ft. pR=2,615,625									

Voussoir	$\cos(\theta_1-\phi_1)-\cos\theta_1$	N_0	$\sin(\theta_1-\phi_1)-\sin\theta_1$	V_0	$R-\frac{t}{2}$	$1-\cos\phi_1$	$(R-\frac{t}{2})\times(1-\cos\phi_1)$	$\frac{M_0}{1,000}$	$\frac{ds}{A}\cos\theta_1$	$\frac{ds}{A}\cos^2\theta_1$	$\frac{ds}{A}\sin\theta_1$	$\frac{ds}{A}\sin^2\theta_1$	$N_0\frac{ds}{A}\cos\theta_1$	$V_0\frac{ds}{A}\sin\theta_1$	$M_0\frac{ds}{I}$	$yM_0\frac{ds}{I}$	Voussoir
1	0	0	0	0	256	0	0	0	0	0	0	0	0	0	0	0	1
2	.00145	3,793	-.04884	-127,747	255.95	.00119	.30458	-797	.54151	.54072	.02933	.00159	2,054	-3,747	-2,440	-75,762	2
3	.01610	42,112	-.16395	-428,832	255.60	.01357	3.46849	-9,072	.72312	.71139	.13184	.02365	30,452	-56,537	-36,535	-1,013,846	3
4	.05223	136,614	-.29301	-766,404	254.95	.04429	11.29774	-29,535	.67756	.64192	.22890	.07326	92,564	-175,430	-109,557	-2,076,105	4
5	.10853	283,874	-.41641	-1,089,172	253.75	.09259	23.49471	-61,453	.60676	.54047	.30961	.14072	172,243	-337,219	-196,973	-1,152,292	5
6	.18368	480,438	-.53043	-1,387,406	252.25	.15755	39.74199	-103,950	.52412	.42723	.37247	.21576	251,807	-516,767	-280,223	+3,376,687	6
7	.27635	722,828	-.63379	-1,657,757	249.85	.23903	59.72165	-156,209	.42599	.30755	.40827	.28249	307,917	-676,812	-325,418	+11,080,483	7
8	.38458	1,005,917	-.72255	-1,889,920	248.15	.33499	83.12777	-217,431	.33886	.20777	.43660	.34491	340,865	-825,139	-378,793	+22,557,123	8
9	.48848	1,277,681	-.78707	-2,058,680	246.10	.42905	105.58921	-276,182	.19408	.09875	.32840	.28272	247,972	-676,071	-291,993	+24,512,812	9
10	.53615	1,402,367	-.80851	-2,114,759	244.75	.47057	115.17201	-301,247	0	0	0	0	0	0	0	0	10
TOTALS										+3.47580		+1.36510	+1,445,874	-3,267,722	-1,621,932	+57,209,100	

Voussoir	$X\cos\theta_1$	N	Xy	M	$\frac{M}{N}$	$X\sin\theta_1$	V	$\frac{144\,t^2}{6}$	M_s lbs/□"	N_s lbs/□"	V_s lbs/□"	M_s+N_s lbs/□" Extrados	M_s+N_s lbs/□" Intrados	Voussoir
1	2,081,343	2,081,343	+64,833,834	+10,317,158	+4.96	0	0	50,784	+203	314	0	+517	+111	1
2	2,078,304	2,082,097	+64,625,700	+9,728,209	+4.67	112,582	-15,165	51,002	+191	314	-3	+505	+123	2
3	2,047,584	2,089,696	+57,757,268	+8,321,641	+3.98	373,408	-55,424	52,564	+158	310	-12	+468	+152	3
4	1,971,864	2,108,478	+39,441,450	+6,174,459	+2.93	666,293	-100,111	55,524	+111	304	-21	+415	+193	4
5	1,853,935	2,137,809	+12,175,857	+1,522,052	+.71	946,188	-142,984	61,206	+25	294	-30	+319	+269	5
6	1,696,565	2,177,003	-25,080,183	-3,718,908	-1.71	1,205,931	-181,475	68,694	-54	283	-36	+229	+337	6
7	1,502,646	2,225,474	-70,869,729	-10,188,362	-4.58	1,440,443	-217,314	81,572	-125	265	-39	+140	+390	7
8	1,276,155	2,282,072	-123,943,976	-18,336,115	-8.03	1,644,561	-245,359	91,368	-201	257	-42	+56	+458	8
9	1,058,967	2,336,648	-174,728,745	-26,302,346	-11.26	1,792,180	-266,500	103,909	-253	247	-42	-6	+500	9
10	958,916	2,388,283	-197,831,652	-30,325,439	-12.70	1,847,680	-267,079	112,614	-269	242	-41	-27	+511	10

$$X = \frac{-1,445,874 + 57,209,100 + 9,803,166}{3.47580 + 23.93087 + 4.09530} = 2,081,343$$

$$Z = \frac{+1,621,932}{.02158231} = 75,150,992$$

FIG. 40.—Computation of stresses in symmetrical compound circular arch under water loading. (See Fig. 51 for layout of arch ring.)

which has opposite action to the potential deflection due to the unit couple, and therefore

$$a' = +\int_0^s \frac{M_0\, ds}{EI}.$$

By substitution of the values of a' and a_1 in Eq. (190), Eq. (193) is obtained.

12-15. Recording Computations.—Owing to the number and importance of the quantities involved in the stress analysis given in this chapter, it is essential that a systematic record be kept of the computations as the work progresses. Figure 40 provides a suitable form for this purpose, together with the complete computations for determining the stresses in a symmetrical thin arch ring of variable thickness. Figure 51 in Chap. XVII shows the layout of the arch ring the computations for which are here presented.

12-16. Earthquake Stresses Due to Mass in Symmetrical Thin Arch Rings of Variable Thickness.—For a discussion and development of formulas relating to this subject see Appendix III.

*

CHAPTER XIII

SYMMETRICAL SINGLE-CENTERED THIN ARCH RINGS OF UNIFORM THICKNESS

INTRODUCTION

13-1. General Statement.—The symmetrical single-centered thin arch ring of uniform thickness is used extensively for low arch dams and for the top rings of high arch dams.

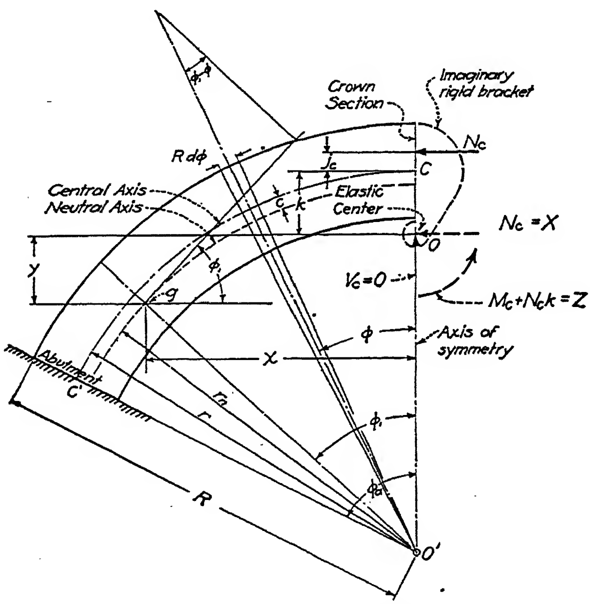


FIG. 41.—Diagram for analysis of symmetrical single-centered thin circular arch ring of uniform thickness.

This type is also used for preliminary layouts for arch dams with rings of variable thickness or with rings of uniform thickness and variable central angles. Formulas for thrust, moment, and shear for this type of arch ring have, therefore, wide use in arch-dam design.

13-2. Coordinates, Signs, and Symbols.—Let Fig. 41 represent the left half, 1 ft. high, of a single-centered thin arch ring of uniform thickness and let it be a duplicate of Fig. 39, except as to thickness of the ring and single-centered intradosal curve. The coordinates, signs, and symbols have the same significance as in the two preceding chapters and the formulas developed in these chapters may be applied to this type of arch ring by introducing into them the conditions of uniform thickness and single-centered intradosal curve.

EQUATIONS FOR THRUST, MOMENT, AND SHEAR

13-3. Stresses in Cantilever Due to Water Load.—Express N, M, and V in Eqs. (198), (199), and (200) for any point (r, ϕ_1) in terms of known geometric and trigonometric quantities. The quantities in these equations requiring such expressions are N_0, M_0, V_0, y, X, Z, ds, and θ_1. For the case here assumed, $pR = p'r$, $R - \dfrac{t}{2} = r$, $ds = r\,d\phi$, and $\theta_1 = \phi_1$ for interior points of the arch ring, and $\theta_1 = \phi_1 = \phi_a$ at the abutment. Substituting the values of $p'r$ for pR, and ϕ_1 for θ_1 in Eqs. (195), (196), and (197), the following values are obtained:

$$N_0 = p'r(1 - \cos\phi_1) = p'r(1 - \cos\phi_a) \text{ at the abutment;} \tag{206}$$

$$M_0 = -p'r^2(1 - \cos\phi_1) = -p'r^2(1 - \cos\phi_a) \text{ at the abutment;} \tag{207}$$

$$V_0 = -p'r\sin\phi_1 = -p'r\sin\phi_a \text{ at the abutment.} \tag{208}$$

From the assumptions of the problem $I = t^3/12$, $A = t$, and $ds = r\,d\phi$; and also from Fig. 41, $x = r\sin\phi_1$ and $y = r\cos\phi_1 - h$, h being the perpendicular distance from the center of curvature onto the x-axis. Now,

$$\int_0^{\phi_a} y\frac{ds}{I} = 0,$$

since the center of coordinates of x and y is taken at the elastic center of the arch ring. Therefore,

$$\int_0^{\phi_a} \frac{y\,ds}{I} = \frac{12r}{t^3}\int_0^{\phi_a}(r\cos\phi - h)\,d\phi = \frac{12r}{t^3}[r\sin\phi_a - h\phi_a] = 0; \tag{209}$$

$$h = \frac{r\sin\phi_a}{\phi_a}; \tag{210}$$

$$y = r\cos\phi_1 - \frac{r\sin\phi_a}{\phi_a}, \tag{211}$$

for point g.

13-4. Evaluation of Redundants.—Substituting the values of N_0, M_0, V_0, y, ds, I, A, and θ_1 in the right-hand members of Eqs. (192) and (193), expanding, integrating between the limits ϕ_a and 0 and rearranging, the following values of X and Z are found:

$$X = \frac{-\left\{rt^2\sin\phi_a - \left[rt^2\left(\frac{1}{2}\phi_a + \frac{1}{4}\sin 2\phi_a\right) + 12r^3\left(\frac{1}{2}\phi_a + \frac{1}{4}\sin 2\phi_a - \frac{\sin^2\phi_a}{\phi_a}\right) + 3rt^2\left(\frac{1}{2}\phi_a - \frac{1}{4}\sin 2\phi_a\right)\right]\right\}p'r}{\left[rt^2\left(\frac{1}{2}\phi_a + \frac{1}{4}\sin 2\phi_a\right) + 12r^3\left(\frac{1}{2}\phi_a + \frac{1}{4}\sin 2\phi_a - \frac{\sin^2\phi_a}{\phi_a}\right) + 3rt^2\left(\frac{1}{2}\phi_a - \frac{1}{4}\sin 2\phi_a\right)\right]}; \tag{212}$$

$$Z = p'r^2\left(1 - \frac{\sin\phi_a}{\phi_a}\right). \tag{213}$$

13-5. Stresses on Arch Ring Due to Water Loads and Redundants.—Now substituting the values of N_0, M_0, V_0, y, N_c, M_e, and θ_1 in Eq. (198), (199), and (200), reducing and arranging terms, the following five fundamental formulas are derived:

$$p'r - N_c = \frac{Kp'rt^2}{12r^2}, \tag{214}$$

where N_c is the crown thrust;

$$K = \frac{2\sin\phi_a}{\left(1+\frac{t^2}{12r^2}\right)\left(\phi_a + \frac{\sin 2\phi_a}{2}\right) - \left(\frac{1-\cos 2\phi_a}{\phi_a}\right) + \frac{t^2}{4r^2}\left(\phi_a - \frac{\sin 2\phi_a}{2}\right)}; \tag{215}$$

$$N = p'r - (p'r - N_c) \cos \phi_1; \tag{216}$$

$$M = -r(p'r - N_c)\left(\frac{\sin \phi_a}{\phi_a} - \cos \phi_1\right); \tag{217}$$

$$V = -(p'r - N_c) \sin \phi_1. \tag{218}$$

By placing $\phi_1 = \phi_a$ in Eqs. (216), (217), and (218), the total thrust, moment, and shear at the abutments are obtained, and by placing $\phi_1 = 0$, the total thrust, moment and shear at the crown are found. In general, these equations give the thrust, moment, and shear at the point (r, ϕ_1).

The unit compressive stresses at any cross-section can be found from Eq. (174), and the unit shear from Eq. (177). It will be noted that Eqs. (216), (217), and (218) are identical (with the exception of nomenclature) with Cain's well-known formulas[1] for thrust, moment, and shear at any point (r, ϕ_1) of the central arc of the arch ring.

13-6. Use cf Tabular Values.—For use in applying the stress formulas herein given for fixed-ended, thin, symmetrical circular arches under uniform normal loads, values of K are given in Table XIV for ratios of t to r, from 0.02 to 0.68 varying by intervals of 0.02, Table XV gives the values of the trigonometric functions

$$1 - \frac{\sin \phi_a}{\phi_a}, \frac{\sin \phi_a}{\phi_a} - \cos \phi_a$$

for angles 30 to 180 deg. varying by intervals of 5 deg. The values of K are useful in finding the thrust at the crown from Eq. (215), and the trigonometric functions are useful in finding the moment at the crown and abutments from Eq. (217).

13-7. Stresses Due to Temperature Changes.—By taking the value of N_c from Eq. (201), and noting that $\theta_1 = \phi_1$ and $ds = r\,d\phi$, Eqs. (202), (203), and (204) are applicable to the case of arch rings of uniform thickness, and temperature stresses can be obtained from them for this type of arch ring.

[1] *Trans. Am. Soc. Civil Eng.*, Vol. 85, pp. 232–241.

TABLE XIV.

VALUES OF K FOR USE IN EQUATION 214 FOR THIN FIXED ENDED CIRCULAR ARCHES UNDER NORMAL LOADS

$\frac{t}{r}$	VALUES OF $2\phi_0$ = TOTAL ANGLE SUBTENDING ARCH RING														
	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	95°	100°
0	7395	5467	3257	1867	1225	840	602	430	322	242.5	188.9	146.4	116.8	93.7	76.1
.02	5751	4626	2974	1780	1166	810	583	423	318	239.7	187.0	145.4	116.1	93.3	75.8
.04	3451	3007	2207	1458	1031	739	550	400	305	232.4	181.6	142.4	114.1	92.0	74.9
.06	2251	1940	1573	1151	854	646	493	369	286	221.0	174.0	137.6	110.8	89.7	73.4
.08	1458	1307	1121	880	696	550	433	334	262.5	206.4	164.8	131.4	106.6	86.8	71.4
.10	977	925	814	677	563	457	372	295.6	238.5	190.0	154.1	124.3	101.8	83.4	69.0
.12	719	676	616	528	454	382	318.5	260.0	213.6	173.9	142.7	116.5	96.3	79.6	66.2
.14	534	514	475	421	369	320	273.2	227.0	190.9	157.7	131.3	108.6	90.6	75.5	63.3
.16	417	404	376	340	304	267.6	233.6	198.6	169.5	142.4	120.1	100.6	84.8	71.3	60.2
.18	334	323.3	304.0	279.3	253.8	226.9	201.2	173.7	150.5	128.4	109.7	92.8	79.1	67.0	57.0
.20	272	264.9	251.5	232.6	214.0	194.0	173.6	152.6	133.9	115.6	99.9	85.5	73.5	62.8	53.8
.22	228	220.3	210.5	196.8	182.4	166.7	151.1	134.3	119.1	104.1	91.0	78.6	68.2	58.8	50.7
.24	191.7	186.8	178.6	168.2	156.8	144.7	132.5	118.9	106.4	94.0	82.9	72.3	63.2	54.9	47.66
.26	164.3	159.5	153.4	145.2	136.1	126.5	116.6	105.6	95.4	85.0	75.5	66.5	58.5	51.2	44.75
.28	142.2	138.2	133.3	126.3	119.4	111.4	103.2	94.2	85.7	77.0	68.9	61.1	54.2	47.72	41.99
.30	123.8	121.0	116.7	111.1	105.1	98.8	92.0	84.5	77.3	69.9	63.0	56.1	50.2	44.50	39.37
.32	109.2	106.8	103.0	98.4	93.4	88.0	82.4	76.0	70.0	63.7	57.7	51.9	46.54	41.50	36.92
.34	96.9	94.7	91.6	87.7	83.5	78.9	74.1	68.8	63.6	58.1	53.0	47.90	43.20	38.72	34.62
.36	86.6	84.6	81.9	78.6	75.0	71.1	67.0	62.4	57.9	53.3	48.75	44.28	40.13	36.15	32.47
.38	77.7	76.0	73.7	70.9	67.4	64.4	60.8	56.9	53.0	48.9	44.95	41.01	37.33	33.77	30.47
.40	70.3	68.7	66.7	64.2	61.5	58.5	55.8	52.0	48.6	45.01	41.55	38.06	34.77	31.60	28.62
.44	58.2	57.0	55.3	53.4	51.3	48.99	46.58	43.90	41.23	38.44	35.70	32.94	30.31	27.73	25.31
.48	48.93	47.92	46.60	45.02	43.37	41.54	39.61	37.49	35.37	33.14	30.94	28.70	26.57	24.46	22.46
.52	41.75	40.89	39.79	38.52	37.15	35.65	34.08	32.37	30.64	28.82	27.02	25.21	23.43	21.68	20.01
.56	36.00	35.27	34.37	33.31	32.16	30.92	29.62	28.20	26.77	25.26	23.77	22.25	20.78	19.31	17.90
.60	31.41	30.75	30.00	29.08	28.11	27.06	25.97	24.77	23.57	22.31	21.05	19.77	18.52	17.28	16.08
.64	27.61	27.05	26.38	25.61	24.77	23.88	22.95	21.93	20.90	19.83	18.75	17.66	16.60	15.53	14.50
.68	24.47	23.97	23.39	22.72	22.00	21.22	20.41	19.53	18.65	17.73	16.81	15.86	14.95	14.03	13.13

Values of K in formula

$$p'r - N_c = \frac{Kp'rt^2}{12r^2}$$

$$K = \frac{2 \sin\phi_0}{(1+t^2/12r^2)(\phi_0 + 1/2 \sin 2\phi_0) - (1-\cos 2\phi_0)/\phi_0 + (t^2/4r^2)(\phi_0 - 1/2 \sin 2\phi_0)}$$

SHEET NO 1 OF 2

TABLE XIV. (*Continued.*)

VALUES OF K FOR USE IN EQUATION 214 FOR THIN FIXED ENDED CIRCULAR ARCHES UNDER NORMAL LOADS

$\frac{t}{T}$	VALUES OF $2\phi_0$ = TOTAL ANGLE SUBTENDING ARCH RING															
	105°	110°	115°	120°	125°	130°	135°	140°	145°	150°	155°	160°	165°	170°	175°	180°
0	62.5	51.7	43.12	36.25	30.69	26.4	22.37	19.26	16.66	14.48	12.63	11.06	9.72	8.57	7.58	6.72
.02	62.3	51.5	43.00	36.17	30.63	26.10	22.33	19.23	16.64	14.46	12.62	11.05	9.71	8.56	7.57	6.72
.04	61.6	51.1	42.67	35.92	30.44	25.95	22.22	19.15	16.57	14.41	12.58	11.02	9.69	8.54	7.56	6.70
.06	60.5	50.3	42.11	35.52	30.13	25.72	22.01	19.02	16.46	14.32	12.51	10.97	9.64	8.50	7.53	6.68
.08	59.1	49.29	41.36	34.96	29.72	25.40	21.80	18.83	16.32	14.21	12.42	10.89	9.58	8.46	7.49	6.65
.10	57.4	48.02	40.44	34.26	29.20	25.01	21.48	18.59	16.14	14.06	12.30	10.80	9.51	8.40	7.44	6.61
.12	55.4	46.57	39.36	33.46	28.59	24.53	21.13	18.31	15.91	13.88	12.16	10.68	9.42	8.32	7.38	6.56
.14	53.2	44.97	38.16	32.55	27.89	24.01	20.73	17.99	15.66	13.68	12.00	10.56	9.31	8.24	7.31	6.50
.16	50.9	43.24	36.87	31.57	27.14	23.42	20.27	17.63	15.37	13.46	11.82	10.41	9.19	8.14	7.23	6.43
.18	48.43	41.45	35.50	30.52	26.33	22.79	19.78	17.24	15.06	13.21	11.62	10.25	9.06	8.03	7.14	6.36
.20	46.05	39.60	34.09	29.43	25.49	22.13	19.25	16.83	14.73	12.94	11.40	10.08	8.92	7.92	7.04	6.28
.22	43.74	37.75	32.66	28.31	24.60	21.44	18.71	16.39	14.38	12.66	11.18	9.88	8.77	7.79	6.94	6.20
.24	41.36	35.91	31.21	27.18	23.72	20.73	18.15	15.94	14.02	12.37	10.94	9.69	8.61	7.66	6.83	6.10
.26	39.08	34.11	29.78	26.08	22.81	20.01	17.57	15.47	13.65	12.06	10.69	9.49	8.44	7.52	6.72	6.01
.28	36.87	32.38	28.38	24.93	21.91	19.28	16.99	15.00	13.26	11.75	10.43	9.28	8.26	7.38	6.60	5.91
.30	34.76	30.65	27.02	23.83	21.02	18.57	16.40	14.53	12.87	11.43	10.17	9.08	8.08	7.23	6.47	5.80
.32	32.76	29.02	25.70	22.75	20.15	17.85	15.82	14.05	12.48	11.11	9.90	8.84	7.90	7.07	6.34	5.70
.34	30.87	27.47	24.42	21.71	19.29	17.18	15.25	13.58	12.09	10.79	9.64	8.62	7.72	6.92	6.21	5.59
.36	29.09	26.00	23.21	20.70	18.46	16.47	14.68	13.11	11.70	10.46	9.37	8.39	7.52	6.76	6.08	5.47
.38	27.41	24.60	22.04	19.74	17.66	15.80	14.13	12.65	11.32	10.14	9.10	8.17	7.34	6.60	5.96	5.36
.40	25.85	23.28	20.94	18.81	16.89	15.15	13.59	12.19	10.94	9.83	8.83	7.94	7.15	6.41	5.81	5.25
.44	23.02	20.88	18.90	17.08	15.43	13.92	12.55	11.32	10.21	9.21	8.31	7.50	6.77	6.12	5.54	5.01
.48	20.56	18.75	17.07	15.52	14.09	12.79	11.58	10.50	9.51	8.61	7.80	7.07	6.40	5.81	5.27	4.783
.52	18.40	16.88	15.45	14.12	12.89	11.74	10.69	9.73	8.85	8.04	7.31	6.65	6.05	5.50	5.00	4.555
.56	16.54	15.24	14.02	12.87	11.79	10.79	9.87	9.02	8.23	7.51	6.85	6.25	5.70	5.20	4.746	4.332
.60	14.92	13.80	12.74	11.76	10.81	9.93	9.11	8.36	7.65	7.01	6.42	5.87	5.37	4.916	4.498	4.115
.64	13.50	12.54	11.62	10.75	9.92	9.16	8.42	7.75	7.12	6.53	6.00	5.52	5.06	4.644	4.259	3.906
.68	12.26	11.42	10.62	9.85	9.13	8.44	7.80	7.20	6.63	6.11	5.63	5.18	4.767	4.384	4.031	3.706

Values of K in formula

$$p'r - N_c = \frac{K p' r t^2}{12 r^2}$$

$$K = \frac{2 \sin \phi_0}{(1 + t^2/12r^2)(\phi_0 + 1/2 \sin 2\phi_0) - (1 - \cos 2\phi_0)/\phi_0 + (t^2/4r^2)(\phi_0 - 1/2 \sin 2\phi_0)}$$

SHEET NO. 2 OF 2

TABLE XV.

FUNCTIONS OF ϕ_a FOR USE IN EQUATION № 217 FOR FIXED ENDED CIRCULAR ARCHES UNDER NORMAL LOADS

$2\phi_a$	ϕ_a	Arc ϕ_a	Sin ϕ_a	Cos ϕ_a	$\frac{\text{Sin}\,\phi_a}{\phi_a}$	$1-\frac{\text{Sin}\,\phi_a}{\phi_a}$	$\frac{\text{Sin}\,\phi_a}{\phi_a}-\text{Cos}\,\phi_a$
30°	15°00'	.261 7994	.25 882	.96 593	.98 862	.01 138	.02 269
35°	17°30'	.305 4326	.30 071	.95 372	.98 454	.01 546	.03 082
40°	20°00'	.349 0659	.34 202	.93 969	.97 981	.02 019	.04 012
45°	22°30'	.392 6991	.38 268	.92 388	.97 449	.02 551	.05 061
50°	25°00'	.436 3323	.42 262	.90 631	.96 857	.03 143	.06 226
55°	27°30'	.479 9655	.46 175	.88 701	.96 205	.03 795	.07 504
60°	30°00'	.523 5988	.50 000	.86 603	.95 493	.04 507	.08 890
65°	32°30'	.567 2320	.53 730	.84 339	.94 723	.05 277	.10 384
70°	35°00'	.610 8652	.57 358	.81 915	.93 896	.06 104	.11 981
75°	37°30'	.654 4985	.60 876	.79 335	.93 012	.06 988	.13 677
80°	40°00'	.698 1317	.64 279	.76 604	.92 073	.07 927	.15 469
85°	42°30'	.741 7649	.67 559	.73 728	.91 078	.08 921	.17 351
90°	45°00'	.785 3982	.70 711	.70 711	.90 032	.09 968	.19 321
95°	47°30'	.829 0314	.73 728	.67 559	.88 933	.11 067	.21 374
100°	50°00'	.872 6646	.76 604	.64 279	.87 782	.12 218	.23 503
105°	52°30'	.916 2979	.79 335	.60 876	.86 582	.13 418	.25 706
110°	55°00'	.959 9311	.81 915	.57 358	.85 334	.14 666	.27 976
115°	57°30'	1.003 5644	.84 339	.53 730	.84 039	.15 961	.30 309
120°	60°00'	1.047 1976	.86 603	.50 000	.82 700	.17 300	.32 700
125°	62°30'	1.090 8307	.88 701	.46 175	.81 315	.18 685	.35 140
130°	65°00'	1.134 4640	.90 631	.42 262	.79 889	.20 111	.37 627
135°	67°30'	1.178 0972	.92 388	.38 268	.78 421	.21 579	.40 153
140°	70°00'	1.221 7305	.93 969	.34 202	.76 915	.23 085	.42 713
145°	72°30'	1.265 3637	.95 372	.30 071	.75 371	.24 629	.45 300
150°	75°00'	1.308 9969	.96 593	.25 882	.73 792	.26 208	.47 910
155°	77°30'	1.352 6302	.97 630	.21 644	.72 178	.27 822	.50 534
160°	80°00'	1.396 2634	.98 481	.17 365	.70 532	.29 468	.53 167
165°	82°30'	1.439 8966	.99 144	.13 053	.68 855	.31 145	.55 802
170°	85°00'	1.483 5299	.99 619	.08 716	.67 150	.32 850	.58 434
175°	87°30'	1.527 1631	.99 905	.04 362	.65 419	.34 851	.61 057
180°	90°00'	1.570 7963	1.00 000	.00 000	.63 662	.36 338	.63 662

13-8. Eccentricity of Thrust.—The eccentricity j of the thrust on any cross-section can be determined by using the values of N and M found by use of Eqs. (216) and (217). The variation of the thrust point follows the same course as given in paragraph 12-12.

DEFLECTION OF ARCH RING

13-9. Deflection at Crown.—A formula for finding the deflection of an arch ring at the crown may be derived from the general deflection formula expressed by Eq. (169) in Chap. XI, by considering a unit load applied at the crown and acting along the axis of symmetry, finding the values of N, M, and V as given in Eqs. (216), (217), and (218) in this chapter and substituting these values with the values of the thrust n, moment m, and shear v, in the arch ring due to the unit load, and integrating the resulting equation over the semiarch. It will be observed that $n = \sin \phi_1$, $m = r \sin \phi_1$, and $v = \cos \phi_1$. The resulting equation for the deflection Δ is then,

$$\Delta = \frac{p'r^2}{Et}\left[\frac{(1-\cos\phi_a)\left(1+\frac{t^2}{12r^2}\right)(\phi_a - \sin\phi_a) + \frac{t^2}{4r^2}(\phi_a + \sin\phi_a)}{\left(1+\frac{t^2}{12r^2}\right)\left(\phi_a + \frac{\sin 2\phi_a}{2}\right) - \frac{1-\cos 2\phi_a}{\phi_a} + \frac{t^2}{4r^2}\left(\phi_a - \frac{\sin 2\phi_a}{2}\right)}\right] \tag{219}$$

By neglecting the deflection due to shear and simplifying, this equation can be reduced to Cain's crown-deflection formula.[1] This equation is also used to find approximate deflections of thin arch rings of variable thickness and thick arch rings of uniform thickness.

13-10. Deflection at Any Point.—The deflections for other points in the arch ring may be found by the same general process, but their usefulness is slight and the equations become very complicated. For these reasons they are not given here.

13-11. Earthquake Stresses Due to Mass in Symmetrical, Single-centered, Thin Arch Rings of Uniform Thickness.—For a discussion and development of formulas relating to this subject see Appendix III.

[1] *Trans. Am. Soc. Civil Eng.*, Vol. 85, p. 241.

CHAPTER XIV

SYMMETRICAL MULTICENTERED THIN ARCH RINGS

INTRODUCTION

14-1. Types Considered.—There will be considered in this chapter three-centered, symmetrical, fixed-ended, thin arch rings of uniform thickness under a uniform symmetrical water load and polycentered, symmetrical, fixed-ended, thin arch rings of variable thickness and loading.

14-2. Method of Procedure.—The analytical procedure will consist, first, in designing the most suitable arch ring of uniform thickness, symmetrically loaded and of symmetrical shape, by means of the formulas in Chap. XIII; second, in finding a multicentered arch ring that will equalize, so far as practicable, the stresses at the crown and the abutments; third, in computing the thrust, moment, and shear in the multicentered arch ring by the voussoir method. By adopting the multicentered arch ring thus determined, the critical stresses in the parent circular arch ring may be reduced. Of course, the multicentered arch ring may be determined by other considerations than its relations to a parent circular arch ring. The number of centers to be used may be varied to suit the accuracy required in the solution—the greater the number of centers the greater the accuracy, and also the greater the necessary calculations. Generally, three centers will be sufficient for all practical purposes. In case three centers are adopted, the water-load formulas can be integrated; but where a greater number is adopted, these load formulas will be more readily solved by arithmetical summations. Formulas and forms for computations will be developed for both types.

THREE-CENTERED ARCH RINGS

14-3. Advantages.—The symmetrical three-centered arch ring of uniform thickness has some marked advantages over the symmetrical single-centered arch ring of uniform thickness, inasmuch as it is possible to make the neutral axis correspond more nearly with the line of thrust due to water load. By the use of the three-centered ring of this type, it is possible greatly to reduce the bending moment at the crown and abutments, and thereby eliminate undesirable tensional stresses that would necessarily exist in a single-centered arch ring.

14-4. Coordinates, Signs, and Symbols.—Let Fig. 42 represent the left half of a symmetrical and symmetrically water-loaded horizontal, three-centered, fixed-ended arch ring of uniform thickness and 1 ft. high, with the entire ring comprised of three circular arcs, half of the central arc and one side arc being as shown in the figure. The coordinates, signs, and symbols will, in general, conform to those given in Chap. XI. The symbols required for use in the formulas, in addition to those given in Chap. XI, are as follows:

R_c = radius of curvature of extrados of central arc.
R_B = radius of curvature of the extrados of side arcs.
r_c = radius of curvature of center axis of central arc.
r_B = Radius of curvature of central axis of side arcs.
ϕ_c = angle between radius at crown and that at left end of the central arc.
ϕ_B = angle between radius at left end of the central arc and abutment radius.

14-5. Stresses for Semi-central Arc Due to Water Load on Cantilever.—The whole arch ring of which Fig. 42 represents the left half is a statically indeterminate structure with the redundant member consisting of the right half of the arch ring. In Fig. 42, therefore, consider the semi-ring as the residual frame swung from the left abutment as a cantilever. The equations for determining the values of N_0, M_0, and V_0 for any point (r_c, ϕ_1) on the arc, whose central angle is ϕ_c, may be written down directly from

Eqs. (195), (196), and (197) by letting $\theta_1 = \phi_1$, $R = R_c$, $\left(R - \frac{t}{2}\right) = r_c$, and $ds = r_c d\phi$, as follows:

$$N_0 = \int_0^{\phi_1} pR_c \sin(\phi_1 - \phi)d\phi = pR_c(1 - \cos\phi_1); \quad (220)$$

$$M_0 = \int_0^{\phi_1} - pR_c r_c \sin(\phi_1 - \phi)d\phi = - pR_c r_c(1 - \cos\phi_1); \quad (221)$$

$$V_0 = \int_0^{\phi_1} - pR_c \cos(\phi_1 - \phi)d\phi = -pR_c \sin\phi_1. \quad (222)$$

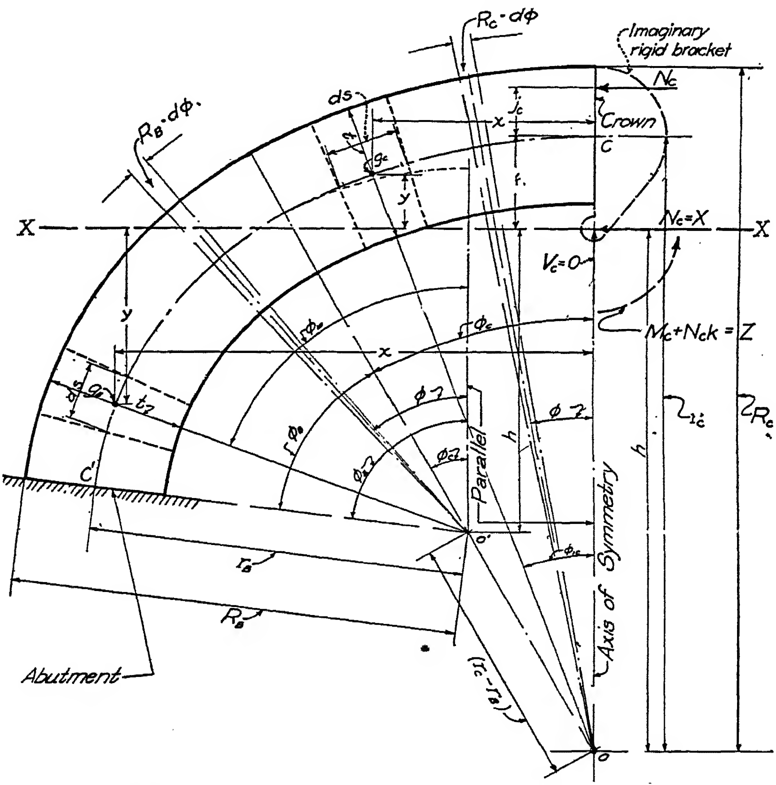


FIG. 42.—Diagram for analysis of symmetrical, three-centered arch ring of uniform thickness.

At the left end of the central arc, $\phi_1 = \phi_c$, and these equations are readily modified for this point by making this substitution in them.

14-6. Stresses for Side Arc Due to Water Load on Cantilever.—For any point (r_B, ϕ_1) on the side arc, whose central angle is ϕ_B, the values of N_0, M_0, and V_0 are obtained by integration of Eqs. (195), (196), and (197) for the entire half central arc between the limits of ϕ_c and 0, and adding to these integrations similar integrations for the portion of the side arc under consideration between the limits of ϕ_1 and ϕ_c. In this process it should be noted as before that $\theta_1 = \phi_c$, $R = R_c$, $R - \frac{t}{2} = r_c$, and $ds = r_c\,d\phi$ for the central arc; and that $\theta_1 = \phi_1$, $R = R_B$, $R - \frac{t}{2} = r_B$, and $ds = r_B\,d\phi$ for the side arc. The resulting equations are as follows:

$$N_0 = \int_0^{\phi_c} pR_c \sin(\phi_1 - \phi)d\phi + \int_{0_c}^{\phi_1} pR_B \sin(\phi_1 - \phi)d\phi = pR_c[\cos(\phi_1 - \phi_c) - \cos\phi_1] + pR_B[1 - \cos(\phi_1 - \phi_c)]; \quad (223)$$

$$M_0 = \int_0^{\phi_c} - pR_c[(r_c - r_B)\sin(\phi_c - \phi) + r_B \sin(\phi_1 - \phi)]\,d\phi + \int_{0_c}^{\phi_1} -pR_Br_B[\sin(\phi_1 - \phi)]d\phi = -\{pR_c[(r_c - r_B)(1 - \cos\phi_c) + r_B\{\cos(\phi_1 - \phi_c) - \cos\phi_1\}] + pR_B\,r_B[1 - \cos(\phi_1 - \phi_c)]\}; \quad (224)$$

$$V_0 = \int_0^{\phi_c} - pR_c \cos(\phi_1 - \phi)d\phi + \int_{0_c}^{\phi_1} - pR_B \cos(\phi_1 - \phi)d\phi = pR_c[\sin(\phi_1 - \phi_c) - \sin\phi_1] - pR_B \sin(\phi_1 - \phi_c). \quad (225)$$

14-7. Redundants and Stresses Due to Water Loads.—The stresses due to the water load on the cantilever having been obtained from Eqs. (220) to (225), the redundant forces, and the total thrust, moment and shear can be determined for the semi-arch rings. Since the semi-ring consists of two sections with different radii, it is simpler, although not necessary, to use the voussoir method of analysis. Let it be assumed, therefore, that the semi-ring

is divided into a convenient number of voussoirs of equal length along the central axis. The redundants X and Z may be determined by noting that in all voussoirs in the

Constants	Constants	Voussoir	ϕ_1	$\cos\phi_1$	$\sin\phi_1$	$1-\cos\phi_1$	N_0 (Center)	V_0 (Center)	M_0 (Center)
p	pR_c	1							
R_c	$pR_c r_c$	2							
r_c	pR_B	3							
R_B	$p(R_c-R_B)$	4							
r_B	$(r_c-r_B)\cdot(1-\cos\phi_c)$	5							
R_c-R_B	$pR_B r_B$	6							
r_c-r_B		7							
ϕ_c		8							
ϕ_B		9							
$\cos\phi_c$		10							
$1-\cos\phi_c$		11							
		12							

Voussoir	$\phi_1-\phi_c$	$\cos(\phi_1-\phi_c)$	$\cos(\phi_1-\phi_c)-\cos\phi_1$	$1-\cos(\phi_1-\phi_c)$	1st Term N_0 (side)	2nd Term N_0 (side)	N_0 (side)	$\sin(\phi_1-\phi_c)$	$\sin(\phi_1-\phi_c)-\sin\phi_1$	1st Term V_0 (side)	2nd Term V_0 (side)	V_0 (side)
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Voussoir	$r_B[\cos(\phi_1-\phi_c)-\cos\phi_1]$	$r_B[1-\cos(\phi_1-\phi_c)]$	1st Term M_0 (side)	2nd Term M_0 (side)	3rd Term M_0 (side)	M_0 (side)	Voussoir Arc		$12ds$
							Degrees	Feet	
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									

Layout No.................. Site.............................. Depth of Water..........Feet
Computed by........................ Checked by........................ Date................

FIG. 43.—Record sheet for analysis of symmetrical three-centered thin arch ring of uniform thickness.

central arc $ds = r_c\, d\phi$ and $\theta_1 = \phi_{1c}$, and for all those in the side arc $ds = r_B\, d\phi$ and $\theta_1 = \phi_1$, and in both arcs $I = t^3/12$ and $A = t$. These values are now to be substi-

tuted together with the appropriate values of N_0, M_0, and V_0 in Eqs. (192) and (193). The values of $X = N_c = N_e$ and $Z = M_e$ having been found, the values of N, M, and V can be computed from Eqs. (198), (199), and (200). The unit compressive stresses can next be determined from Eq. (174) and shearing stresses from Eq. (177).

14-8. Stresses Due to Temperature Changes.—Stresses due to temperature changes can be obtained by the process and equations given in paragraph 12–11 and noting that $\theta_1 = \phi_1$.

14-9. Eccentricity of Thrust.—The eccentricity of thrust j, at any voussoir center, can be found by dividing N into M, using the values of N and M as determined from the method given in paragraph 14–7.

14-10. Recording Computations.—The computations should be carefully recorded as the work progresses. A convenient form for recording the values of N_0, M_0, and V_0 will be found in Fig. 43.

SYMMETRICAL, POLYCENTERED, THIN ARCH RINGS OF VARIABLE THICKNESS

14-11. Advantages.—The polycentered arch ring of variable thickness offers a more complete means of reducing the tensile stresses that often exist in a single-centered or three-centered arch ring of the same rise and span. It is, therefore, convenient to have a form of analysis for this type of ring. Such a form is presented in the following paragraphs. In this method, a symmetrical and symmetrically loaded, fixed-ended, thin arch ring 1 ft. high is assumed with polycentered circular central axis, intrados, and extrados. This method is especially adapted to the inclined arches of multiple-arch dams, whereon the water loading is not uniform.

14-12. Coordinates, Signs, and Symbols.—In the development of the equations for this method of computing the thrust, moment, and shear in the arch ring, let the following designations be given to the symbols used, in addition to those given in Chap. XI:

L_m = length of any chord of extrados.
l_n = length of part of chord of extrados to right of k_n.
j_m = length of perpendicular from axis of symmetry to any chord L_m at its center.
k_n = length of perpendicular from axis of symmetry to any chord of neutral axis at its center.
n = serial number of any voussoir to left of crown.
m = serial number of any load voussoir to left of crown and to right of voussoir n.
ϕ_m = angle between perpendicular j_m and axis of symmetry.
θ_n = angle between perpendicular k_n and axis of symmetry.
e_{n-m} = length of arm between center of stress voussoir n and line of action of load on any specified load voussoir m, to right thereof.
e_0 = length of arm between center of any stress voussoir n and line of action of load on it to right of center thereof.
w = unit water pressure on any voussoir.

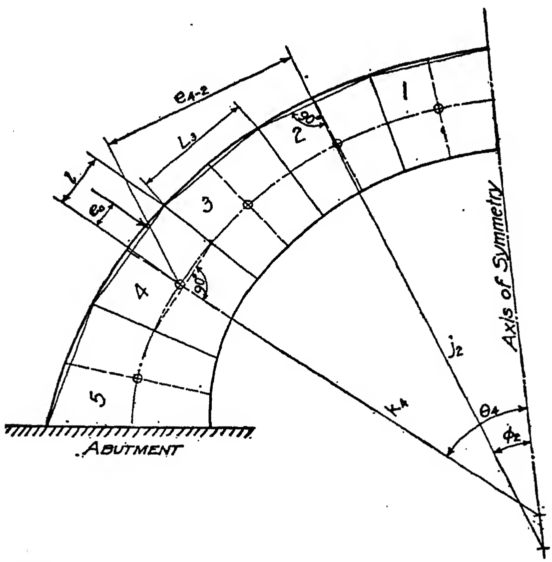


FIG. 44.—Diagram for analysis of symmetrical polycentered thin arch ring of variable thickness.

14-13. Stresses Due to Water Load on Cantilever.—Let Fig. 44 represent the left half of a symmetrical and symmetrically water-loaded, polycentered, fixed-ended, thin arch ring of variable thickness. Consider the semi-ring

swung from the left abutment as a cantilever. Since the extrados is not circular, it is necessary to determine values of N_0, M_0, and V_0 by a process of arithmetical summation. In this solution the left-hand semiarch is drawn to a suitable scale, voussoirs of convenient length are laid off, and the neutral axis of the ring is drawn half-way between the extrados and intrados. Chords L_m of the extradosal arcs for each voussoir are drawn; perpendicular lines j_m are erected to them at their centers and extended to meet the axis of symmetry of the arch ring; chords of the neutral-axis arcs for each voussoir are then drawn; and perpendiculars k_n are erected to them at their centers and extended to meet the central axis and the axis of symmetry as shown in the figure. A division into five voussoirs is shown, but any number may be used. The forces acting on the right section of any voussoir n at its center, may be found by resolving onto it the water load acting on the half voussoir to the right of the center together with that acting on all of the voussoirs to the right thereof. From the geometry and trigonometry of Fig. 44, the water pressure on any voussoir m will be wL_m, and that on the voussoir n, to the right of the perpendicular k_n, will be wl_n. The following equations will hold for the forces acting at the center of voussoir n:

$$N_0 = wl_n \sin(\theta_n - \phi_n) + \int_{m=0}^{m=n-1} wL_m \sin(\theta_n - \phi_m); \quad (226)$$

$$M_0 = -wl_n e_0 - \int_{m=0}^{m=n-1} wL_m \, e_{n-m}; \quad (227)$$

$$V_0 = -wl_n \cos(\theta_n - \phi_n) - \int_{m=0}^{m=n-1} wL_m \cos(\theta_n - \phi_m). \quad (228)$$

In applying Eqs. (226), (227), and (228) for the purpose of determining the values of N_0, M_0, and V_0, w is computed from the known water depth and chords l_n and L_m, the angles θ_n and ϕ_m and the lever arms e are measured from the scale drawing and tabulated in the spaces provided therefor on the form given in connection with Fig. 45. These

fundamental quantities having been determined, the necessary subsidiary values are readily set down on the form and the final values N_0, M_0, and V_0 computed.

Voussoir	w	L	l	wL	wl	θ	ϕ	$\theta-\phi_1$	$\theta-\phi_2$	$\theta-\phi_3$	$\theta-\phi_4$	$\theta-\phi_5$	$\theta-\phi_6$	$\theta-\phi_7$	$\theta-\phi_8$	$\theta-\phi_9$	$\theta-\phi_{10}$	$\theta-\phi_{11}$	$\theta-\phi_{12}$
1																			
2																			
3																			
4																			
5																			
6																			
7																			
8																			
9																			
10																			
11																			
12																			

Voussoir	Sines of Angles given below												Cosines of Angles given below											
	$\theta-\phi_1$	$\theta-\phi_2$	$\theta-\phi_3$	$\theta-\phi_4$	$\theta-\phi_5$	$\theta-\phi_6$	$\theta-\phi_7$	$\theta-\phi_8$	$\theta-\phi_9$	$\theta-\phi_{10}$	$\theta-\phi_{11}$	$\theta-\phi_{12}$	$\theta-\phi_1$	$\theta-\phi_2$	$\theta-\phi_3$	$\theta-\phi_4$	$\theta-\phi_5$	$\theta-\phi_6$	$\theta-\phi_7$	$\theta-\phi_8$	$\theta-\phi_9$	$\theta-\phi_{10}$	$\theta-\phi_{11}$	$\theta-\phi_{12}$
1																								
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Voussoir	Leverages of Loads wL (or wl) about Voussoir Centers												Thrusts ΔN_0 at Voussoir Centers due to Voussoir Loads												N_0
	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	1	2	3	4	5	6	7	8	9	10	11	12	
1																									
2																									
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5																									
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9																									
10																									
11																									
12																									

Voussoir	Shears ΔV_0 at Voussoir Centers due to Voussoir Loads												V_0
	1	2	3	4	5	6	7	8	9	10	11	12	
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													

Voussoir	Moment ΔM_0 at Voussoir Centers due to Voussoir Loads												M_0
	1	2	3	4	5	6	7	8	9	10	11	12	
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													

Layout No.	Site
Depth of Water	Ft.
Pressure =	Lbs. per sq. ft.
pR =	Lbs.
x = =	Z = =
Computed by	Checked by

FIG. 45.—Record sheet for analysis of symmetrical polycentered thin arch ring of variable thickness.

14-14. Stresses Due to Water Loads.—The total net thrust, moment, and shear for each voussoir of the semiarch may now be determined as for the three-centered arch rings

by means of Eqs. (198), (199), and (200), the maximum unit compressive stresses by Eq. (174), and the maximum unit shearing stresses from Eq. (177).

14-15. Stresses Due to Temperature Changes.—As in the three-centered arch rings, stresses due to temperature changes can be obtained by the process outlined in paragraph 12–11, the value of the angles θ_1 being measured from the scale diagram.

14-16. Eccentricity of Thrust.—The eccentricity of thrust is equal to $M \div N$ as computed for each voussoir center.

CHAPTER XV

SYMMETRICAL, SINGLE-CENTERED THICK ARCH RINGS OF UNIFORM THICKNESS

INTRODUCTION

15-1. General Statement.—When an arch ring is relatively thick the neutral axis and the center line are an appreciable distance apart; and, as indicated by Table XVII, when t/r and $2\ \phi_1$ are both relatively small or large,

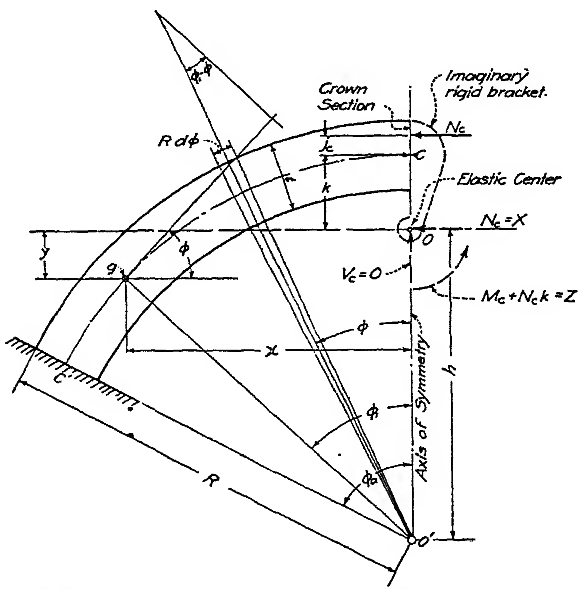

FIG. 46.—Diagram for analysis of symmetrical single-centered thick arch ring of uniform thickness.

appreciable errors are involved in the stress formulas in considering these lines coincident as is done in thin arches. Such conditions rarely exist, however, in dams of usual proportions; but in very high dams it becomes necessary for accuracy either to use arch rings of variable thickness

as discussed in Chap. XII or to use arch rings of considerable thickness. Generally, the former type of ring is preferable. Stress formulas for thick arch rings are, however, sometimes useful, convenient, and necessary.

STRESS FORMULAS

15-2. Coordinates, Signs, and Symbols.—Let Fig. 46 represent the left half, 1 ft. high, of a symmetrical and symmetrically water-loaded, single-centered, fixed-ended, thick arch ring of uniform thickness, and let it be a duplicate of Fig. 41 except as to ring thickness and intradosal curve. Also, let the coordinates, signs, and symbols used in this chapter have the same significance as those in Chap. XIII, except that r_n shall represent the radius of the neutral axis, and r_i the radius of the intrados.

15-3. Stress Due to Water Load and Redundants.—Formulas for thrust, moment, and shear can be deduced from Eq. (169) of Chap. XI and the equations in Chap. XII by using the procedure given in Chap. XIII in the development of Eqs. (216), (217), and (218) for thin arch rings. The differences in the procedure are insufficient to warrant its repetition in this chapter. Attention, however, is directed to the need for using pR instead of $p'r$; $R - \left(\frac{t}{2} - c\right) = r_n$ instead of $R - \frac{t}{2} = r$; and $ds = r_n\, d\phi$ instead of $ds = r d\phi$. It should also be noted that $I = t^3/12$ may be used for I_n, as $I_n = I$ very nearly, since the distance c between the neutral axis and the central axis is always small compared with the thickness t of the arch ring. Noting these differences and carrying out the computations, the following formulas may be derived, which are the same except as to nomenclature as Cain's formulas,[1] derived by a different method;

$$pR - N_c = \frac{K'pR}{12}; \tag{229}$$

$$K' = \frac{\frac{2t^2}{r_n^2}\sin\phi_a}{\left(1 + \frac{t^2}{12r_n^2}\right)\left(\phi_a + \frac{\sin^2\phi_a}{2}\right) - \frac{1-\cos 2\phi_a}{\phi_a} + \frac{rt^2}{4r_n^3}\left(\phi_a - \frac{\sin 2\phi_a}{2}\right)}; \tag{230}$$

[1] *Trans. Am. Soc. Civil Eng.*, Vol. 90, pp. 522–546.

$$N_{\phi 1} = pR - (pR - N_c) \cos \phi_1; \tag{231}$$

$$M_{\phi 1} = -r_n(pR - Nc)\left(\frac{\sin \phi_a}{\phi_a} - \cos \phi_1\right); \tag{232}$$

$$V_{\phi 1} = -(pR - N_c) \sin \phi_1. \tag{233}$$

15-4. Use of Tables.—For use in applying the stress formulas given in this chapter, there are given in Table XVI, values of K' for t/r ranging from 0.02 to 0.68, and for central angles from 30 to 180 deg. The trigonometric functions given in Table XV are also helpful in finding the moment at the crown and the abutments.

15-5. Computation of Thrust, Moment, and Shear.—With the values of r_n and K' available, N, M, and V can easily be computed for any section of the arch ring from Eqs. (231), (232), and (233), the maximum unit compressive stresses from Eqs. (188) and (189), and the maximum unit shearing stresses from Eq. (174).

15-6. Stresses Due to Temperature Changes.—Stresses due to temperature changes can be found by use of Eqs. (201) to (204), noting that $\theta_1 = \phi_1$, and $ds = r_n\, d\phi_1$. The value of I may be used without change as it has been shown that $I_n = I$ very nearly for all practical cases.

15-7. Eccentricity of Thrust.—The eccentricity of the thrust on any section of the arch ring with respect to the neutral axis is found by dividing the moment by the thrust, but in this case the distance is measured from the neutral axis and not from the central axis as in the thin arch ring.

15-8. Error Resulting from Thick-arch Assumption.—As a means of determining the error resulting from the assumption that the neutral axis of an arch ring coincides with the central axis, Table XVII has been prepared showing the ratio of K to $K' \div (t/r)^2$ derived from a comparison of Eqs. (214) and (229). This table provides a means of comparing the values of the thrust, moment, and shear as computed from the equations for thin arch rings with those computed from the equations for thick arch rings, both being of uniform thickness, circular, and uniformly water loaded.

TABLE XVI.

VALUES OF "K'" FOR USE IN EQUATION 229 FOR THICK FIXED ENDED CIRCULAR ARCHES UNDER NORMAL LOADS

$\frac{t}{r}$	$\frac{r_e}{r_i}$	$\log_e \frac{r_e}{r_i}$	VALUES OF $2\phi_1$ = TOTAL ANGLE SUBTENDING ARCH RING														
			30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	95°	100°
0	1.000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.02	1.0202	.01999	2.916	1.756	1.096	0.707	0.4735	0.3272	0.2327	0.1696	0.1264	0.0960	0.0691	0.0582	0.04621	0.03723	0.03013
.04	1.0408	.04000	6.57	4.787	3.366	2.356	1.665	1.193	0.869	0.643	0.4855	0.3719	0.2930	0.2280	0.1817	0.1467	0.1191
.06	1.0619	.06006	8.53	6.98	5.450	4.155	3.119	2.338	1.761	1.337	1.027	0.797	0.631	0.4969	0.3979	0.3232	0.2633
.08	1.0833	.08002	9.60	8.37	6.994	5.66	4.493	3.520	2.743	2.138	1.594	1.321	1.062	0.842	0.680	0.555	0.4544
.10	1.1053	.10012	10.09	9.12	7.979	6.78	5.62	4.582	3.695	2.961	2.372	1.904	1.550	1.245	1.014	0.834	0.686
.12	1.1277	.12017	10.50	9.71	8.734	7.65	6.56	5.52	4.575	3.757	3.066	2.510	2.071	1.683	1.385	1.147	0.951
.14	1.1505	.14021	10.69	10.02	9.198	8.25	7.26	6.26	5.32	4.468	3.729	3.097	2.592	2.132	1.772	1.481	1.236
.16	1.1739	.16033	10.85	10.28	9.56	8.73	7.81	6.88	5.96	5.10	4.336	3.657	3.100	2.583	2.169	1.827	1.537
.18	1.1978	.18048	10.97	10.45	9.81	9.07	8.24	7.37	6.50	5.66	4.877	4.173	3.579	3.020	2.562	2.176	1.844
.20	1.2222	.20065	11.01	10.55	9.99	9.31	8.56	7.75	6.93	6.12	5.35	4.636	4.022	3.432	2.940	2.518	2.150
.22	1.2472	.22091	11.07	10.65	10.13	9.52	8.83	8.08	7.30	6.52	5.77	5.06	4.431	3.821	3.302	2.852	2.453
.24	1.2727	.24115	11.11	10.72	10.25	9.68	9.04	8.34	7.59	6.86	6.13	5.43	4.805	4.180	3.644	3.171	2.746
.26	1.2989	.26153	11.15	10.79	10.34	9.82	9.22	8.56	7.85	7.16	6.45	5.76	5.14	4.512	3.963	3.474	3.030
.28	1.3256	.28186	11.18	10.83	10.42	9.92	9.36	8.74	8.07	7.41	6.72	6.05	5.44	4.813	4.258	3.758	3.299
.30	1.3529	.30226	11.21	10.87	10.49	10.01	9.48	8.90	8.27	7.63	6.96	6.32	5.74	5.08	4.55	4.027	3.553
.32	1.3809	.32273	11.23	10.90	10.54	10.08	9.58	9.00	8.44	7.82	7.17	6.55	6.04	5.33	4.80	4.271	3.792
.34	1.4096	.34332	11.23	10.93	10.56	10.14	9.66	9.13	8.55	7.96	7.35	6.74	6.16	5.56	5.01	4.496	4.014
.36	1.4390	.36395	11.24	10.94	10.59	10.18	9.72	9.22	8.66	8.10	7.51	6.92	6.35	5.76	5.22	4.706	4.223
.38	1.4691	.38465	11.25	10.96	10.62	10.23	9.78	9.30	8.76	8.22	7.65	7.07	6.52	5.94	5.41	5.00	4.416
.40	1.5000	.40546	11.26	10.98	10.64	10.26	9.83	9.36	8.85	8.32	7.77	7.21	6.67	6.11	5.58	5.08	4.596
.44	1.5641	.44730	11.27	11.00	10.68	10.32	9.92	9.47	8.99	8.50	7.98	7.45	6.93	6.39	5.88	5.39	4.916
.48	1.6316	.48956	11.28	11.01	10.71	10.36	9.97	9.55	9.10	8.63	8.14	7.63	7.14	6.63	6.13	5.65	5.26
.52	1.7027	.53222	11.29	11.03	10.73	10.40	10.02	9.62	9.18	8.74	8.27	7.79	7.31	6.82	6.34	5.88	5.42
.56	1.7777	.57533	11.29	11.04	10.75	10.42	10.06	9.67	9.25	8.82	8.37	7.91	7.45	6.97	6.51	6.06	5.62
.60	1.8571	.61903	11.29	11.04	10.76	10.44	10.09	9.70	9.30	8.88	8.45	8.00	7.56	7.10	6.66	6.22	5.79
.64	1.9412	.66331	11.29	11.04	10.76	10.45	10.10	9.73	9.34	8.94	8.51	8.08	7.65	7.21	6.78	6.35	5.93
.68	2.0303	.70819	11.30	11.05	10.77	10.46	10.12	9.76	9.37	8.98	8.57	8.15	7.73	7.30	6.88	6.46	6.05

$$pR - N_c = \frac{K'pR}{12}$$

$$K' = \frac{2\sin\phi_a \cdot \frac{t^2}{r_n^2}}{\left(1+\frac{t^2}{12r_n^2}\right)\left(\phi_a + \frac{\sin 2\phi_a}{2}\right) - \frac{1-\cos 2\phi_a}{\phi_a} + \frac{rt^2}{4r_n^3}\left(\phi_a - \frac{\sin 2\phi_a}{2}\right)}$$

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TABLE XVI. (*Continued.*)

VALUES OF "K'" FOR USE IN EQUATION 229 FOR THICK FIXED ENDED CIRCULAR ARCHES UNDER NORMAL LOADS

$\frac{t}{r}$	VALUES OF $2\phi_1$ = TOTAL ANGLE SUBTENDING ARCH RING															
	105°	110°	115°	120°	125°	130°	135°	140°	145°	150°	155°	160°	165°	170°	175°	180°
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.02	0.02487	0.02063	0.01717	0.01445	0.01225	0.01041	0.00893	0.00770	0.00666	0.00579	0.00505	0.00441	0.003884	0.003426	0.003029	0.002687
.04	0.0984	0.0818	0.0682	0.0574	0.04865	0.04141	0.03555	0.03066	0.02652	0.02305	0.02012	0.01762	0.01550	0.01367	0.01209	0.01072
.06	0.2181	0.1816	0.1519	0.1280	0.1087	0.0926	0.0796	0.0686	0.0594	0.0517	0.04516	0.03956	0.03481	0.03071	0.02717	0.02411
.08	0.3777	0.3159	0.2645	0.2236	0.1900	0.1623	0.1395	0.1205	0.1044	0.0909	0.0795	0.0697	0.0613	0.0542	0.04792	0.04254
.10	0.574	0.4815	0.4048	0.3430	0.2923	0.2500	0.2154	0.1863	0.1616	0.1409	0.1232	0.1082	0.0953	0.0841	0.0745	0.0662
.12	0.798	0.673	0.568	0.483	0.4124	0.3535	0.3052	0.2645	0.2297	0.2005	0.1756	0.1542	0.1360	0.1202	0.01065	0.0947
.14	1.044	0.884	0.749	0.639	0.548	0.4709	0.4073	0.3536	0.3077	0.2689	0.2358	0.2074	0.1830	0.1620	0.1436	0.1277
.16	1.306	1.112	0.947	0.810	0.611	0.6008	0.521	0.4532	0.3951	0.3458	0.3037	0.2674	0.2363	0.2093	0.1858	0.1653
.18	1.576	1.350	1.154	0.992	0.856	0.7404	0.644	0.561	0.4904	0.4299	0.3782	0.3334	0.2950	0.2616	0.2324	0.2070
.20	1.850	1.592	1.369	1.182	1.024	0.888	0.774	0.677	0.593	0.521	0.4587	0.4051	0.3588	0.3186	0.2834	0.2526
.22	2.124	1.838	1.588	1.377	1.197	1.042	0.911	0.799	0.701	0.617	0.545	0.4817	0.4274	0.3800	0.3384	0.3020
.24	2.392	2.082	1.808	1.574	1.374	1.200	1.053	0.925	0.814	0.718	0.635	0.562	0.500	0.4449	0.3967	0.3544
.26	2.655	2.322	2.027	1.773	1.553	1.361	1.198	1.055	0.931	0.823	0.729	0.647	0.576	0.514	0.4585	0.4101
.28	2.906	2.556	2.241	1.968	1.731	1.523	1.344	1.187	1.050	0.931	0.826	0.735	0.655	0.585	0.523	0.4682
.30	3.145	2.781	2.448	2.158	1.905	1.683	1.490	1.319	1.170	1.040	0.925	0.825	0.736	0.658	0.589	0.528
.32	3.372	2.996	2.648	2.344	2.076	1.845	1.635	1.451	1.291	1.151	1.025	0.916	0.818	0.733	0.658	0.590
.34	3.591	3.203	2.847	2.532	2.252	2.001	1.782	1.588	1.415	1.263	1.128	1.009	0.905	0.812	0.729	0.655
.36	3.793	3.399	3.033	2.708	2.417	2.155	1.925	1.720	1.537	1.375	1.231	1.103	0.990	0.890	0.801	0.721
.38	3.983	3.583	3.211	2.877	2.577	2.305	2.065	1.850	1.657	1.486	1.333	1.198	1.077	0.978	0.873	0.788
.40	4.162	3.758	3.380	3.039	2.731	2.451	2.202	1.978	1.776	1.596	1.435	1.292	1.164	1.050	0.947	0.855
.44	4.483	4.076	3.691	3.341	3.020	2.727	2.463	2.225	2.007	1.812	1.637	1.478	1.337	1.209	1.095	0.991
.48	4.761	4.355	3.969	3.613	3.285	2.982	2.708	2.457	2.227	2.020	1.831	1.661	1.507	1.368	1.242	1.129
.52	5.00	4.598	4.213	3.855	3.523	3.213	2.932	2.672	2.433	2.215	2.017	1.836	1.672	1.518	1.386	1.263
.56	5.21	4.809	4.427	4.070	3.737	3.424	3.137	2.872	2.625	2.400	2.193	2.003	1.830	1.672	1.527	1.395
.60	5.38	4.992	4.615	4.260	3.927	3.614	3.324	3.054	2.803	2.571	2.358	2.161	1.981	1.815	1.663	1.524
.64	5.53	5.15	4.778	4.428	4.096	3.783	3.493	3.221	2.966	2.730	2.512	2.309	2.123	1.952	1.793	1.647
.68	5.66	5.29	4.922	4.575	4.247	3.935	3.645	3.372	3.115	2.876	2.654	2.447	2.257	2.080	1.916	1.765

$$pR - N_c = \frac{K'pR}{12}$$

$$K' = \frac{2 \sin \phi_a \cdot \frac{t^2}{r_n^2}}{\left(1+\frac{t^2}{12 r_n^2}\right)\left(\phi_a + \frac{\sin 2\phi_a}{2}\right) - \frac{1-\cos 2\phi_a}{\phi_a} + \frac{r t^2}{4 r_n^3}\left(\phi_a - \frac{\sin 2\phi_a}{2}\right)}$$

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TABLE XVII.

TABLE SHOWING COMPARISON OF VALUES OF K AND $K' \div \left(\frac{t}{r}\right)^2$ IN FORMULAS FOR FIXED ENDED CIRCULAR ARCHES UNDER NORMAL LOADS

(See Tables 14 and 16 for complete Tables of values of K and K')

$\frac{t}{r}$	$\left(\frac{t}{r}\right)^2$	$2\phi = 30°$				$2\phi = 105°$				$2\phi = 180°$			
		K'	$K'/\left(\frac{t}{r}\right)^2$	K	$\frac{K}{K'/\left(\frac{t}{r}\right)^2}$	K'	$K'/\left(\frac{t}{r}\right)^2$	K	$\frac{K}{K'/\left(\frac{t}{r}\right)^2}$	K'	$K'/\left(\frac{t}{r}\right)^2$	K	$\frac{K}{K'/\left(\frac{t}{r}\right)^2}$
.02	.0004	2.916	7290	5751	.789	0.02487	62.2	62.3	1.002	0.002687	6.72	6.72	1.000
.04	.0016	6.57	4106	3451	.840	0.0984	61.5	61.6	1.002	0.01072	6.70	6.70	1.000
.06	.0036	8.53	2369	2251	.950	0.2181	60.6	60.5	1.000	0.02411	6.70	6.68	.999
.08	.0064	9.60	1500	1458	.972	0.3777	59.0	59.1	1.002	0.04254	6.65	6.66	.999
.10	.0100	10.09	1009	977	.968	0.574	57.4	57.4	1.000	0.0662	6.62	6.61	.999
.12	.0144	10.50	729	719	.986	0.798	55.4	55.4	1.000	0.0947	6.56	6.56	.999
.14	.0196	10.69	545	539	.989	1.044	53.3	53.2	.998	0.1277	6.51	6.50	.998
.16	.0256	10.85	424	417	.983	1.306	51.0	50.9	.998	0.1653	6.46	6.43	.996
.18	.0324	10.97	339	334	.985	1.576	48.64	48.43	.996	0.2070	6.39	6.36	.996
.20	.0400	11.01	275	272	.989	1.850	46.25	46.05	.996	0.2526	6.32	6.28	.995
.22	.0484	11.07	229	228	.996	2.124	43.88	43.74	.996	0.3020	6.24	6.20	.994
.24	.0576	11.11	192.9	191.7	.994	2.392	41.53	41.36	.996	0.3544	6.15	6.10	.992
.26	.0676	11.15	164.9	164.3	.996	2.655	39.28	39.08	.995	0.41014	6.07	6.01	.991
.28	.0784	11.18	142.6	142.2	.997	2.906	37.07	36.87	.995	0.4682	5.97	5.91	.990
.30	.0900	11.21	124.5	123.8	.994	3.145	34.94	34.76	.995	0.528	5.87	5.80	.989
.32	.1024	11.23	109.7	109.2	.995	3.372	32.93	32.76	.995	0.590	5.76	5.70	.989
.34	.1156	11.23	97.1	96.9	.998	3.591	31.06	30.87	.994	0.655	5.67	5.59	.985
.36	.1296	11.24	86.7	86.6	.999	3.793	29.27	29.09	.994	0.721	5.56	5.47	.983
.38	.1444	11.25	77.9	77.7	.998	3.983	27.58	27.41	.997	0.788	5.46	5.36	.982
.40	.1600	11.26	70.4	70.3	.998	4.162	26.01	25.85	.995	0.855	5.34	5.25	.981
.44	.1936	11.27	58.2	58.2	1.000	4.483	23.16	23.02	.996	0.991	5.12	5.01	.980
.48	.2304	11.28	48.96	48.93	1.000	4.761	20.66	20.55	.994	1.129	4.900	4.783	.976
.52	.2704	11.29	41.75	41.75	1.000	5.00	18.49	18.40	.997	1.263	4.671	4.565	.977
.56	.3136	11.29	36.00	36.00	1.000	5.21	16.61	16.54	.996	1.395	4.448	4.332	.975
.60	.3600	11.29	31.36	31.41	1.002	5.38	14.94	14.92	.999	1.524	4.233	4.115	.973
.64	.4096	11.29	27.56	27.61	1.002	5.53	13.50	13.50	1.000	1.647	4.021	3.906	.972
.68	.4624	11.30	24.44	24.47	1.002	5.66	12.24	12.26	1.001	1.765	3.817	3.706	.972

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CHAPTER XVI

UNSYMMETRICAL THIN ARCH RINGS OF VARIABLE THICKNESS

INTRODUCTION

16-1. General Statement.—Generally it is possible to confine the centers of curvature of the various arch rings of an arch dam to either a single vertical or oblique plane, so that the rings will all be symmetrical with respect to that plane. However, it sometimes happens that the canyon walls are so irregular as to make this impossible for arch rings of variable thickness or variable central angles; and in such cases it is necessary to be able to find the stresses in unsymmetrical arch rings. Of these cases the one involving variable thickness is of most extensive and practical application. To provide for this case, there is presented here in brief form a solution of the unsymmetrical arch ring of variable thickness.

16-2. Symbols Used.—The symbols used in connection with the unsymmetrical-arch analysis here given are, so far as applicable, used with the same significance as those given in Chap. XI and other preceding chapters. Where a difference exists full explanations are given in the text.

16-3. General Deflection Formula for Unsymmetrical Arch.—Let Fig. 47 represent an unsymmetrical water-loaded, fixed-ended, thin arch ring with circular extrados of radius R and a compound circular intrados of the form shown in the figure. Then, this arch ring will form a statically indeterminate structure in which the left abutment may be considered as the redundant member. Solution, therefore, of the arch ring may be made by means of the dummy unit load method and the general deflection equation (Eq. [169]). In applying Eq. (169), it is well to omit

in the general solution the term involving temperature changes. With this omission, Eq. (169) becomes,

$$\Delta_q = \int_0^s \frac{Nn\,ds}{AE} + \int_0^s \frac{Mm\,ds}{EI} + C\int_0^s \frac{Vv\,ds}{AE'}. \qquad (234)$$

Equation (234) then represents the general deflection formula for any point q in any specified direction for any length of the neutral axis of the portion of the arch ring between the limits of s and 0 when the thickness of the ring is not too great, relative to the radius of curvature.

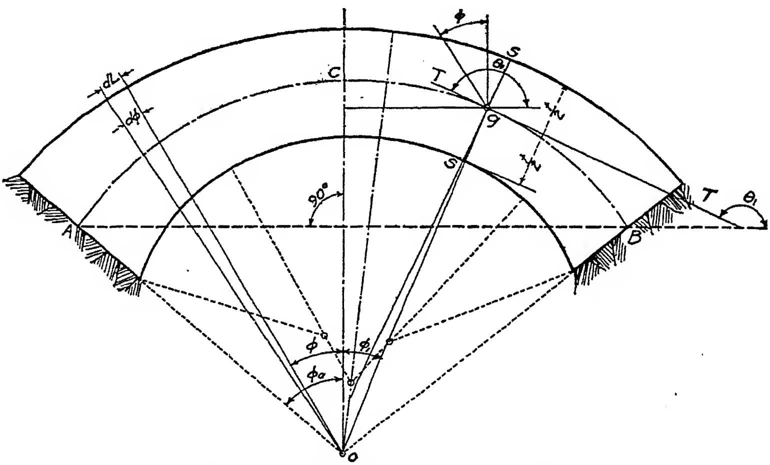

FIG. 47.—Unsymmetrical thin arch ring of variable thickness.

16-4. Location of Elastic Center of Unsymmetrical Arch.—In Fig. 48 draw a compound circular arc ACB conforming as nearly as possible to the neutral axis and consider it as representing that axis. Draw the chord AB and a perpendicular line JJ through the right end of the neutral axis. Let x' and y' be the coordinates of points of the neutral axis measured parallel to AB from JJ and perpendicular to AB from AB, respectively. Also let $\bar{x}$ and y be the similarly measured undetermined coordinates of the elastic center O. Divide the neutral axis into convenient equal lengths ds, or into such length that ds/I is constant, where I is the moment of inertia about the

neutral axis of a radial section of the arch ring at the center of length ds. Then the center of gravity of elastic weights will be located at the left of JJ in a line YY perpendicular to AB determined by the equation,

$$\bar{x} = \frac{\int^{s}\frac{x'ds}{I}}{\int_0^{s}\frac{ds}{I}}; \tag{235}$$

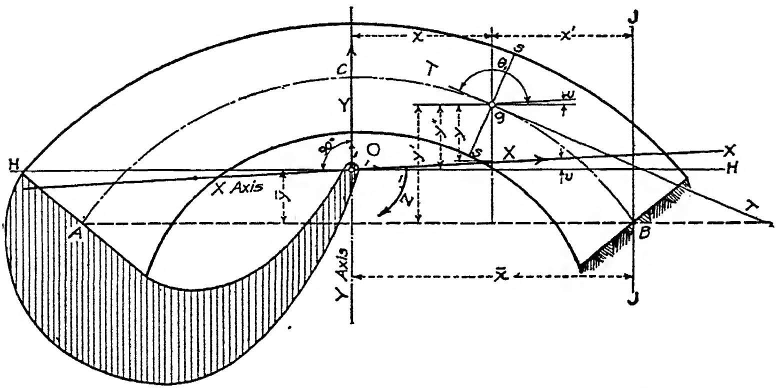

FIG. 48.—Diagram for analysis of unsymmetrical thin arch ring of variable thickness.

and above AB in the line YY a distance determined by the equation,

$$\bar{y} = \frac{\int_0^{s}\frac{y'ds}{I}}{\int^{s}\frac{ds}{I}}. \tag{236}$$

16-5. Location of Conjugate Axes of Elipse of Elasticity. It is necessary, not only to find the center of gravity of elastic weights by Eqs. (235) and (236), but also to find the direction of the conjugate axes of the elipse of elasticity. To do this, draw through O the line HH perpendicular to the Y-axis, which is drawn parallel to JJ, and give the coordinates of the point g in the neutral axis of the arch ring ACB the designations shown in Fig. 48. Draw the axis XX

through O making an angle u with and measured upward from HH. Let the coordinates of the point g be referred to X-axis and Y-axis, x being taken positive towards the right and y positive on the crown side. Now, in order to eliminate the centrifugal inertial moment from the deflection equation,

$$\int_0^s \frac{xy \cos u \, ds}{I}$$

must equal zero. From Fig. 48, it will be noted that $y = y'' - x \tan u$. Substituting the value of y in the integral expression and dividing by the constant, $\cos u$, the simplification equation becomes,

$$\int_0^s \frac{xy\,ds}{I} = \int_0^s \frac{xy''\,ds}{I} - \tan u \int_0^s \frac{x^2\,ds}{I} = 0, \qquad (237)$$

or

$$\tan u = \frac{\displaystyle\int_0^s \frac{xy''\,ds}{I}}{\displaystyle\int_0^s \frac{x^2\,ds}{I}} \qquad (238)$$

DETERMINATION OF REDUNDANTS FOR UNSYMMETRICAL ARCH

16-6. Replacement of Left Abutment by Rigid Bracket.—Consider the left abutment of the arch ring shown in Fig. 48 removed and the arch ring held in equilibrium by the reaction forces of the abutment, which are respectively equal to the thrust, moment, and shear on the arch ring at the abutment. Now replace the effect of the thrust N, moment M, and shear V at the left abutment by three redundant forces applied at the assumed center of coordinates or pole O, by means of a rigid bracket fastened to the left end of the arch. These forces consist of first, a linear force designated as X acting along XX; second, another linear force designated as Y acting along YY; and, third, a couple designated as Z acting about O; each of these redundant forces being taken positive when acting in the direction shown in Fig. 48. Also let unit forces be applied at O

acting in the same direction and with the same sign as the redundant forces, X, Y, and Z; and let the thrust, moment, and shear on any section of the arch ring due to the unit loads be represented respectively by n, m, and v with the subscript designation x, y, and z indicating the line of action of the unit forces at O. Thrusts are considered positive when acting toward the section; moments, when acting clockwise; and shears, when acting toward the extrados.

16-7. Deflection of Center of Coordinates of Redundants.—Under the water load and the forces, X, Y, and Z, the arch ring will undergo linear and angular deformations. Let it be assumed that as a result the pole O tends to be deflected a distance Δ_x parallel to XX, a distance Δ_y parallel to YY, and through an angle Δ_z from its original position. The general deflection formula given in Eq. (234) can now be applied. Since the deflections are to be found in three distinct directions, Eq. (234) must be applied three times, giving three simultaneous equations for determining the redundant forces X, Y, and Z. These applications give the following general deflection equations:

$$\Delta_x = \int_0^s \frac{Nn_x ds}{AE} + \int_0^s \frac{Mm_x ds}{EI} + C\int^s \frac{Vv_x ds}{AE'}; \quad (239)$$

$$\Delta_y = \int_0^s \frac{Nn_y ds}{AE} + \int_0^s \frac{Mm_y ds}{EI} + C\int_0^s \frac{Vv_y ds}{AE'}; \quad (240)$$

$$\Delta_z = \int_0^s \frac{Nn_z ds}{AE} + \int_0^s \frac{Mm_z ds}{EI} + C\int_0^s \frac{Vv_z ds}{AE'}. \quad (241)$$

In Eqs. (239), (240), and (241), A, E, I, and ds can all be determined either by computation or by measurement of the arch ring at the voussoir centers, and

$$\Delta_x = \Delta_y = \Delta_z = 0$$

because the arch ring is assumed to be fixed at the abutment and the bracket rigidly attached thereto. The values of n, m, and v, and N, M, and V need amplification.

16-8. Stresses Due to Unit Forces.—For the purpose of determining the value of n, m, and v, let reference be had to

Fig. 48, where g is any point in the neutral axis of the arch ring ACB, TT is the tangent to this axis at g, and SS is the radial section of the arch ring at g. Now resolve the unit forces acting at O normal to, parallel to, and around the point g, remembering that x is to be taken positive to the right, y positive upward, and θ_1 positive counterclockwise. From the geometry of the figure, it is then clear that

$$n_x = 1 \cdot \cos[180° - (\theta_1 - u)] = -\cos(\theta_1 - u); \quad n_y = -1 \cdot \sin(180° - \theta_1) = -\sin\theta_1; \quad n_z = 0; \quad m_x = -1 \cdot y \cos u = -y\cos u; \quad m_y = 1 \cdot x = x; \quad m_z = 1 \cdot 1 = 1;$$
$$v_x = 1 \cdot \sin[180° - (\theta_1 - u)] = \sin(\theta_1 - u); \quad v_y = 1 \cdot \cos(180° - \theta_1) = -\cos\theta_1; \quad v_z = 0. \tag{242}$$

16-9. Stresses Due to Water Load and Redundants.—In finding the values of N, M, and V representing the total thrust, moment, and shear at any cross-section of the arch ring, such as g, it will be noted that the thrust N_0, the moment M_0, and the shear V_0 due to the water pressure, and the thrust, moment, and shear due to the action of the forces X, Y, and Z are the total forces acting on the section. Moreover, since the thrust due to the forces X, Y, and Z are respectively equal to X, Y, and Z times the thrust, moment, and shear at the cross-section caused by the unit forces at the pole O and acting in the same direction as X, Y, and Z, then

$$N = N_0 + n_xX + n_yY + n_zZ = N_0 - \cos(\theta_1 - u)X - \sin\theta_1 Y; \tag{243}$$

$$M = M_0 + m_xX + m_yY + m_zZ = M_0 - y\cos u\, X + xY + Z; \tag{244}$$

$$V = V_0 + v_xX + v_yY + v_zZ = V_0 + \sin(\theta_1 - u)X - \cos\theta_1 Y. \tag{245}$$

16-10. Evaluation of Redundants.—Substituting the values of n, m, and v, and N, M, and V in Eqs. (239), (240, and (241) from Eqs. (242) to (245), inclusive, noting that

$$\Delta_x = \Delta_y = \Delta_z = 0,$$

and collecting and transposing terms, the following general equations are derived:

$$\left[\int_0^s \frac{\cos^2(\theta_1 - u)ds}{AE} + \int_0^s \frac{y^2 \cos^2 u\, ds}{EI} + C\int_0^s \frac{\sin^2(\theta_1 - u)ds}{AE'}\right]X + \left[\int_0^s \frac{\cos(\theta_1 - u)\sin\theta_1\, ds}{AE} - \int_0^s \frac{xy\cos u\, ds}{EI} - C\int_0^s \frac{\sin(\theta_1 - u)\cos\theta_1\, ds}{AE'}\right]Y + \left[-\int_0^s \frac{y\cos u\, ds}{EI}\right]Z = \int_0^s \frac{N_0\cos(\theta_1 - u)ds}{AE} + \int_0^s \frac{M_0 y\cos u\, ds}{EI} - C\int_0^s \frac{V_0\sin(\theta_1 - u)ds}{AE'}; \quad (246)$$

$$\left[\int_0^s \frac{\cos(\theta_1 - u)\sin\theta_1\, ds}{AE} - \int_0^s \frac{xy\cos u\, ds}{EI} - C\int_0^s \frac{\sin(\theta_1 - u)\cos\theta_1\, ds}{AE'}\right]X + \left[\int_0^s \frac{\sin^2\theta_1\, ds}{AE} + \int_0^s \frac{x^2ds}{EI} + C\int_0^s \frac{\cos^2\theta_1\, ds}{AE'}\right]Y + \left[\int_0^s \frac{x\, ds}{EI}\right]Z = \int_0^s \frac{N_0\sin\theta_1\, ds}{AE} - \int_0^s \frac{M_0 x\, ds}{EI} + C\int_0^s \frac{V_0\cos\theta_1\, ds}{AE'}; \quad (247)$$

$$\left[-\int_0^s \frac{y\cos u\, ds}{EI}\right]X + \left[\int_0^s \frac{x\, ds}{EI}\right]Y + \left[\int_0^s \frac{ds}{EI}\right]Z = -\int_0^s \frac{M_0 ds}{EI}. \quad (248)$$

The angular and linear coordinates involved in Eqs. (246), (247), and (248) having been determined in the usual manner either by computations or measurement, and noting that the angle u and its functions are constant and that

$$\int_0^s \frac{x\cos u\, ds}{I} = \int_0^s \frac{y\cos u\, ds}{I} = \int_0^s \frac{xy\cos u\, ds}{I} = 0,$$

the integrals can be computed and the redundants determined in terms of known quantities, and of unknowns N_0, M_0, and V_0, which are to be determined as shown in paragraph 16–11. However, Eqs. (246), (247), and (248) can be considerably simplified without involving serious error by assuming the terms

$$\int_0^s \frac{\cos(\theta_1 - u)\sin\theta_1}{AE}$$

and

$$c\int_0^s \frac{\sin(\theta_1 - u)\cos\theta_1}{AE'}$$

each equal to zero. These assumptions are approximately true because u is small, and when the point g is to the right of OC, $\cos\theta_1$ is negative and $\sin\theta_1$ is positive, and when the point g is to the left of OC, both $\cos\theta_1$ and $\sin\theta_1$ are negative. Considering these integrals as zero, they may also be canceled and Eqs. (246), (247), and (248) reduce to the following equations by noting that $c/E' = 3/E$:

$$X = \frac{\int_0^s \frac{N_0\cos(\theta_1 - u)ds}{A} + \int_0^s \frac{M_0 y\cos u\,ds}{I} - 3\int_0^s \frac{V_0\sin(\theta_1 - u)ds}{A}}{\int_0^s \frac{\cos^2(\theta_1 - u)ds}{A} + \int_0^s \frac{y^2\cos^2 u\,ds}{I} + 3\int_0^s \frac{\sin^2(\theta_1 - u)ds}{A}}; \quad (249)$$

$$Y = \frac{\int_0^s \frac{N_0\sin\theta_1\,ds}{A} - \int_0^s \frac{M_0 x\,ds}{I} + 3\int_0^s \frac{V_0\cos\theta_1\,ds}{A}}{\int_0^s \frac{\sin^2\theta_1\,ds}{A} + \int_0^s \frac{x^2 ds}{I} + 3\int_0^s \frac{\cos^2\theta_1 ds}{A}}; \quad (250)$$

$$Z = \frac{\int_0^s \frac{M_0 ds}{I}}{\int_0^s \frac{ds}{I}}. \quad (251)$$

16-11. Stresses Due to Water Load on Cantilever.—For the purpose of determining the values of N_0, M_0, and V_0 used in Eqs. (246), (247), and (248), let the left abutment in Fig. 47 be removed so that the arch ring will swing under water pressure as a cantilever from the right abutment. Let g be any point on the neutral axis and assume lines to

be drawn as shown in the figure. Now the water pressure on a length dL of the extrados for a thickness vertically of 1 ft. is pdL, where p is the unit water pressure per square foot. Resolve the water pressure on dL perpendicular to the section SS, parallel to the section SS, and around the point g in a clockwise direction. In this process, it follows that

$$dN_0 = pdL \sin (180° - \theta_1 + \phi) = -pR\,d\phi \sin (\phi - \theta_1);$$

$$dM_0 = -pdL\left(R - \frac{t}{2}\right) \sin (\phi - \phi_1) = -pRd\phi\left(R - \frac{t}{2}\right) \sin (\phi - \phi_1)$$

very closely; and

$$dV_0 = -pdL \cos (180° - \theta_1 + \phi) = pRd\phi \cos (\phi - \theta_1).$$

Taking the integrals of these expressions with respect to ϕ between the limits of ϕ_a and ϕ_1, the following equations are derived:

$$N_0 = -\int_{\phi_1}^{\phi_a} pR \sin (\phi - \theta_1)d\phi = -pR\,[\cos (\phi_1 - \theta_1) - \cos (\phi_a - \theta_1)]; \quad (252)$$

$$M_0 = -\int_{\phi_1}^{\phi_a} pR\left(R - \frac{t}{2}\right) \sin (\phi - \phi_1)d\phi = -pR\left(R - \frac{t}{2}\right)[1 - \cos (\phi_a - \phi_1)]; \quad (253)$$

$$V_0 = \int_{\phi_1}^{\phi_a} pR \cos (\phi - \theta_1)d\phi = pR \sin [(\phi_a - \theta_1) - \sin (\phi_1 - \theta_1)]. \quad (254)$$

These values may now be substituted in Eqs. (246), (247), and (248), or in Eqs. (249), (250), and (251), for complete evaluation of redundant forces X, Y, and Z.

16-12. Unit Stresses.—The total thrust, moment, and shear at the center of any voussoir can now be found by substituting the values of N, M, and V, and X, Y, and Z, and the known values of the trigonometric and geometric quantities in Eqs. (243), (244), and (245). The maximum

compressive stresses can then be computed from Eq. (174) and the maximum unit shearing stresses from Eq. (177).

16-13. Stresses Due to Temperature Changes.—Temperature stresses can best be found with sufficient accuracy by construction of a symmetrical arch ring as nearly conforming to the unsymmetrical ring as possible, and finding the stresses in this symmetrical ring by the method given in Chap. XII.

CHAPTER XVII

DESIGN OF SINGLE-ARCH DAMS

INTRODUCTION

17-1. General Statement.—The design of single-arch dams involves the selection of arch rings of suitable shape to resist the water pressure and to fit into the canyon walls in such manner as to give an economical, stable, and presentable structure. Consideration has to be given also to the base and abutment foundations of the dam and to the accessories necessary for its proper functioning structurally and practically, for the purpose for which it is constructed.

DETERMINATION OF TENTATIVE TYPE AND DIMENSIONS OF ARCH

17-2. Dimensions of Arch Rings.—The required shape of the arch ring depends on the length of its radius, the size of the central angle, the length of the neutral axis of the ring, and the water pressure to which the ring will be subjected. For a given water load, the stress in a thin arch ring increases as the radius increases, and decreases as the thickness increases. Consequently, a reduction in the radius causes a reduction in the stresses and this permits a reduction in the thickness of the ring, but it also increases the length of the ring for a given span. It has been shown that the gain in economy due to a reduction in thickness reaches a maximum when the radius equals about five-ninths the span, which corresponds to a central angle of about 134 deg. The theoretical advantage of the large central angle changes slowly for values near the economical angle, and the slenderness ratio of length to thickness indicates a practical limit of about 120 deg. The slenderness ratio generally ranges from about 30 to 50. A rough rule that is sometimes applied to the ratio of span to radius is four-

thirds for the bottom arch, the ratio increasing gradually to nine-fifths for the top arch. This will correspond to a range of central angle varying from about 84 deg. at the bottom to about 120 deg. at the top. With these rough limitations placed on the radius and central angle, the thickness of the ring is usually first determined roughly by Eq. (168), then checked and modified by the equations of Chap. XIII covering symmetrical, single-centered, thin arch rings of uniform thickness. A ring of this type with reasonable thickness and freedom from tension at the crown or abutments will generally be found applicable to reasonable spans at the top of high dams and to most spans of low dams. The lower rings for high dams will have to be modified to secure economy of material and to avoid tension in the masonry at the crown and abutments. Tension in cases of moderate pressure and length of span may be obviated by use of three-centered or polycentered rings of uniform thickness; but the greatest economy for the heavier pressures in high dams will be obtained by use of the arch ring of variable thickness. In any event the final shape will of necessity be the result of several trials and readjustments.

17-3. Symmetry of Arch Rings.—One of the difficult problems encountered in design is that of securing simplicity of analysis through the selection of the layout that will make all of the arch rings symmetrical, or nearly so. If the cross-section of the canyon at the dam site is an isosceles triangle, an isosceles trapezoid, or an equiangular parallelogram, or any symmetrical curve with a vertical axis, the crown points in the neutral axes of the arches and the centers of gravity or of symmetry of the neutral axes will all lie in a vertical plane. If the triangle or trapezoid is oblique, the parallelogram is non-equiangular, or the curve is symmetrical with respect to an oblique plane, then the plane containing the crown points of the neutral axes and their centers of gravity or of symmetry will be in an oblique plane. In either of these cases the arch rings will be symmetrical and symmetrically loaded under the proper layout. If, however, the cross-section of the canyon does

not conform to the rectilinear boundary of the triangle, trapezoid, or parallelogram, or to the boundary of a curve symmetrical with respect to an oblique plane, the various arch rings cannot be made symmetrical with reference to a single vertical or inclined plane. Usually, however, the irregularities in the sides of the canyon walls are so small that the distance of the crown points on the neutral axes, or their centers of curvature from a vertical or an inclined plane, is so small as to involve only a slight error in considering the arch rings symmetrical with reference to the plane. In such cases all of the arch rings are generally computed as though symmetrical with reference to the plane of centers. Of course, it is evident that no difficulty arises in regard to symmetry for arch rings of uniform thickness and constant curvature, but that difficulties regarding symmetry do arise in arch rings of variable thickness and variable curvature.

17-4. Unsymmetrical Arch Rings.—Cases sometimes occur where the canyon walls are very irregular, causing the arch rings to be so unsymmetrical with reference to a single plane that the results obtained by the methods applicable to symmetrical rings are considerably in error for rings of variable thickness and variable curvature. In these cases it is necessary to abandon the symmetrical layout idea and resort to the use of unsymmetrical rings. In these cases the layout will consist in placing the thinnest parts of arch rings of variable thickness and the same points of curvature in arch rings of variable curvature in an assumed plane of canyon symmetry, making the rings symmetrical on each side of this plane for a distance equal to that to the nearest abutment, and then extending the rings to the more distant abutment with suitably increased thicknesses and changes in radii of curvature. The method of analysis of such arch rings is described in Chap. XVI.

17-5. Freeboard, Cutoff Walls, Contraction Joints, Water Stops, and Grouting.—The freeboard, cutoff wall, contraction joint, water stop, and grouting requirements

for arch dams are similar to those for non-overflow gravity dams. In this connection reference should be made to paragraphs 8–14 to 8–17, inclusive.

ANALYSIS OF STRESSES

17-6. General Analysis.—After deciding on the general type of arch ring to be used for the design, and reducing the shape to the approximate shape desired by trial methods through the employment of the cylinder formula and the single-centered thin-arch-ring formulas, it is then necessary to make two or more trial analyses for the type selected by the formulas applicable thereto. The stresses found in these general analyses must take into consideration certain adjustments for conditions at the abutments that are variable for different dam sites and different levels in the same site. These adjustments will now be considered.

17-7. Changes in Volume of Arch Rings.—As stated in paragraph 6–16, the volume of the concrete changes with variations in temperature and water content. In setting, the concrete shrinks from about 0.0003 to 0.0005 of its length, which corresponds to a drop in temperature of from 50 to 83°F.

The stresses produced in arch rings by temperature changes are analogous to those caused by rib shortening, a drop in temperature corresponding to a negative value for T in Eq. (201). The laws governing the variation of temperature in mass concrete are not well known; consequently it has been customary in arch design to assume that the temperature in the arch rings is uniform from one surface to the other. This, it will be observed, is an approximation because when the reservoir is full the temperature of the upstream face will approximate the temperature of the water in the reservoir, while that of the downstream face will approximate the temperature of the air, and that within the body of the dam will be different from either of these. It will, however, be reasonably safe to assume that the range of temperature within the dam is the same as that of the water in the reservoir, which is approximately the

same as the difference between the mean-summer and mean-winter temperatures. From consideration of the few temperature records kept for mass concrete, the following empirical equation may be taken as expressing approximately the relation of the outside range of temperature to the interior temperature of the concrete.

$$y = \frac{C}{\sqrt{x}}, \tag{255}$$

in which y is the range of temperature at any point within the dam in degrees Fahrenheit, x the distance in feet of that point from the face of the dam, and C is a constant. C should be varied to suit the climatical conditions at the dam site, but for average conditions it may be taken as 60. Since in design the mean temperature of the concrete must be taken, it is necessary to find the mean value of y over the half thickness of the arch ring under consideration. Thus,

$$\bar{y} = \int_0^x y\,dx \div \int_0^x dx = \int_0^x \frac{C\,dx}{\sqrt{x}} \div \int_0^x dx = 2C\sqrt{x} \div x$$

or

$$\bar{y} = \frac{2C}{\sqrt{x}}. \tag{256}$$

If $C = 60$ and $x = 4$ for an arch ring 8 ft. thick, $y = 60°$, and if $C = 60$ and $x = 49$ for an arch ring 98 ft. thick $\bar{y} = 17°$. Equation (256) applies to dams with expansion joints grouted after dissipation of the setting heat of the concrete. If the dam is grouted when the concrete is at the mean seasonal temperature, one-half of the value of y as computed from Eq. (256) may be taken, the stresses being computed for a drop in temperature of that amount and combined with the stresses due to full water load. For a complete discussion of this subject see Houk on "Temperature Variations in Concrete Dams."[1]

[1] *Western Construction News*, Vol. 5, pp. 601–608, Dec. 10, 1930.

The change of volume from water soaking is also variable throughout the cross-section of the dam due to the different degrees of hydration for the different parts of the rings. The expansion due to water-soaking concrete is approximately equal to the shrinkage in setting and, inasmuch as dams probably never completely dry out in curing of the concrete and are liable to be wet on the upstream face, it has been customary to reduce the effects of shrinkage in design. Consequently, it is generally considered in the design of arch dams that the shrinkage of concrete due to the combined effect of curing and water soaking corresponds to a drop of temperature of from 10° to 20°F. It is probable that the total change due to temperature and change in water content should never be taken greater than 20 to 30°.

17-8. Effect of Oblique Abutments on Length of Arch Ring.[1]—In the analysis of stresses in arch dams, either with rings of variable thickness or rings of uniform thickness, account has to be taken of the effects on the stresses due to stepped abutments. This condition is generally encountered in the design of arch dams, and provisions for determining its effects must be considered. In computing the stresses in the arch rings of arch dams, the canyon walls against which the arch rings abut are considered to be inelastic.[2] While this assumption is not exact, it is sufficiently so for all practical purposes and is provided against by the use of a factor of safety large enough to cover it. As a matter of economy, excavations for the arch rings in the canyon walls are not made to lines perpendicular to the neutral axes of the rings, but are made either to planes oblique to the neutral axes of the rings or are stepped along such planes. As a result the ends of the segments of the arch ring are not bounded by planes radial to the neutral axes. The question of whether the final radial plane shall be passed through the point where the intrados, extrados,

[1] *Western Construction News*, Vol. 3, pp. 574–577, Sept. 10, 1928.

[2] Vogt, Frederick, "Computations of the Deformations of Foundations," Oslo, 1925.

or neutral axes meet the canyon wall, must be settled before the rings can be properly analyzed.

In Figure 49*a*, representing a fixed-ended, horizontal arch ring, 1 ft. thick, of an arch dam under water load with extrados, intrados, and neutral axis of any suitable shape, let the arch ring be fixed to the abutments along the vertical planes through DB'' and KL'' making angles with radial lines OD and OL, and an angular wedge DBB'' at the left end of the ring and a similar one at the right end. If the computation of stresses in the semiarch ring is based on the assumption of the ring extending to the radial plane through $D''B''$, there will be a large wedge of inelastic material included in the ring, and incorrect results will follow. If it is based on the assumption that the ring extends to the radial plane through $D'B'$, there will be a semi-wedge of inelastic material included in, and a semi-wedge of elastic material excluded from, the ring, which is not entirely satisfactory. If the computation of stresses is based on the assumption that the ring ends at the radial plane through DB, there will be a wedge of elastic material excluded from the ring. It would appear, therefore, that the most accurate results would be obtained by finding the elastic displacement through the internal work done by the ring between the planes DB and KL and the abutments.

17-9. Internal Work of Arch Wedges at Abutments.—Referring to Fig. 49*b* it may be proved that the internal work due to the deflection of the triangular voussoir DKH is approximately equal to that of a rectangular voussoir whose width is equal to the median line of the triangle DKH. If the internal work in these two voussoirs is equal, it follows that the deflections are approximately equal and, therefore, that the rectangular voussoir may be substituted for the triangular one without serious error throughout the arch analysis, or that the arch may be considered to extend to the radial line $D'B'$, except that the water load may be taken only as far as the point D, if a water-tight joint exists between the arch and the abutment. The approximate equivalency of the internal work in the triangular and

rectangular voussoirs will be proven independently for thrust, moment, and shear by the method of least work.

a. In order to find the internal work done by the thrust N on the voussoir DKH, consider an elementary area

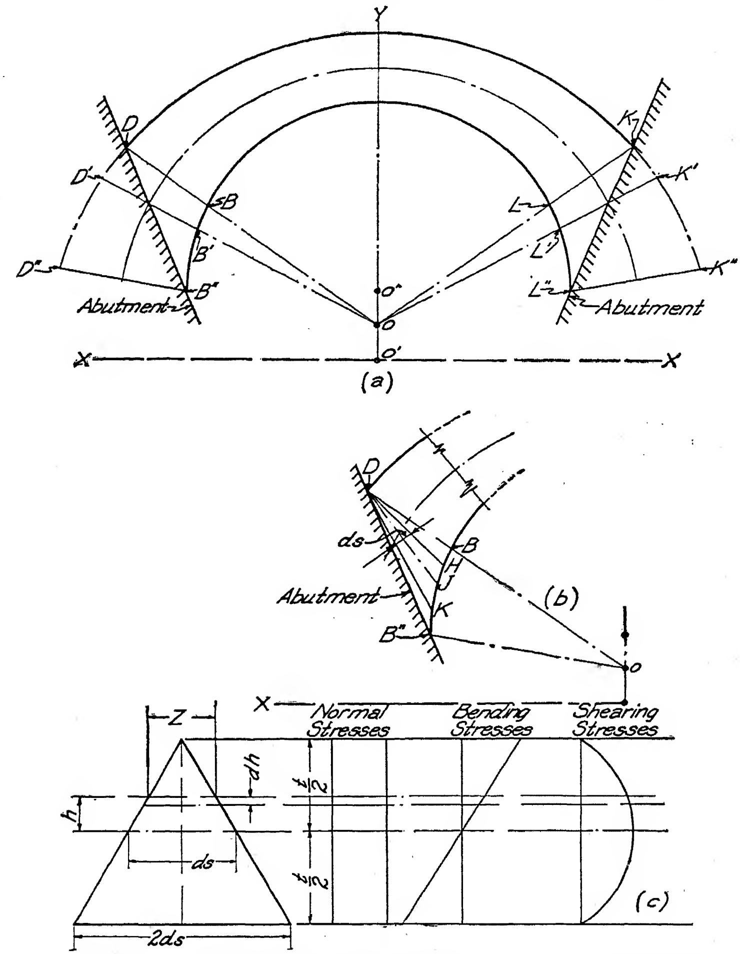

FIG. 49.—Diagram for analysis of triangular voussoirs at abutments.

$1 \cdot dh$ as shown in Fig. 49*c*. The intensity of stress on this area is

$$f_n = \frac{N}{A} = \frac{N}{t}$$

for an arch ring 1 ft. high; and the total stress on it is

$$f_n dh = \frac{N\,dh}{t}.$$

From Hooke's law, the deformation

$$\Delta_n = \frac{\frac{N\,dh\,z}{t}}{dh\,E} = \frac{N\,z}{t\,E}.$$

From similar triangles,

$$z:ds::\frac{t}{2} - h:\frac{t}{2}, \text{ or } z = \left(1 - \frac{2h}{t}\right)ds.$$

Substituting the value of z in the right-hand member of the expression giving the value of Δ_n, it is found that

$$\Delta_n = \frac{N}{tE}\left(1 - \frac{2h}{t}\right)ds.$$

Now the internal work on the entire face of the triangular voussoir is expressed by the equation

$$W_n = \int_{-\frac{t}{2}}^{\frac{t}{2}} \tfrac{1}{2} f_n\,dh\Delta_n. \tag{257}$$

Substituting the value of $f_n\,dh$ and Δ_n in Eq. (257), integrating between the given limits, reducing and noting that $t = A$ since the face of the voussoir is 1 ft. high and t ft. long, the following equation is derived:

$$W_n = \frac{N^2 ds}{2\,AE}. \tag{258}$$

Equation (258), it will be noted, is the expression for the internal work of a rectangular voussoir of length ds and end area A, under the thrust N.

b. The internal work on the triangular voussoir due to moment may be found by considering the infinitesimal area dh, as in the case of the thrust. From the principle of flexure due to moment, the intensity of stress f_m on dh can be found in terms of the intensity of stress at the edges of

the arch ring, under the assumption made, by observing similarity of triangles. If f is taken as the intensity of stress at the edges, then

$$f_m : f :: h : \frac{t}{2},$$

or

$$f_m = \frac{2fh}{t}.$$

The total stress on dh is therefore

$$f_m dh = \frac{2fh\,dh}{t}.$$

The deflection of the prisms with end area dh due to this load is from Hooke's law

$$\Delta_m = \frac{\frac{2fh\,dh\,z}{t}}{dh\,E} = \frac{2fhz}{tE}.$$

From the law of internal work for a gradually applied force,

$$dW_m = \tfrac{1}{2} f_m dh \Delta_m$$

substituting the values of $f_m dh$ and Δ_m as just found in this expression and expressing the integration between the limits of $\frac{t}{2}$ and $-\frac{t}{2}$, there results

$$W_m = \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{2f^2h^2z\,dh}{t^2E}. \tag{259}$$

From a principle of mechanics,

$$f = \frac{M\frac{t}{2}}{I} = \frac{Mt}{2I}$$

and as before

$$z = \left(1 - \frac{2h}{t}\right)ds.$$

Substitution of these values in Eq. (259) integrating the variable h between the limits of $\frac{t}{2}$ and $\frac{-t}{2}$, noting that the integration of

$$\left(h^2 - \frac{2h^3}{t}\right)dh$$

between the given limits equals $\frac{1}{12}t^3$ or I, Eq. (259) reduces to the following equation:

$$W_m = \frac{M^2 ds}{2\,EI}. \tag{260}$$

Equation (260) expresses the internal work due to moment in a rectangular voussoir of length ds and moment of inertia I under a total moment force of M.

c. The internal work on the triangular voussoir due to shear may be found by considering the infinitesimal area dh as in the case of the thrust and moment. If f_v is taken to be the intensity of shear on this elementary area, the shearing force on it will be $f_v dh$. By Hooke's law, the deformation of the prism with an end area dh and length z will be given by the expression

$$\Delta_v = \frac{f_v dh\, z}{dhE'} = \frac{f_v z}{E'}.$$

Since the internal work equals one-half the applied force times the deformation,

$$dW_v = \frac{1}{2}\left(f_v dh \frac{f_v z}{E'}\right) = \frac{f_v^2 dh\, z}{2E'}. \tag{261}$$

The distribution of shear may be taken to conform to a parabolic curve with the intensity of shear at the neutral axis equal to $3V/2t$ and at a distance h from the neutral axis equal to

$$\frac{V}{t}\left(\frac{3}{2} - \frac{6h}{t^2}\right),$$

where V is the total shear on the section. Substituting the value of f_v in Eq. (261), and expressing the integration

between the limits of $\frac{t}{2}$ and $\frac{-t}{2}$, the following equation is derived:

$$W_v = \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{V^2\left(\frac{3}{2} - \frac{6h^2}{t^2}\right)^2 dh\left(1 - \frac{2h}{t}\right)ds}{2t^2E'}. \tag{262}$$

Then, by taking the constants from under the integral sign, Eq. (262) may be written as follows:

$$W_v = \frac{V^2ds}{2t^2E'}\int_{-\frac{t}{2}}^{\frac{t}{2}}\left(\frac{3}{2} - \frac{6h^2}{t^2}\right)^2\left(1 - \frac{2h}{t}\right)dh. \tag{263}$$

By performing the integration between the expressed limits, and noting that $t = A$, Eq. (263) may be reduced to the following:

$$W_v = \frac{CV^2\,ds}{2AE'}, \tag{264}$$

the coefficient C, which equals six-fifths, being applied because of a lack of uniform distribution of shear and inexact radial deformation. Equation (264), however, expresses the internal work due to shear in a rectangular voussoir of length ds and end area A under a total shearing force V.

Having established the principle of approximate equivalency of the deformation of the triangular and the rectangular voussoirs, the wedge of material DBB'' and KLL'' of Fig. 49*a* may be considered to be rectangular portions of the arch ring of widths equal to the median of the triangular voussoir. This is equivalent to assuming the arch ring to extend to the point of intersection of the neutral axis with the abutment. The portions of the ring between D and D' and between K and K' may not be subjected to water pressure, and consequently the redundant forces should be computed for the water load over the arch to the points D and K only. Although the arch ring is loaded only between points D and K, it is nevertheless elastic between points D' and K', and the effect of this

length of arch should be considered in determining the thrust, moment, and shear in the arch ring. However, since the lengths DD' and KK' are usually small and it will be on the side of safety to consider the water load to extend to the points D' and K', the arch is generally taken with full water load to these points. This is the usual practice in the design of an arch dam.

17-10. Adjustment for Abutment Extradosal Tension.[1]—In relatively flat and thick arch rings of arch dams where the water pressure is intense, it is difficult to entirely elimi-

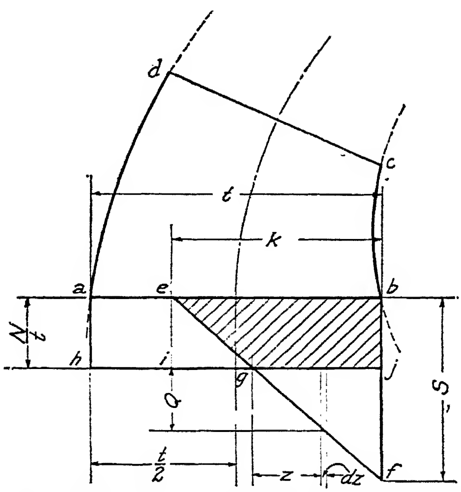

FIG. 50.—Diagram for adjustment of extradosal tension.

nate tension at the extrados at the abutments; and since it is customary to assume that the masonry is incapable of withstanding tension and since the non-resistance to tension affects the internal work of the bending moment, the deficiency of support must be compensated for by increased compression in the ring section. The method for determining the approximate increased compressive stress will now be described.

17-11. Formula for Adjustment for Abutment Extradosal Tension.—Let Fig. 50 represent a segment of a horizontal arch ring of an arch dam bounded by normal

[1] *Western Construction News*, Vol. 3, pp. 574–577, Sept. 10, 1928.

planes ab and cd, and assume that tension has been found at the extrados on the plane ab by the ordinary elastic displacement theory. Let N, M, S, t, and k represent, respectively, the normal thrust and the moment computed by the usual method, the revised maximum normal unit stress, the total thickness of the arch ring on the plane ab, and the compressively stressed thickness on this plane. Let it further be desired to find the value of S in terms of N, M, and t. Let the triangle bef represent the unit normal stresses on the compressively stressed portion of ab, based on the ordinary theory that sections plane before bending are plane after bending, for the compressively stressed portion thereof. Now, it is evident from Fig. 50 that

$$N = \tfrac{1}{2}kS; \tag{265}$$

$$M = \tfrac{1}{2}kS(\tfrac{1}{2}t - \tfrac{1}{3}k); \tag{266}$$

whence by eliminating S from Eqs. (265) and (266),

$$k = 3\left(\frac{t}{2} - \frac{M}{N}\right); \tag{267}$$

and by solving for S from Eq. (265),

$$S = \frac{2N}{k}. \tag{268}$$

Substituting the value of k from Eq. (267) in Eq. (268),

$$S = \frac{2N}{3\left(\frac{t}{2} - \frac{M}{N}\right)}. \tag{269}$$

Equation (269) provides a means for finding the approximate normal maximum stress on the section ab, and if the maximum inclined stress is desired, this value must be multiplied by the square of the cosecant of the angle between the plane ab and the tangent to the intrados at the point b.

17-12. Accuracy of Formula for Adjustment for Abutment Extradosal Tension.—In order to show the accuracy of Eq. (269) for practical purposes, let ds be the length of the neutral axis or of the arch segment $abcd$, of Fig. 50,

to which the total thrust N is applied first, and the total moment M subsequently. Let the rectangle *abjh* represent the total stress N on *ab*; then the deflection Δ_n, due to the uniformly distributed thrust for voussoir *abcd* only as found by the unit-load method is

$$\frac{Nnds}{AE} = \frac{Nnds}{tE}.$$

When the moment is applied, the compressive stresses due to the thrust designated by the trapezoid *aegh* are neutralized and the remaining compressive stresses are increased by the triangle *fgj*, thus changing the stress figure from the rectangle *abjh* to the triangle *ebf* and creating an eccentricity of thrust necessary to develop the required moment resistance.

The unit stresses Q subtracted from or added to the stresses which would correspond to a uniformly distributed thrust, and caused by the separate application of the moment M, may be considered as represented by the rectangle *aeih* plus the triangle *egi* plus the triangle *fgj*, leaving the trapezoid *ebjg* undisturbed. Now, by reference to Fig. 50 and the application of the unit load method, Δ_m, the portion of the deflection due to the moment effect in the length ds only, may be expressed in the following general terms:

$$\Delta_m = \int\left(\frac{Q}{E}\cdot ds\right)\left(\frac{m}{M}\cdot Q\right)dz = \frac{ds}{E}\int Q^2\frac{m}{M}dz. \tag{270}$$

In the rectangle *aeih*, $Q = N/t$; in the triangle *egi*, $Q = zS/k$ by similar triangles; and in the triangle *fgj*, similarly, $Q = zS/k$. Substituting the different values of Q in Eq. (74), and noting its application to each stress diagram, separately,

$$\Delta_m = \frac{ds}{E}\left[\left(\frac{N}{t}\right)^2\frac{m}{M}(t-k) + \int_0^{\frac{kn}{St}}\left(\frac{zS}{k}\right)^2\frac{m}{M}\cdot dz + \int_0^{k-\frac{kN}{St}}\left(\frac{zS}{k}\right)^2\frac{m}{M}\cdot dz\right]. \tag{271}$$

By integration between the limits given and reduction,

$$\Delta_m = \frac{ds}{E}\frac{m}{M}\left(\frac{N^2}{t} + \frac{kS^2}{3} - \frac{kSN}{t}\right). \tag{272}$$

Solving for S in Eq. (266),

$$S = \frac{M}{\frac{1}{2}k\left(\frac{t}{2} - \frac{k}{3}\right)}; \tag{273}$$

and finding N from a combination of Eqs. (265) and (273),

$$N = \frac{M}{\frac{t}{2} - \frac{k}{3}}. \tag{274}$$

Substituting the values S and N from Eqs. (273) and (274) in Eq. (272),

$$\Delta_m = \frac{ds}{E} \cdot Mm\left(\frac{48t - 36k}{kt(3t - 2k)^2}\right). \tag{275}$$

From Eq. (275),

$$\Delta_m = \frac{Mm\,ds}{EI},$$

for $k = t$; and

$$\Delta_m = \frac{1.04Mm\,ds}{EI},$$

for $k = 3t/4$; and

$$\Delta_m = \frac{1.25Mm\,ds}{EI},$$

for $k = t/2$. Since the value k is rarely less than $3t/4$ and since tension rarely exists over any large proportion of the arch ring, Δ_m may without appreciable error be taken equal to $Mm\,ds/EI$, as in the usual elastic-displacement formulas. Consequently no recalculation of the entire ring stresses is necessary and the maximum stress on the section ab is given by Eq. (269). The safety of the approximation contained in Eq. (269) is further augmented by the fact that the increase of internal work, due to the narrowing of the section, results in an increase of deflection tending to bring into action an increased thrust and shear not included

in Eq. (269). Attention is directed to the fact that tensional cracks in the extrados may admit water pressure into the crack between the arch ring and the abutment which may tend to increase the stresses in the ring, unless guarded against by carrying an upstream cutoff trench with drains up to an elevation above which no tensional stresses occur in the arch.

As an illustration of the increase of the compressive stress due to the adjustment under discussion, assume a unit stress per square inch of 600 lb. compression and 100 lb. tension in an arch ring. The general formula for finding stresses in a right section of an arch ring after the thrust and moment have been determined, is

$$f = \frac{N}{t} \pm \frac{6M}{t^2}, \tag{276}$$

whence

$$-100 = \frac{N}{t} - \frac{6M}{t^2}; \tag{277}$$

and

$$+600 = \frac{N}{t} + \frac{6M}{t^2}, \tag{278}$$

whence

$$N = 250t \tag{279}$$

and

$$M = \frac{350t^2}{6}. \tag{280}$$

Substituting the values of N and M in Eq. (269), $S = 625$, which shows that the compressive stress is increased from 600 lb. per square inch to approximately 625 lb. per square inch, due to the neglect of tensile stresses. From Eq. (267), k will be found equal to $0.8t$, that is the compressive stress extends from the intrados over 0.8 of the distance to the extrados under this assumption.

17-13. The Effect of Overhanging Arch Rings.—The effect of overhanging of the arch ring downstream at the crown will be found to be beneficial in the control of the central angles and radii, and in carrying the load to

the abutments by inclining the arch action. However, it complicates the arch-ring analyses. An equally important help can be obtained in the adjustment of the radii and central angles through overhang in an upstream direction adjacent to the abutments. Abutment overhang, however, inclines the arch rings upward and carries the stresses upward at the abutments. This overhang usually occurs near the base of the dam, however, where the weight of the superstructure and the shortness of the arches counterbalance the bad effect. Judicious handling of the combination of an overhang downstream at the crown near the top of the dam and of an overhang upstream at the abutments near the base of the dam can be made to be advantageous in the design of arch dams. The overhang in any case should, however, never be enough to involve the gravitational stability of the structure.

17-14. Earthquake Shocks.—The forces of earthquake shocks on an arch dam for reservoir full or reservoir empty are to be found, the same as for gravity dams, as discussed in Chap. V. In resistance, however, there is some difference between the two types of dams. The arch dam is probably more capable of withstanding the shock of the earthquake when the reservoir is full than the gravity dam; but when the reservoir is empty it has less resisting power against failure under upstream vibrations on account of its lesser thickness. In earthquake regions, investigations of the arch dam's resistance against failure under the upstream force of earthquakes should be made in accordance with the principles given in Chap. V.

17-15. Holes through Arch Dams.—Holes through arch dams are sometimes necessary for draining the reservoir and as general outlets for water for irrigation, power, and flow control, but they are undesirable when tunnel outlets or spillways can be provided to supplant them. The stresses produced by such holes are similar to those in a gravity dam discussed in Chap. VIII, except that the line of the principal action is at right angles to that in the gravity type.

17-16. Spillways for Arch Dams.—Spillways for reservoirs controlled by arch dams should, if practicable, be independent of the arched section of the dam. Often they are combined with one or both of the abutments of the arch or built at a distance from the dam at some low point in the reservoir boundary. Sometimes, however, they may have to be placed on top of the arch. In such cases provision should be made for admission of air behind the overflowing sheet of water to prevent the forming of a partial vacuum; a water cushion should be provided to break the impact of the falling water at the toe of the arch, or a riprap of very heavy stones provided to absorb the impact of the water; and the arch should be thickened at the top to withstand the impact of logs and ice, and the added pressure of the water. In case gates are to be installed on top of the arch for control of water level, the overturning moment of the water pressure on them should be provided against by reinforcing the gate piers with steel bars and carrying them down the upstream face of the arch far enough to prevent overstressing the upper arch rings and serious tension in the concrete on horizontal sections. The subject of gates is fully discussed in Chap. XXI.

17-17. Working Stresses for Single-arch Dams.—The allowable working stresses for the foundation materials for single-arch dams should be the same as those for gravity dams as given in paragraph 6-7; and those in the plain concrete part of the dam should be the same as those just referred to, except that on account of the difficulty of avoiding tension at the crown and the abutments, maximum tensional stresses to the extent of 100 lb. per square inch may be allowed in the arch. For any reinforced concrete used in connection with a single-arch dam, the working stresses given in paragraph 10-13 should be used.

FOUNDATION AND ABUTMENTS

17-18. General Statement.—As the foundation at the base of the dam must be capable of withstanding the weight of the dam, the base width of the structure must be

made great enough to bear the pressures to which it will be subjected. The foundation should also be excavated for a cutoff wall at the heel and should be grouted thoroughly to prevent excessive seepage or piping. The abutments should be excavated to solid rock, fully capable of carrying the thrust of the arch rings, and should be properly shaped to receive the thrust of the arch. Care should be taken to seal the arch ring to the abutment walls so that seepage cannot pass around the ends of the arch.

17-19. Artificial Abutments.—The excessive weathering of the canyon walls near the top of most dam sites so increases the canyon width as to make the design of the upper part of a high arch dam either uneconomical or unfeasible without the construction of abutments to take the place of the missing canyon walls. Without such abutments, the arch has to be made too flat for economy in order to intersect the canyon walls at suitable angles. Two types of artificial abutments are used to overcome this difficulty, namely: the tangential abutment and the forked abutment.

17-20. Artificial Tangential Abutment.—The tangential abutment is built in the line of the arch thrust and carried to the full height of the arch. In this case the abutment is usually made thicker than the arch ring so as to become relatively inelastic, and the stress line is carried in the abutment both vertically and laterally in such manner as to keep the center of pressure within the middle-third limit.

17-21. Artificial Forked Abutment.—Where the artificial forked abutment is used, the upstream fork usually serves in part as a gravity dam of short length to close the reservoir and also sometimes, in part, as a buttress to resist the arch thrust, and the downstream fork serves only as a buttress to take the remaining thrust of the arch. The buttress is designed with a downward sloping top so that its weight, when combined with the arch thrust taken by it above any horizontal plane, will meet that plane at an angle less than the angle of friction. The practice of taking advantage of inclined arch action to assist the buttress resistance is

questionable as the amount carried by such inclined arch action is not easily determined and the extra cost of the buttress due to embodying this additional factor of safety is usually not great. The downward slope of a buttress designed to take the arch thrust will usually be found to lie between 2:1 and 2.5:1 where the whole arch thrust is

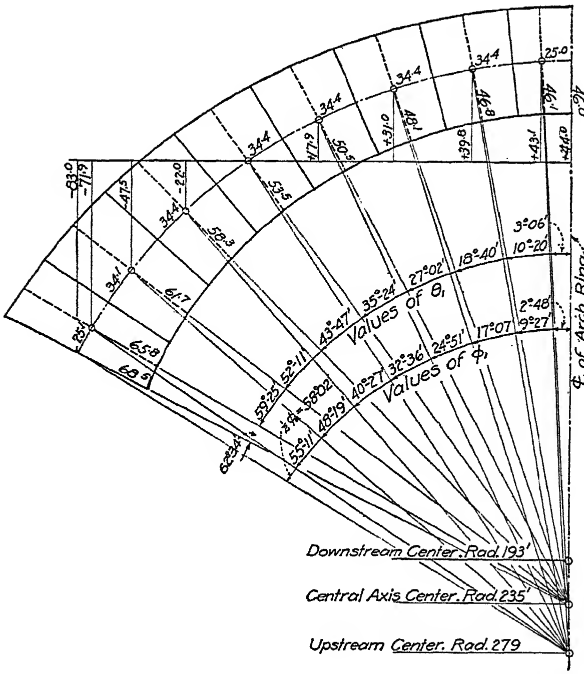

FIG. 51.—Layout of symmetrical compound circular arch ring (see Fig. 40 for computations of stresses in arch ring).

taken into the buttress. In some cases the three-forked abutment is used. In such cases a gravity section closing the reservoir is run perpendicularly to the canyon wall to secure the shortest line; a second fork, perpendicular to the arch ring at the springing line, is used to take the radial thrust; and a third fork slopes from the ends of the arch in line with the remaining thrust.

17-22. Computing and Recording Stresses in Single-arch Dams.—In order to set forth the principles of computation

and the methods of recording them in a logical manner, an analysis has been made and the results recorded in Fig. 40, in Chap. XII, for a typical symmetrical and symmetrically water-loaded, single-centered, fixed-ended thin arch ring. Figure 51 shows the complete layout of this arch ring.

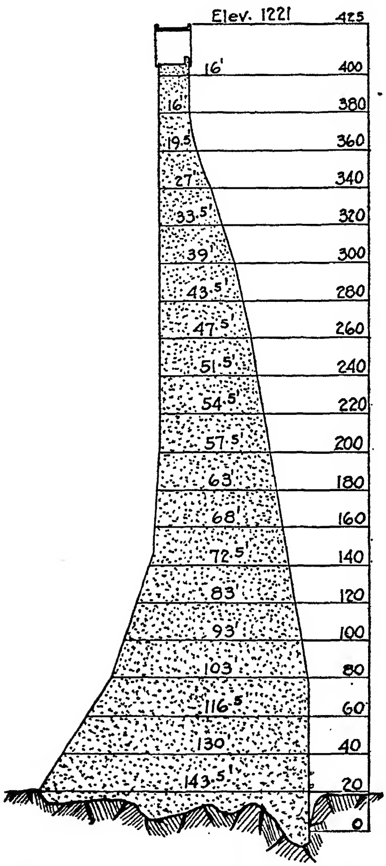

FIG. 52.—Section of Diablo arch dam (Seattle, Wash.).

DIABLO DAM

17-23. Location and Use.—The Diablo Dam is located on the Skagit River about 102 miles northwesterly from Seattle in Washington and is being constructed by that city as an adjunct to its municipal power system. The initial development at the dam contemplates the installed capacity of 90,000 kw. with a proposed future extension to a total of 180,000 kw.

17-24. Character of Foundation.—The dam site is located in a precipitate canyon of glacial-stripped gneissic rock. The canyon walls are nearly vertical for 180 ft. above the stream bed and are only from 40 to 100 ft. apart. Above this elevation the canyon gradually widens. As a result it will be observed that the character of the rock and of the site are adapted to the construction of a single-arch dam with the upper part supported by artificial abutments.

17-25. Type and Dimensions of Dam.—The dam as constructed is comprised of an arch with constructed abutments in the top 125 ft., which are designed to act as abut-

ments for the upper part of the arch and as gravity dams and spillways. The arch portion of the dam is 587.68 ft. long on the crest and has an upstream radius of 390 ft. It will be observed from Fig. 52 that the dam is vertical at the crown intrados from the top down 205 ft., and from there to the bottom is battered downstream. The following table gives the extradosal and intradosal radii, the crown and abutment thicknesses, and the central angle at various levels from the top to the bottom of the dam.

DIMENSIONS OF DIABLO DAM

Height above foundation, ft.	Radius of extrados, ft.	Radius of intrados, ft.	Thickness at crown, ft.	Thickness at abutments, ft.		Central angle, deg.
				Right	Left	
405	390	374	16	16	16	88
400	390	374	16	16	16	88
380	390	374	16	16	16	88
360	378	358.5	19.5	20.5	20.5	91.5
340	358	331	27	29	29	97.5
320	340	306.5	33.5	38	38	104
300	327	288	39	48	48	109
280	315	271.5	43.5	55	57	112
260	304	256.5	47.5	57.5	60	109
240	289.5	238	51.5	64.5	66	109.5
220	274	219.5	54.5	71	71.5	109
200	258.5	201	57.5	76	84	108
180	243.5	180.5	63	86	38	106
160	223	155	68	102	106	102
140	208	135.5	72.5	87	106	74
120	197.5	114.5	83	89	112	67
100	187.5	94.5	93	96	117	67
80	178	75	103	104	123	67
60	163.5	.....	116.5			
40	148.6	.....	130			
20	148.6	.....	143.5			
0		Cutoff trench 20 ft. deep at heel				

17-26. Contraction Joints and Grouting System.—The dam is divided into eight sections by contraction joints spaced 73.46 ft. apart on the center line of the crest. The joints are not in single planes but are keyed together by

offsets 6 ft. wide and 1 ft. deep. Vertical grout pipes are provided in the centers of the keyways and these are connected by horizontal pipes at various levels. The connecting pipes are carried to the downstream face of the dam. A metal stop is provided in each contraction joint at each face and immediately behind the upstream stop there is placed an asphaltic water stop.

17-27. Character of Concrete.—The dam was designed on the basis of a compressive strength in concrete of 2,000 lb. per square inch at 28 days. Careful tests were made on standard cylinders during the construction of the dam which indicated a 28-day strength in the concrete of 3,800 lb. per square inch. This high strength was secured through careful control of the concrete mix and of the water-cement ratio. The proportioning of the cement and aggregates was secured by automatic weighing machines and the water used was carefully measured. The maximum computed compressive stress in the arch is 750 lb. per square inch and the maximum computed tensile stress is 200 lb. per square inch in one of the lower arch rings.

17-28. Outlets.—Four outlet pipes, 8 ft. in diameter, were constructed in the dam with their center lines at 180 ft. below the top of the dam. These outlet pipes are controlled by Broome gates at the upstream end protected with trash racks. Three of the valves are provided with 78-in. butterfly valves and the other with a 96- by 72-in. needle valve at the downstream end. Water is carried through the dam in riveted steel pipes, which are air vented immediately below the Broome gates with 12-in. pipes carried to the top of the dam.

17-29. Spillway.—The spillway of the dam consists of 20 openings 20 ft. wide with their bottoms 18 ft. below the top of the dam. These openings are controlled by Taintor gates anchored to piers 20 ft. apart face to face, connected at the top with a reinforced-concrete slab with suitable reinforced-concrete parapet walls. A 17-ft. roadway passes over the concrete slab to provide for a traveling gate-hoist for operating the Taintor gates. The roadway is carried

TABLE XVIII.—IMPORTANT SINGLE-ARCH DAMS IN THE UNITED STATES

Name of dam	Location	River	Year completed	Height above foundation, ft.	Height above stream-bed, ft.	Crest length, ft	Upstream radius		Thickness at top, ft.	Thickness at bottom, ft.	Volume of dam, cu. yd.
							Top, ft.	Bottom, ft.			
Diablo	Washington	Skagit	1929	400	...	547.7	390	150	16	98	275,000
Pacoima	California	Pacoima	1929	380	375	450	320	115	8	96	151,000
Shoshone	Wyoming	Shoshone	1910	328	243	200	155	...	10	108	78,600
Horse Mesa	Arizona	....	1927	311	...	764	...	...	..	....	126,000
Lake Spaulding	California	South Yuba	1916	295	275	800	428	250	11	186	192,000
Lake Cushman	Washington	....	1926	280	250	340	200	131	8	46	85,000
Copco No. 1	Oregon	Klamath	1922	257	...	420	356	...	30	95	
Big Santa Anita	California	Santa Anita	1928	225	...	460	270	100	7	41	60,300
Mormon Flat	Arizona	....	1924	224	...	444	...	...	..	...	43,500
Santeetlah	Pennsylvania	Tallassee	....	...	202	340	300	...	10	48	
Salmon River	Idaho	Salmon	1912	220	...	490	225	225	12		
Melones	California	Stanislaus	1926	210	185	590	240	240	10	...	91,000
Bullards Bar	California	Yuba	1923	183	...	440	240	94	6	43	
Salmon Creek	Alaska	Salmon Creek	1913	168	...	545	331	148	6	48	52,000
Sawpit	California	....	1927	...	160	...	265	265	..	...	42,300
Upper Hubbard	Montana	....	1923	131	...	400	220	194	5	22	17,000
Emigrant Creek	Oregon	....	1924	115	...	360	165	132	5	19	
Lost Creek	California	Lost Creek	....	112	...	380	200	90	4	24	11,000
Grizzly Forebay	California	Grizzly Creek	1927	...	88	420	205	140	6	22	11,000

13 ft. above the crest of the main arch for flood clearance. In order to bring the crest of the main arch to the elevation of the roadway over the spillway, 16 reinforced concrete arches of 32-ft. clear spans and a 17-ft. width were constructed over the top of the main arch.

17-30. List of Important Single-arch Dams.—In Table XVIII there are given some of the important single-arch dams. While this list is not exhaustive as to number, it contains some of the most successful single-arch dams in America and the designing engineer contemplating the design of a single-arch dam will find it of value to give this list careful study.

CHAPTER XVIII

DESIGN OF MULTIPLE-ARCH DAMS

INTRODUCTION

18-1. General Statement.—A multiple-arch dam consists essentially of a series of arches carried by buttresses spaced at suitable intervals. The upstream faces of the buttresses usually have considerable slope; and stability against sliding and distribution of foundation pressure are largely controlled by the water weight on the arch extradosal areas. The yardage of concrete is on this account reduced considerably over that required for solid gravity dams although the unit costs are increased at the same time by thinness of sections and necessity for a large amount of forms. The stresses in this type of dam are similar to those in the deck type so far as the buttresses are concerned and to those in the single-arch type so far as the arches are concerned.

18-2. Foundation.—Multiple-arch dams require good foundation materials for the buttresses and arch footings and relatively impervious materials in the foundation area to prevent seepage under the structure. Grouting may be resorted to, to secure these conditions where practicable, and an effective cutoff trench should be provided at the arch footings and buttress heels.

18-3. Buttresses.—The design of buttresses for multiple-arch dams is similar to the design of buttresses for deck dams. The method of finding the vertical and principal stresses at the toe and heel in the body of the buttresses is identical with that for the buttresses of deck dams. The main difference between these two types of dams is that the principal stresses at the upstream faces of the buttresses in the deck dam are carried vertically to the faces of the

buttresses from the deck, whereas those in the multiple-arch dam are derived from the thrust of the arches on the buttresses which may not be perpendicular to their faces but which may be resolved into forces perpendicular, and others parallel, to them. The parallel forces will be counterbalanced by similar forces from the adjacent arch. The Noetzli "H" buttress differs from the solid-type buttress in that two parallel walls, each forming a seat for the adjacent arch, are used. These walls are joined together at the upstream face of the buttress with a small arch, at the downstream face with a face wall, and at an intervening point with a cross-web. Analysis of this type of buttress does not differ from the other type except that consideration must be given to the integrity of the integral parts.

18-4. Arches.—Multiple-arch dams have been built with vertical springing lines and horizontal circular sections; others have been built with sloping springing lines and the arch elements circular in horizontal planes; but the present practice is to build them with the springing lines parallel to the buttress faces. The requirements for stability of the dam necessitate sloping upstream faces for the buttresses. Consequently, since the principle of least work requires the water pressure to be transmitted to the buttresses by the shortest route, the axes of the arch rings should be perpendicular to the buttress faces, thus securing proper resistance to sliding and distribution of foundation pressure of the buttresses. The present practice is to give a slope of from 45 to 55 deg. to the upstream faces of the buttresses, and to make the central angles of the arches from 130 to 150 deg.

PRINCIPLES GOVERNING STRESS ANALYSIS

18-5. Acting Forces on Arch Ring.—In the analysis of arch stresses of the multiple-arch dam, the arch barrel is assumed to be divided into elementary rings by parallel planes, 1 ft. apart and perpendicular to the buttress faces. The forces acting on these rings are those due to (*a*) water pressure, (*b*) weight of the arch, (*c*) shrinkage of con-

crete, and (*d*) changes in temperature. It is incorrect to base the arch analyses on horizontal sections because arch barrels are constructed parallel to the upstream faces of the buttresses and will, therefore, be thinnest where the stresses indicate that they should be thickest, and *vice versa*. Moreover, if it is assumed that stresses on horizontal planes can be determined, the stresses thus found would not be the principal stresses because they would be oblique to the upstream face of the buttresses, which are either actual or assumed planes of principal stresses. The unit stresses due to the acting forces should be computed separately and their sum taken to obtain the total unit stresses in the arch rings.

18-6. Distribution of Water Pressure on Arch Ring.—The water load on an inclined arch ring is generally taken as the mean water pressure on the arch ring inasmuch as the arch ring at the crown is considerably higher than at the abutments. This procedure greatly simplifies the calculations. Greater accuracy, however, may be secured by dividing the water pressure on the extrados into two parts: (*a*) a uniform unit pressure corresponding to the head at the crown; and (*b*) a variable unit pressure increasing from zero at the crown to a maximum at the abutments. In this latter method a complete analysis is made for each of these two loadings and the results combined to obtain the stresses under maximum head. It is not necessary to consider the case where the top arch rings are only partly covered, as the arches are usually thickened at the top of the dam to meet minimum requirements.

18-7. Shape of Arch Ring.—The arches may be made (*a*) of variable thickness with a circular extrados and a circular or compound circular intrados, (*b*) of uniform thickness and circular in section, or (*c*) of uniform thickness and of compound circular section. In general the first type will be found most acceptable for resisting the stresses in the arches of multiple-arch dams.

18-8. Method of Analysis for Uniform Water Pressure.—The stresses in these arch rings may be computed directly

by the formulas given under single-arch dams where the mean water depth is used and for the stresses produced by uniform depth at the crown referred to in the second preceding paragraph, care being taken to apply the proper formulas for the type of arch under design, namely: arch rings of variable thickness with circular extradoses and circular or compound circular intradoses, circular arch rings of uniform thickness, and compound circular arch rings of uniform thickness.

18-9. Method of Analysis for Variable Water Pressure.—The analysis for the stresses produced by the water load assumed to be zero at the crown of the arch and increasing to the abutments requires some modification of the formulas given for single-arch dams. If the arch ring is of variable thickness with a circular extrados and a circular or compound circular intrados, the equations and the record form of Chap. XII may be applied, and if it is circular with uniform thickness, the equations of Chap. XIII may be applied. If the arch ring is a compound circular one of uniform thickness, or variable thickness, the equations and the record form of Chap. XIV may be applied. While it is possible to obtain integral expressions for the variable water load for the circular arch ring of uniform thickness, this process gives long cumbersome equations, and it is better to use the voussoir method for this type of ring also. In applying these equations, it is first necessary to develop an expression for the variable water pressure in terms of the water pressure at the level of the neutral axis at a specific elevation.

18-10. Stresses Due to Variable Water Load on Cantilever.—Let Fig. 53 represent horizontal and vertical projections of the left half of the fixed-ended, symmetrical, thin arch ring under water load which is shown in Fig. 54 and the plane of which is inclined at an angle a with the horizontal plane. Also, let p be the unit water pressure on the extrados at any point in the semiarch ring varying from zero at the crown to p_1 at the level of the horizontal plane through the center of curvature of the extrados.

Then at any point g_1, on the surface of the extrados angularly distant ϕ from the axis of symmetry, it is evident from Fig. 53 that $p' = wh'$, $p = wh' - wh = w(h' - h)$, and $p_2 = wh' - wh_2 = w(h' - h_2)$. Now

$$h' = R \sin a,\ h = R \cos \phi \sin a,$$

whence

$$p = w(R \sin a - R \cos \phi \sin a) = wR \sin a(1 - \cos \phi)$$

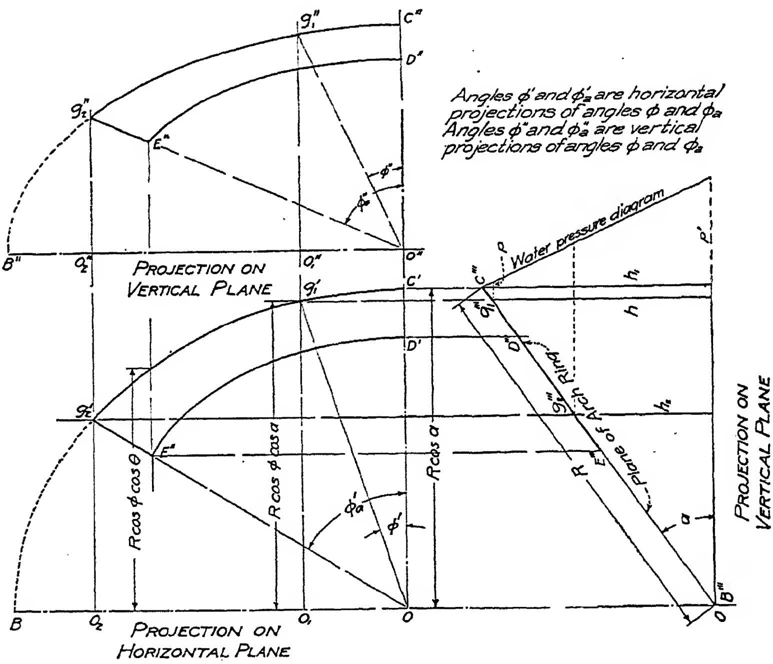

FIG. 53.—Multiple-arch ring of variable thickness.

and

$$p' = wR \sin a,$$

and, therefore,

$$\frac{p}{p'} = \frac{wR \sin a\ (1 - \cos \phi)}{wR \sin a} = 1 - \cos \phi,$$

or

$$p = p'(1 - \cos \phi). \tag{281}$$

Considering the bracket removed and the semiarch ring swung as a cantilever from the left abutment under water load in Fig. 54, employing Eqs. (195), (196), and (197) of Chap. XII, and substituting the value of p from Eq. (281), it follows that,

$$N_0 = \int_0^{\phi} Rp'(1 - \cos\phi)\sin(\theta_1 - \phi)d\phi = Rp'\left[\cos(\theta_1 - \phi_1) - \cos\theta_1 - \frac{\sin\theta_1 \sin\phi_1 \cos\phi_1 + \cos\theta_1 \sin^2\phi_1 - \phi_1 \sin\theta_1}{2}\right]; \quad (282)$$

$$M_0 = \int_0^{\phi} -Rp'(1 - \cos\phi)\left(R - \frac{t}{2}\right)\sin(\phi_1 - \phi)d\phi = -Rp'\left(R - \frac{t}{2}\right)\left[1 - \cos\phi_1 - \frac{\phi_1 \sin\phi_1}{2}\right]; \quad (283)$$

$$V_0 = \int_0^{\phi_1} -Rp'(1 - \cos\phi)\cos(\theta_1 - \phi)d\phi = Rp'\left[\sin(\theta_1 - \phi_1) - \sin\theta_1 + \frac{\cos\theta_1 \cdot \sin\phi_1 \cos\phi_1 + \sin\theta_1 \sin^2\phi_1 + \phi_1 \cos\theta_1}{2}\right]. \quad (284)$$

18-11. Evaluation of Redundants for Variable Water Load.—The values of N_0, M_0, and V_0 may now be used in Eqs. (192), (193), and (194) for finding the redundant forces $X = N_c$, $Z = M_c + N_c k$, and $Y = V_c$. If the arch ring is to be solved by the voussoir method, the integral signs in Eqs. (192) and (193) should be used as summation signs and the semiarch ring divided into a convenient number of voussoirs by radial lines of the neutral axis. Let the semiarch ring in Fig. 54 be divided into voussoirs of equal elastic weights. In arch rings of uniform thickness this will give voussoirs of equal length, and in arch rings of variable thickness it will give voussoirs of variable length. It will be well in this case to make the lengths of the voussoirs such that ds/I is constant or its equivalent

12 ds/t^3 constant. The elastic center can then be located on the axis of symmetry of the arch ring, so that

$$\int \frac{y\,ds}{I} \text{ and } \int y = 0.$$

As in the solution for the redundant forces in Eqs. (192), (193), and (194), all centers, angles, and coordinates of divisions should be drawn to scale, and the angles, coordinates, and other quantities computed and tabulated on the form given in Fig. 40.

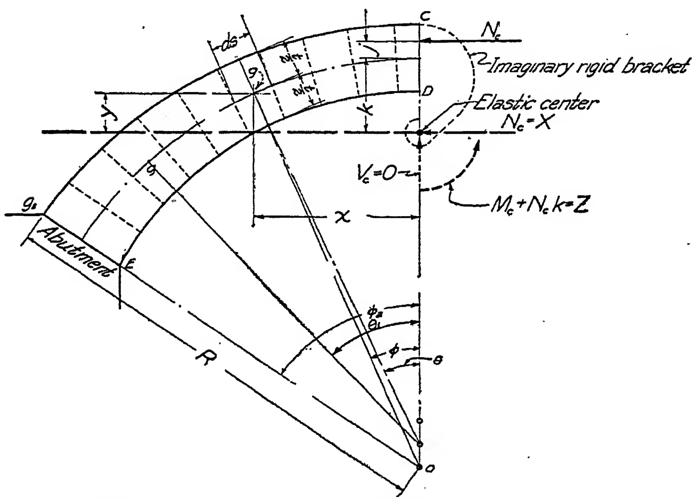

FIG. 54.—Diagram for analysis of multiple-arch ring of variable thickness.

It will now be evident that the thrust, moment, and shear, N, M, and V, at a point g, of the arch ring due to the water load and the redundant forces at the elastic center resolved on the radial section at this point, can be computed by the use of Eqs. (198), (199), and (200) of Chap. XII, the unit compressive stress f, due to the variable water load can be computed by use of Eq. (174), and the unit shearing stress by use of Eq. (177), unless reinforcing steel is used to take part of the stress. This case is considered in paragraph 18–17. In the case of the multiple-centered arch ring, Eqs. (226), (227), and (228) and the method outlined in

connection therewith in Chap. XIV should be applied and the whole process carried out as there suggested.

18-12. Action of Weight of Arch Ring.—In addition to the usual water load given in the formulas for single-arch dams, it is necessary to make an analysis where the voussoir weights of the arch ring alone are considered, the loading being parallel to the axis of symmetry of the arch ring and consisting of the components acting in a direction normal to the upstream edges of the buttress. This analysis should be made separately and the stresses obtained by it should be combined with those obtained from water pressure. These weights of the voussoirs of the arch ring should each be resolved into two forces, one parallel to the arch barrel and the other parallel to the arch axis and in the plane of the arch ring. These latter forces will then be in planes perpendicular to the buttresses. In a symmetrically formed arch, the arch loading, due to the weight of the concrete, will be symmetrical with respect to the axis of the arch ring and uniform for the circular or compound circular types of uniform thickness, and variable for the type with a circular extrados and a circular or compound circular intrados.

18-13. Method of Analysis Applicable.—In the analysis of stresses due to the weights of the arch ring, Eq. (169) and Eqs. (190) to (194) may be applied for the arch ring of variable thickness with a circular extrados and circular or compound circular intrados, and to the circular arch ring of uniform thickness. Integral expressions can be obtained for the weight loads on the latter type and these values can be substituted in Eqs. (192), (193), and (194), and integrated. Such a solution is given in Appendix IV. This process gives long cumbersome equations. Consequently, it is better to use the voussoir method for both of these types of rings.

18-14. Stresses Due to Arch-ring Weight on Cantilever.—Let Fig. 54 represent the left half of an arch ring of the two types under discussion acting for the time being as a cantilever and let it be divided into voussoirs of equal elastic weight. In the arch rings of variable thickness, this

will give voussoirs of variable lengths. Now, if w' is taken as the weight per cubic foot of concrete in the arch ring, and t as the mean thickness thereof normal to the neutral axis between the crown and the point g, then by well-known principles of trigonometry and mechanics it follows that,

$$N_0 = \int_0^{\theta_1} w'tr\, d\theta \sin a \sin \theta_1 = w'tr\theta_1 \sin a \sin \theta_1; \quad (285)$$

$$M_0 = \int_0^{\theta_1} -w'tr^2\, d\theta \sin a(\sin \theta_1 - \sin \theta) = -w'tr^2 \sin a$$
$$(\theta_1 \sin \theta_1 + \cos \theta_1 - 1) \text{ approximately}; \quad (286)$$

$$V_0 = \int_0^{\theta_1} - w'tr\, d\theta \sin a \cos \theta_1 = -w'tr\theta_1 \sin a \cos \theta_1. \quad (287)$$

18-15. Evaluation of Weight Redundants.—The values N_0, M_0, and V_0 may now be substituted in Eqs. (192), (193), and (194) to compute the redundant forces, $X = N_c$, $Z = M_c + N_c k$, and $Y = V_c$.

18-16. Stresses Due to Weight Load and Redundants.—The thrust, moment, and shear at any point g of the arch ring are those due to the weight of the arch ring and the redundant forces at the elastic center resolved on to the radial section through the point g, whence

$$N = N_0 + X \cos \theta_1, \quad (288)$$

$$M = M_0 - Xy + Z, \quad (289)$$

and

$$V = V_0 + X \sin \theta_1. \quad (290)$$

The arch ring being divided into voussoirs as suggested, the elastic center is then located on the axis of symmetry, so that

$$\int \frac{y\, ds}{I} \text{ and } \int y = 0,$$

for all points on the neutral axis at the longitudinal center of the division. As in the case of the variable unit pressure, all center angles and coordinates of the division are drawn to scale, the angles and coordinates scaled and computed and the redundant and stress forces computed. The

elements of the computations are, as in the case of a single-arch dam, recorded on the form given in Fig. 40. N, M, and V having been determined for the various voussoirs, the unit compressive stress can be ascertained for each of them by means of Eq. (174) of Chap. XI and the unit shearing stress from Eq. (177) of the same chapter.

In case of the multiple-centered arch ring, the method outlined in Chap. XIV should be applied, and the whole process carried out as there suggested.

18-17. Reinforced-concrete Arch Rings.—In general the barrels of multiple-arch dams are slightly reinforced with steel bars to provide against temperature and shrinkage cracks, the amount of this reinforcing being too small to materially affect the unit stresses or the deflections of the arches. In some cases, however, designers have considered it desirable to include sufficient reinforcement to make its consideration in stress analysis necessary. If it is desired to follow this procedure, the value of A, the cross-sectional area, and of I, the moment of inertia of the section, in the stress and deflection formulas for the arch rings, should be replaced by the value

$$A' = A_c + nA_s,$$

and

$$I' = I_c + nI_s.$$

In these expressions, A_c and I_c apply to the concrete and A_s and I_s to the steel, n being the ratio of the moduli of elasticity of steel and concrete, of E_s/E_c. For unit stresses, the principles used in reinforced-concrete beams are applicable, treatment of which may be found in works on this subject.

18-18. Stresses Due to Temperature Changes.—The temperature and shrinkage stresses produced in an arch ring of a multiple-arch dam are similar to those for single-arch dams. Methods for computing stresses due to temperature in single arch dams are given in Chap. XII, and Eqs. (201) to (204) of that chapter may be applied to the solution of the temperature and shrinkage problem for multiple-arch

dams. The stresses due to temperature and shrinkage should be considered in conjunction with those of both the loaded and unloaded arch ring.

18-19. Working Stresses.—The working stresses in multiple-arch dams for the arches should be the same as for single-arch dams, except that where the computations show any tensile stresses, reinforcing steel should be used. Owing to the thinness of the arch barrel, reinforcing steel will often be required in the arch to take care of these stresses. The working stresses for the buttresses should be the same as those employed for deck dams.

18-20. Earthquake Stresses Due to Mass in Multiple-arch Dams.—For a discussion and development of formulas relating to this subject see Appendix III.

MOUNTAIN DELL MULTIPLE-ARCH DAM

18-21. General Description.—The Mountain Dell Dam was constructed by Salt Lake City for storage purposes in connection with its water supply in Parley's Canyon about 12 miles north of the city. It was originally constructed in 1916 and 1917 to a height of 110 ft. and later, in 1925, was raised to the full contemplated height of 150 ft. The Mountain Dell Dam is of the multiple-arch type, 560 ft. in length, comprised of 16 arch spans supported on buttresses. Figure 55 shows the principal features of the design of this dam.

18-22. Arches.—The arch rings are constructed normal to the buttress faces and consist of circular arcs of 120 deg. The arch rings are 4.9 ft. thick at the base and 15 in. thick at the top. The upstream faces of the arch barrels have a slope of 10:12. The lower arches are of the three-hinged type but the upper ones are fixed and reinforced with ½-in. square bars, 18 in. on centers on both faces. The arches are constructed of 1:2:4 concrete.

18-23. Buttresses.—The buttresses are spaced 35 ft. on centers, have a minimum thickness at the crest of 18 in., and a maximum base thickness of 8 ft. The downstream slope is 1:5. The buttresses are reinforced with layers of

1-in. square bars placed 4-ft. apart. At the bottom of the buttresses there are four bars in each layer, but the number diminishes with elevation. The buttresses are tied together with 23 horizontal beams spaced 20 ft. apart vertically and 29 ft. horizontally, beginning at the haunches or corbels of the arches. These beams are T-shaped and reinforced with steel extending through the buttresses from end to

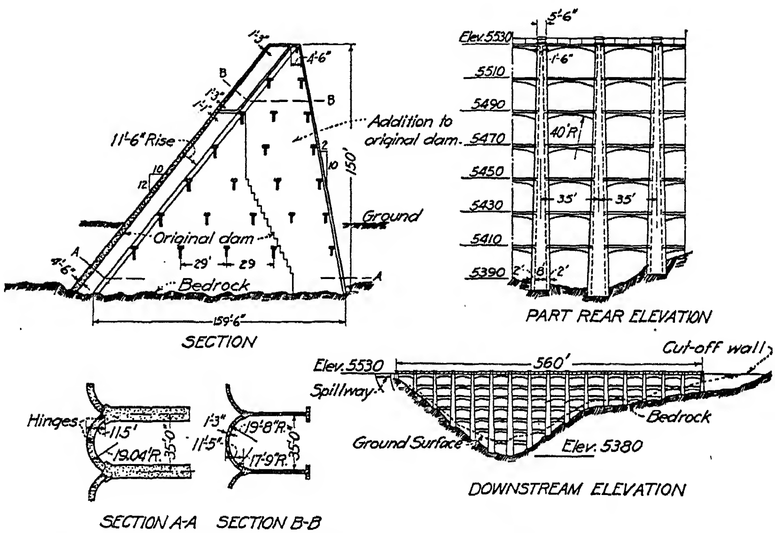

FIG. 55.—Mountain Dell multiple-arch dam. (Salt Lake City, Nev.).

end of the dam. The downstream faces of the buttresses are stiffened with slabs 2 ft. wider on each side than the thicknesses of buttresses.

18-24. List of Important Multiple-arch Dams.—Table XIX contains a list of some of the important well-designed multiple-arch dams in the United States.

TABLE XIX.—IMPORTANT MULTIPLE-ARCH DAMS IN THE UNITED STATES

Name of dam	Location	River	Year completed	Height above foundation, ft.	Height above stream bed, ft.	Crest length, ft.	Arch thickness at top, ft.	Arch thickness at bottom, ft.	Slope of upstream face, deg.	Volume of dam, cu. yd.
Bartlett	Arizona	Verde	1938	286	194	750	2.0	7.0	42	157,725
Sutherland	California	Santa Isabel	1928	...	180	780	....	2.02		
Lake Pleasant	Arizona	Auga Fria	1927	201	170	1,850				
Palmdale	California	....	1924	175	...	724	1.25	3.0	45	
Lake Hodges	California	San Dieguito	1919	150	...	558	1.0	2.6	45	
Florence Lake	California	South San Joaquin	1926	149	...	3,160	....	....	..	57,500

CHAPTER XIX

DESIGN OF ROCK-FILL DAMS

INTRODUCTION

19-1. General Statement.—Rock-fill dams, as well as the now almost obsolete brush and timber-crib dams, had their beginning in pioneer history when construction materials for masonry dams were not economically available. The rock-fill dam, however, has a permanency that still makes it suitable for use in places remote from railroads or highways. The essential features to be taken into consideration in the design of a rock-fill dam are:

a. An impervious solid foundation.
b. A well-compacted suitable rock body.
c. Suitable slopes.
d. An effective cutoff membrane.
e. Abundant freeboard.
f. Ample spillway capacity.

PRINCIPLES OF DESIGN

19-2. Character of Foundation.—Rock-fill dams are usually constructed on sound rock or indurated foundations for the purpose of getting a good bearing for the rock body and the securing of imperviousness. Grouting should be resorted to, to secure imperviousness when necessary.

19-3. Materials for Body.—A rock-fill dam is generally constructed of the rock materials obtainable in the vicinity of the dam. These materials must be such as will resist crushing and exposures to air and water; and they generally consist of the granitic or volcanic rocks or of hard, durable sandstone or limestone, secured by blasting ledges in the neighborhood of the structure. The quarry run is generally used and therefore usually consists of angular fragments of various sizes up to the largest pieces that can be handled by the construction equipment.

19-4. Top Width and Height.—The top width of rock-fill dams is usually based on the height of the structure and is generally taken at from one-tenth to one-fifth the height with a minimum of 10 ft. and a maximum of 20 ft. Formerly rock-fill dams of 100 ft. in height were considered extreme, but at the present time heights of as much as 300 ft. or better are considered entirely feasible.

19-5. Side Slopes.—One of the most important things in the design of a rock-fill dam is to obtain slopes that will not ravel out or slip under pressure, which is not always easy to do in the process of construction. The angle of repose for different characters of rock varies somewhat and experiments should be conducted to determine the proper angle to use in design. However, a downstream slope of about 1½ horizontal to 1 vertical is generally found to be adequate. Downstream slopes as steep as 1:1 and as flat as 2:1 have been used with success. Where the heights are considerable, berms should be provided. The upstream slope is often made considerably steeper than the downstream slope, but it should always be sufficient to prevent crushing or distortion from the water pressure at the toe. Practice varies from about ½:1 to 1:1, with ¾:1 predominating. In the higher dams it is customary to flatten both slopes for the lower levels. The slopes of both faces are sometimes steepened to the steeper values given here and raveling is prevented by hand placing of the facing materials for several feet of face thickness.

19-6. Water-tight Membrane.—Another very essential element to the safety of a rock-fill dam is the water-tight membrane. There are several types of these membranes in use. Some of the earlier dams provided such a membrane in a masonry upstream face placed on a rather steep incline; and in other cases the impervious wall has been thus placed in the middle of the dam along the axis. The thickness of these walls has generally varied from about 5 to 25 ft. at the bottom and from about 5 to 15 ft. at the top. Plank facing spiked to timbers buried in the rock-fill or steel-sheet facing riveted to I-beams so buried have also

been employed. In modern practice the water-tight membrane has generally been made of concrete supported on a hand-placed facing over the main rock-fill body. In other cases the membrane has been secured with an earth-fill placed on the upstream side of the dam with a cutoff wall dividing it from the rock-fill. The better practice, however, is to place a cutoff wall at the upstream toe of the rock-fill carried well into the foundation and supplemented with an impervious apron of plain or reinforced concrete over the upstream slope of the dam resting on a hand-placed backing. The slope should be so inclined and so placed as to secure stability against sliding or compressing under the water pressure; and the concrete apron should be either flat or slightly concave towards the face of the rock-fill, so that in case the rock-fill should move slightly the membrane could follow it.

19-7. Freeboard and Spillway Capacity.—Some of the failures of rock-fill dams have been due to overtopping and for this reason ample freeboard and spillway capacity should always be provided for a rock-fill dam.

DIX RIVER DAM

19-8. General Description.—The Dix River Dam was constructed by the Kentucky Hydroelectric Company on the Dix River, a branch of the Tennessee River in Tennessee, for the purpose of developing hydroelectric power. A cross-section of this dam is shown in Fig. 56. This dam is at the present time the highest completed rock-fill dam in the world, being 275 ft. from the bottom to top of the crest. It is, 1,032 ft. long on top and 20 ft. thick at the crest. The dam is curved upstream in plan on a compound curve of 1,250 ft. and 1,700 ft. radii. The downstream slope is 1.4 horizontal to 1 vertical to within 55 ft. of the top where it changes to 1:1. The upstream slope is 1.2 horizontal to 1 vertical at river-bed level, gradually changing to a slope of 1:1 as it approaches the crest. The upstream face is covered with derrick-placed dry cyclopean masonry on which rests a reinforced-concrete apron. The apron

terminates in the river bed and the sides of the canyon in a cutoff wall carried into the solid rock with a grout curtain below it.

19-9. Character of Foundation.—The foundation consists of a dense, crystalline limestone free from fossils, and was prepared for the reception of the dam by removing all dirt, clay, and loose material.

19-10. Materials for Body.—The bulk of the rock required for the dam was obtained from the excavation for the spillway where it was blasted and loaded with steam shovels and hauled in dump cars to the dry rock-fill and

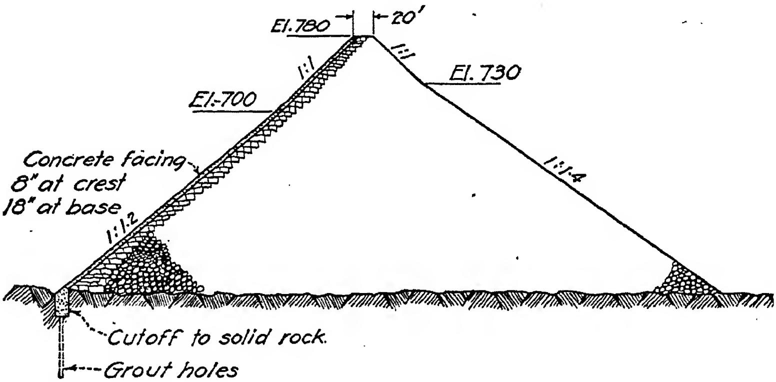

FIG. 56.—Dix River rock-fill dam. (*Kentucky Hydroelectric Company.*)

dumped into place without further work except sluicing and soaking to promote settlement. The upstream face of the dam was covered with a rubble facing built of large derrick-placed stones weighing from 1 to 8 tons. This wall is 14 ft. thick at the bottom and 7 ft. at the top and is covered with a reinforced-concrete slab divided into panels of 48 by 70 ft. with expansion joints running down the slope and horizontally. The concrete slab is 18 in. thick at the bottom and 8 in. at the crest and is reinforced with 0.5 per cent of steel in each direction. At the expansion joints the abutting slabs rest on concrete ribs formed in grooves built in the sloping rubble wall. The slabs are joined to the ribs with U-shaped copper strips with compressed cork filling

in the joint above them. In order to secure additional precautions against percolation the top forms of the concrete slabs were left in place for the lower 160 ft. of the dam, which will always be submerged.

19-11. Cutoff Wall.—A cutoff wall was carried into the foundation at the upstream toe of the apron to a depth of from 25 to 30 ft. Three-inch grout holes were drilled through the cutoff wall to varying depths up to 250 ft. at intervals of 10 ft. and in some cases of 5 ft. These holes were grouted under pressure as drilled.

19-12. Outlet Works.—The water is let out of the reservoir through a tower connected with a 24-ft.-diameter horseshoe section, concrete-lined pressure tunnel driven through the rock around the right end of the dam. The tower at the intake end of the tunnel is built of reinforced concrete, is 260 ft. high, and has an internal diameter of 22 ft. The flow from the tower into the tunnel is controlled by a balanced needle valve 17 ft. in diameter, placed about 60 ft. above the floor of the reservoir and operated by an electrically driven oil-pressure hoist.

19-13. Spillway.—The spillway is located in a rock cut around the left end of the dam and has a capacity of 200,000 sec.-ft., which equals about 500 cu. ft. per second per square mile of drainage area. It is 250 ft. wide at the bottom with side slopes of $\frac{1}{4}$:1, and has its crest 35 ft. below the top of the dam. The spillway discharges back into the river below the dam.

19-14. List of Important Dams.—Below is given in Table XX, a list of some of the important rock-fill dams in the United States.

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TABLE XX.—IMPORTANT ROCK-FILL DAMS IN THE UNITED STATES

Name of dam	Location	River	Year completed	Height above foundation, ft.	Crest length, ft.	Top width ft.	Up-stream slope	Down-stream slope	Volume of dam, cu. yd.
Salt Springs	California	Mokelumne	1931	330	1,300	15	1.3 :1	1.4:1	3,000,000
Dix River	Kentucky	Dix	1925	275	1,032	20	1.2 :1 to 1 :1	1:1, 1.4:1	1,747,000
Morena	California	Cottonwood Creek	1911	265	505	16	½ :1, 0.9 :1	1½:1	306,000
Relief	California	Stanilaus	1909	140	509	13	½ :1	2:1	
Strawberry	California	Stanilaus	1916	173	612	15	1.1 :1, 1.2 :1	1.3:1	400,000
Lake Fordyce	California	....	1927	140	1,030	10	1 :1	1:1	

CHAPTER XX

DESIGN OF EARTH DAMS

INTRODUCTION

20-1. General Statement.—An earth dam is a structure composed of a mixture of silt, clay, loam, sand, and gravel or rock in suitable proportions to secure stability and water tightness, deposited in layers, and compacted with heavy rollers or other suitable equipment, or sluiced into place with water. There are three general types of earth dams each of which is formed in one of the following ways: Type *A*, an embankment with a relatively impervious puddle core of earth or a central wall of masonry and coarser outer parts; Type *B*, an embankment with a relatively impervious upstream part and central core and a coarser downstream part; Type *C*, an embankment with a relatively impervious homogeneous body.

20-2. Essential Features of Earth Dams.—In the design of an earth dam the following essential features must be taken into consideration: (*a*) water-tightness and weight; (*b*) stability of slopes; (*c*) protection of water slope from wave action; (*d*) protection of structure from overtopping either by floods or waves; (*e*) imperviousness of foundation.

DETAILS OF DESIGN OF EARTH DAMS

20-3. Percolation.—The materials used for the construction of earth dams, in spite of any precautions which may be taken, are pervious, and this must be recognized in the design and construction of the embankment. Infiltration of water will decrease the weight of the submerged part of the embankment by the weight of water displaced by it, and will diminish its stability in proportion to the porosity of the construction material. On the other hand, the heavier the material of which the embankment is

constructed, the more stable it will be, whence the necessity for the use of materials of high specific gravity.

20-4. Effect of Saturation of Embankment Materials.—If the embankment is constructed of coarse, pervious material, the hydraulic gradient through it will approach horizontality; and the more impervious the material is, the steeper the hydraulic gradient will be. The slope of the hydraulic gradient of percolation measures the resistance of the material in the embankment to the flow of water through it, and the height at any point of the hydraulic gradient above the ground surface measures the decrease in its stability. The saturation of the material in the embankment increases the tendency to slide, which is one of the most likely possibilities of failure. Sliding is due, of course, partly to loss of friction between the particles and partly to loss of adhesion of the particles. Friction is greatest for coarse material and least for the fine soils. The addition of moisture in small quantities increases friction, but the addition of moisture in considerable quantities diminishes it. Adhesion is an extremely varying force and, like friction, it is increased by a moderate amount of moisture but diminished by an excessive amount. An embankment will, therefore, be most stable when slightly damp and least stable when wet. Consequently it is of the greatest importance to construct the upstream section of the embankment with impervious material to keep the water out and the downstream part of the embankment with pervious material to let the water out.

20-5. Percolation through Embankment and Foundation.—Since the flow of water through the embankment is due to the slope of the hydraulic gradient, and since the hydraulic gradient varies directly as the distance between the free outlet and inlet of subsoil flow, the more pervious the embankment the flatter should be its slopes in order to increase this distance of water travel and to decrease the rate of percolation.

The flow of water through the foundation of the dam takes a somewhat different course than it does through the

dam itself. When the embankment rests on homogeneous soil and has equal slopes, the lines of flow approximate a series of confocal ellipses. The ellipses are not perfect because of the increase in the weight of the embankment from the heel and the toe to the center line of the dam, and the resistance offered by the foundation cutoff. The rate of flow is greatest for the shortest elliptical curve and is, therefore, a maximum at the exit at the downstream toe. This is shown in Fig. 57. Placing a foundation cutoff under the dam disturbs the symmetry of the ellipses by disturbing the homogeneity of the foundation material, and more particularly if it is not placed under the central part of

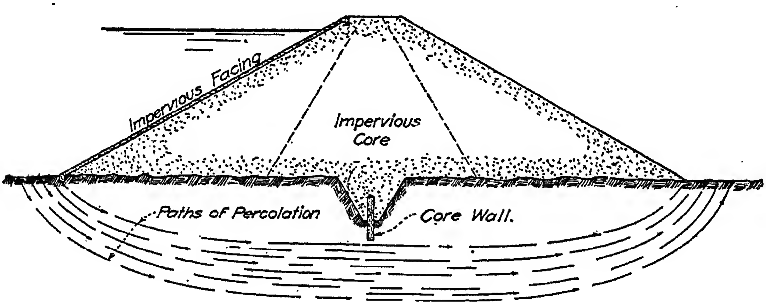


FIG. 57.—Percolation under earth dam

the embankment. So far as the foundation alone is concerned the efficiency of the foundation cutoff would be improved by placing it upstream from the center of the foundation, but, since the dam itself is permeable, water would pass through it over the top of the foundation cutoff and the flow through the foundation would not be greatly helped by this change of position of the cutoff. Moreover, if the embankment core is placed centrally, the foundation cutoff should be placed directly under it to secure the best connection between them. For this reason the foundation cutoff is usually placed centrally under the embankment where it will make a good connection with the embankment core and will also receive the greatest amount of compression and most symmetrical settlement.

20-6. Piping of Embankment or Foundation.—It is important to know whether percolation through the foundation or through the body of the dam is likely to cause piping. From a well-known principle of hydraulics the pressure of the flowing water, through the dam or its foundation, on a particle of soil at the toe of the dam or at any point on the downstream slope of the embankment is given by the equation,

$$F = \frac{2\,wav^2}{2g} = 2\,av^2, \text{ nearly;} \tag{291}$$

in which F is the pressure in pounds per square foot, w the weight of water per cubic foot, a the cross-sectional area of the particle in square feet, v the velocity of the water in feet per second, and g the acceleration of gravity in feet per second per second.

Now, in order to move the particle, F must be equal to its submerged weight in pounds. Let this submerged weight be taken as 99 times the volume of the particle, let the particle be assumed to be a sphere with diameter d. The volume of the particle will be $\frac{\pi}{6}d^3$, its weight $99 \times \frac{\pi}{6}d^3$, and its impinged area $\frac{\pi}{4}d^2$. Solving for v in Eq. (291), and substituting the weight and impinged area of the particle in it,

$$v = \sqrt{\frac{F}{2\,a}} = \sqrt{\frac{99 \times \frac{\pi}{6}\,d^3}{2 \times \frac{\pi}{4}d^2}} = \sqrt{33\,d}. \tag{292}$$

The smallest particle of clay that may be dislodged by the water has a diameter of 1/12,500 in. or 1/150,000 ft. Substituting this value in Eq. (292), $v = 0.015$ ft. per second or $V = 0.9$ ft. per minute. With a factor of safety of 4 against the dislodgment of this most minute particle, the safe velocity may be assumed to be about 0.25 ft. per minute. A minimum value of 0.5 ft. per minute is sometimes used.

20-7. Thickness of Embankment Required.—The required horizontal thickness of the embankment to reduce the velocity of water through it to a safe value for the material used in its construction can be obtained by the use of the Slichter formula as discussed in Chap. II. For the present purposes this equation may be written

$$Q = \frac{KHA}{L}. \tag{293}$$

Let A equal one square foot and note that Q equals PV, the porosity times the velocity in feet per minute. Substituting these values in Eq. (293) and solving for L,

$$L = \frac{KH}{PV}. \tag{294}$$

Having the porosity P of the material in the dam, V the safe velocity of flow, H the difference between the level of the water behind the dam and the horizontal level under consideration, and K from Tables III and IV of Chap. II, it is possible to compute L, the required length of path of travel to prevent piping. Generally, the thickness found in this manner will be much less than that required by other conditions unless the foundation of the dam is quite pervious, in which case the dam must be made thick enough at all points to satisfy the required value of L. In the foundation the path of travel, or L, may be increased and the thickness of the dam diminished by the use of a sheet piling or other type of cutoff wall. Although the actual path of travel is curvilinear, it should for the sake of simplicity, be measured from the heel of the dam to the bottom of the cutoff wall and from there to the toe of the dam as discussed in paragraph 4–15 for a pervious foundation. For an assumed or fixed desirable base thickness of the dam, Eq. (294) provides a means of finding the required depth of cutoff wall. If b is the base thickness of the dam, d the unknown depth of the cutoff wall and with the cutoff wall in the center of the base, it is evident that

$$d = \tfrac{1}{2}\sqrt{L^2 - b^2}. \tag{295}$$

Many attempts have been made to develop a mathematical process of finding the line of saturation in earth dams. Probably the most complete and most accurate method is the Justin method given in the 1924 *Transactions of the American Society of Civil Engineers*. However, the laws governing the flow of underground water are only roughly known, the variability of the materials in the cross-section of any earth dam is so great, and the character and variability of the foundation strata so indeterminate that the application of mathematical equations is very unsatisfactory. Consequently, it is better to depend on thorough investigations of the foundation strata, careful selection of the materials for the embankment, and proper methods of construction of the fill as developed by long experience and sound judgment.

PREPARATION OF THE FOUNDATION FOR EARTH DAMS

20-8. Preliminary Investigations.—In selecting the site for an earth dam, the character and stability of the foundation should be investigated to determine whether it is capable of bearing the weight of the structure and whether it can be made sufficiently impervious by economical treatment. The investigation of the foundation can best be made by digging test pits to an impervious substratum; or sinking holes with an ordinary well-drilling outfit. In either case, a log of the materials encountered should be kept and an examination of them made to determine their porosity, weight, plasticity, and general suitability for the foundation of the dam. A mechanical analysis may be very useful in making this study.

20-9. The Puddle Trench.—If the examination of the logs of materials encountered in the test pits and drill holes indicates that pervious strata of sand, gravel, or other porous material underlie the dam site, a puddle trench should be dug parallel to the axis of the dam for its entire length. This should be located under the center of the puddle core with reference to the cross-section of the dam. The trench must either connect directly with an impervious

stratum or be sufficiently deep to produce abundant frictional resistance to the passage of water.

The bottom width of the puddle trench should be made to depend on the nature of the strata passed through (especially those near the ground level), the nature and the manner of securing consolidation of the filling, and the depth of water at trench bottom. In important rolled-fill dams, it is desirable that the trench width be sufficient to permit of consolidation of the fill by rolling, but, in smaller, less important dams, the consolidation may be secured by ramming, and the width reduced accordingly. The side slopes of the puddle trench should be made flat enough to stand during excavation and refilling, and to produce ample trench width for proper resistance to percolation at all levels. There should be no sudden change in the section of the trench, either transversely or longitudinally, as such changes are liable to cause unequal settlement and rupture of the filling. All changes should be made gradually so that the material will settle towards the center both ways and thus assist in compacting the material.

20-10. Puddle Material Required.—The puddle filling should consist of the most impervious material procurable at the site, and should be compacted as completely as possible. To insure compactness, water should be used sparingly and to prevent cracking, the different layers should be constructed as rapidly as possible.

20-11. Concrete Cutoff Wall.—It is sometimes desirable to connect the impervious stratum at the bottom of the cutoff trench with the puddle core by means of a concrete cutoff wall. The thickness of the wall should be made to depend on the construction necessities and on the porosity of the strata lying above the trench bottom. The concrete in the wall should be of good quality and reasonably watertight. The cutoff wall may be terminated in the puddle material in the trench or carried up to any desired height.

20-12. Treatment of Springs in Dam Site.—A spring in the dam site is objectionable and the dam site should not

be used if at all possible to avoid it. If this is not possible, the spring should be drained out from the site. The spring should be followed back to its source, if possible, and confined in a pipe leading to the downstream face or above water level in the reservoir. The drain pipe should be strong enough to support the material above it and should have tight joints to prevent entrance of seepage water from the reservoir.

20-13. Treatment of Ground Surface before Starting Embankment.—The foundation of the dam should be stripped of all unsuitable material to a depth sufficient to remove all soil containing objectionable quantities of humus, and to reach below all burrow holes and cracks. It should then be trenched and ridged in lines about 20 ft. apart parallel to the axis of the dam. The trenches thus formed on the water side of the puddle core should be filled with puddle material thoroughly compacted in place, and those on the opposite side of the puddle core should be filled with porous material capable of collecting water and delivering it into a system of tile drains running perpendicular to the axis of the dam, for carrying the water out from under the area below the puddle core. Tile drains may also be placed in the longitudinal trenches, but they should not be continuous and should be joined to the cross-drains. All the drains should be surrounded with sand, which itself should be surrounded with crushed rock or gravel. The tile should be sufficiently strong to carry the weight of the embankment above it. Drains of 4- or 6-in. diameter are the preferred sizes, and additional capacity should be secured by increasing the number rather than the size of the drains. The cross-drains should be arranged to discharge into a larger drain below the dam and paralleling its toe, this larger drain conveying the water into a water conduit below the structure. It may also be necessary to drain, with ordinary farm tile, the area immediately below the toe of the dam to prevent it from becoming waterlogged. The purpose to be attained in the entire foundation treatment is to diminish as far as possible the flow of

water under the dam on the upstream side of the puddle core, to facilitate its escape on the downstream side, and to lead it away from the dam into a natural water course without permitting saturation or erosion of the ground surface.

EMBANKMENT MATERIALS FOR EARTH DAMS

20-14. Examination of Available Materials.—It is essential to examine the soils and subsoils at the dam site to determine whether suitable materials for the construction of the dam exist in the locality of the site within economical reach and in abundance of quantity. Here it is necessary to make a careful mechanical analysis of the materials. Two essential elements must be kept in mind in determining whether the materials are suitable for the construction of the dam: first, it is essential that the materials have suitable qualities to give water-tightness; and second, that they have suitable qualities to give proper weight. To determine these requisites, careful consideration must be given to the size and form of the soil grains, the amount of pore space in the various soils, the weight of a unit volume of combinations of the various soils, and the ingredients in the soils subject to solvency, change in volume, or decay. The general problem involved is to determine and eliminate solvent and vegetable-laden soils and then to find the combination of the remaining soils that will give the greatest compactness of mass, which will result generally in a mass of maximum weight and minimum porosity. The soil samples should first be air dried; second, subdivided into particles of various sizes; and then mixed in combinations that will give the maximum compactness.

20-15. Classification of Soils.—The Bureau of Soils of the United States Department of Agriculture classifies all soils on the basis of the size of the grains, into seven different groups ranging from clay to fine gravel. The classes and sizes are given in the following table:

TABLE XXI.—CLASSIFICATION OF SOILS

No.	Description	Size of soil grains	
		Millimeters	Inches
1	Fine gravel	2 to 1	$\frac{1}{12}$ to $\frac{1}{25}$
2	Coarse sand	1 to 0.5	$\frac{1}{25}$ to $\frac{1}{50}$
3	Medium sand	0.5 to 0.25	$\frac{1}{50}$ to $\frac{1}{100}$
4	Fine sand	0.25 to 0.1	$\frac{1}{100}$ to $\frac{1}{250}$
5	Very fine sand	0.1 to 0.05	$\frac{1}{250}$ to $\frac{1}{500}$
6	Silt	0.05 to 0.005	$\frac{1}{500}$ to $\frac{1}{5000}$
7	Clay	0.005 to 0.0001	$\frac{1}{5000}$ to $\frac{1}{12500}$

20-16. Advantages of Combinations of Soils.—None of the materials given in the table, when used alone, is suitable for the construction of an earth dam. The weight and stability of gravel are both satisfactory, but gravel lacks the essential feature of water-tightness as it often contains as much as from 35 to 40 per cent of pore space. Sand, on the other hand, while less porous than gravel, is much less stable and much more seriously affected by moisture. Loam or earth, containing a high percentage of organic matter, is relatively impervious but it lacks weight and stability to give the best results. Silt and clay are quite impervious, but they lack weight and stability and tend to swell and slump when saturated with water and to shrink and crack when dry. Consequently, clay or silt alone are not suitable for the construction of an earth dam, although clay is excellently suited for the core wall when held in place by heavier materials in the outer thirds of the cross-section of the dam, and is also excellently suited for an admixture with other materials.

Although there are serious objections to the construction of an earth dam out of any one class of material, suitable combinations of them can readily be made by so proportioning them that the pore space in the sand is filled with finer grains of sand, silt, and clay, and the pore space in the gravel filled with this combination of sand, silt, and clay. By using the proper proportions of these materials it is

readily possible to reduce the pore space to less than 15 per cent of the total volume and to secure a product more stable and heavier than gravel and nearly as impervious as clay. It is, of course, impossible to find materials in nature exactly suitable for making an ideal combination and it is likewise impossible to separate materials thus found into the necessary constituents to form such a combination. However, the various materials available at a dam site may be selected and so mixed as to approach somewhat closely the ideal combination.

20-17. Pore Space of Typical Soils.—In the following table are given the percentages of pore space of some of the typical soils that will be found in the vicinity of a dam site and out of which it will be necessary to construct the dam.

TABLE XXII.—APPROXIMATE PORE SPACE OF SOILS

Description of Material	Pore Space, Per Cent
Fine gravel	39
Coarse sand	36
Medium sand	34
Fine sand	34
Sandy soil	34
Sandy loam	39
Loam	44
Clayey loam	47
Clay soil	48

DIMENSIONS OF EARTH DAMS

20-18. Common Practice Regarding Top Widths and Slopes.—The dimensions of an earth dam depend on the angle of repose and porosity of the materials out of which the dam is to be constructed, the height of the dam, the manner in which the dam is to be constructed, and the extent of the water surface of the reservoir controlled by the dam.

The top width of the dam varies with the height and importance of the structure, and ranges from 10 ft. in smaller dams to 30 ft. or more in large and important ones.

An approximate formula for top width is

$$\sqrt{h} + 0.2\,h,$$

where h is the height above the ground surface.

The downstream slope usually varies from 2:1 to 2½:1, depending upon the character of the material and the height of the dam. For materials of low angle of repose and for high dams, the larger value is used. The upstream slope usually varies from 2½:1 to 3:1 also depending upon the angle of repose of the material, the height of the dam and the nature of the protection that is to be provided against wave action. In case of earth dams of about 100-ft. height, it is generally desirable to provide berms on the slopes at intervals of about 50 ft. in elevation, to spread the load on the foundation and at the same time to provide against erosion of the embankment from rainfall on the downstream slope.

20-19. Freeboard Requirements for Earth Dams.—Ample freeboard should be allowed between the high water level in the reservoir and the top of the dam. The amount of freeboard that must be provided is generally based on the computed wave heights and depth of frost action, but is also related to the problem of maximum runoff of the watershed and the spillway capacity. In cold climates, it is desirable to have the water level in the reservoir kept below the frost line in the upper part of the dam, on account of the possibility of fissures due to freezing and on account of ice pressure. Danger from overtopping the embankment by waves is a serious one and ample freeboard must be provided to care for this contingency. An approximate value of the proper freeboard to allow is $\sqrt{h}$, where h is the height of the dam above the ground surface.

20-20. Recommended Dimensions for Earth Dams.—Below is given a tabulation of the slopes for the downstream and upstream faces, the top width, and the freeboard for dams varying from 20 to 100 ft. These values are approximations that are to be varied by the character of the material at hand, but they are ample for average conditions.

TABLE XXIII.—RECOMMENDED DIMENSIONS OF EARTH DAMS

Height of dam, ft.	Downstream slope	Upstream slope	Top width, ft.	Freeboard, ft.
20	2 : 1	2½ : 1	10	5
30	2 : 1	2½ : 1	12	6
40	2 : 1	2¾ : 1	14	6
50	2 : 1	2¾ : 1	16	7
60	2½ : 1	3 : 1	20	8
70	2½ : 1	3 : 1	22	8
80	2½ : 1	3 : 1	25	9
90	2½ : 1	3 : 1	30	9
100	2½ : 1	3 : 1	30	10

PROTECTION OF SURFACES OF EARTH DAMS

20-21. Necessity for and Types of Slope Protection.—It is necessary to protect the upstream surface of an earth dam from wave action, which will ultimately destroy it unless prevented by proper precautions. It is best to place the protection on the slope of the dam after it has attained its final consolidation so that further small settlements will not affect it. In general the following four types of protection are used for this purpose: (*a*) dry riprap, (*b*) hand-placed riprap, (*c*) reinforced concrete, and (*d*) crude oil.

20-22. Dry Riprap.—Loose riprap may be composed of broken rock or gravel deposited loosely on the slope of the dam. In applying this type of slope protection, it is necessary to use an abundance of material of large size because the finer parts will be washed out and deposited at the heel of the dam, and the larger particles will be left on the slope to take their natural angle of repose under wave action. Where rock or suitable gravel is plentiful

and cheap transportation is available, this forms a serviceable protection for the embankment.

20-23. Hand-placed Riprap.—Hand-placed riprap is very generally used where stone or boulders are available in abundance. In this method of protection, a layer of from 1 to 3 ft. of gravel or small broken rock is first placed on the earth slope, following which a course of broken rock or boulders ranging from 18 to 24 in. thick is placed. The pieces of rock or boulders for this course are selected as to size, hand-placed, and neatly spalled together, or the interstices are filled with concrete mortar. This forms a very durable and satisfactory protection for the water slope.

20-24. Concrete Paving.—Where rock and gravel are not available in large quantities, concrete is resorted to and the entire water slope of the embankment is paved with concrete usually from 4 to 6 in. in thickness. The pavement is placed in sections running horizontally across the dam and up the slope. These sections usually vary from about 20 to 50 ft. in dimensions, and are reinforced with steel in both directions in order to prevent temperature cracks and to prevent breaking up through unequal settlement and wave action. The blocks are placed with their four edges on concrete sills set into the face of the dam, these sills extending into the face of the dam a distance of from 1 to 1½ ft. One side of the slab is connected integrally with the concrete sill and the other is rested on top of the sill with an asphaltic or tar-paper joint between them.

20-25. Crude-oil Paving.—Where the dam is of not too great height and where crude oil is cheap, it is possible to protect the water slope with a coating of crude oil thoroughly incorporated into the earth surface. This gives a fairly good protection if the slope of the dam is not too steep.

20-26. Downstream-slope Protection.—The downstream slope of the dam must also receive consideration to prevent it from gullying from rainfall or eroding under wind action. These difficulties are sometimes overcome by placing a layer of gravel on the downstream slope of the dam some

12 to 24 in. in thickness, depending upon the availability of the material, and in other cases by seeding the slope with a hardy grass which has a strong root system that will resist wind and water erosion. If the dam is an exceptionally high one, berms should be placed on the downstream slope at vertical intervals of about 50 ft. to prevent rain water from running in continuous channels down the face of the dam. These berms should be sloped parallel to the axis of the dam to convenient points, usually at the abutments where the water can be taken into suitable drains and carried down the slope. The berm should be provided with a non-erosive gutter adjacent to the rising slope of the dam.

SELECTION OF TYPES

20-27. Factors Determining Selection of Type.—The selection of Type A, Type B, or Type C earth dams depends

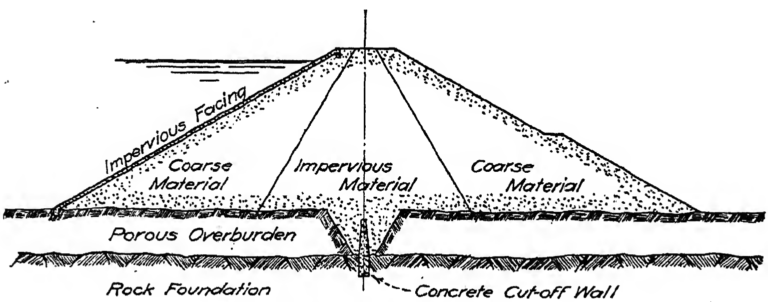


FIG. 58.—Earth dam, Type A.

on the nature of the site, the nature of the materials available for the construction of the embankment, and, in some instances, the facilities available for use in construction.

Where the foundation of the dam rests on rock near the ground surface, or rock with pervious strata adjacent to it, and where there is little impervious material near the site for construction of the embankment, it is desirable to select the Type A dam with a concrete cutoff wall and either to continue the concrete cutoff wall to the top of the dam or to connect it with a puddle core to be constructed of the limited puddle material available and carried to the top

of the dam. If the impervious stratum to which the cutoff trench is to be carried is overlain with relatively impervious strata, the concrete cutoff wall may be omitted (see Fig. 58).

If there is an abundance of impervious material at the dam site, and also of coarse pervious material, it is desirable

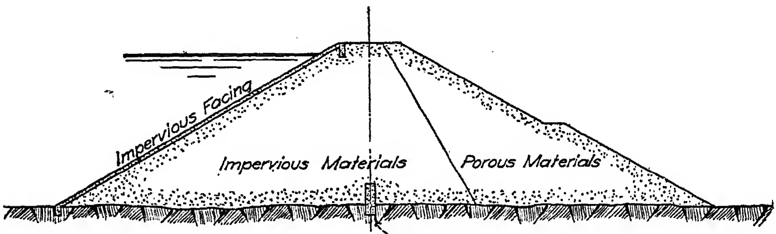


Fig. 59.—Earth dam, Type *B*.

to select the Type *B* dam. In this case, if rock exists near the ground surface or if the rock to which the cutoff trench is to be excavated is overlain with previous strata, a concrete cutoff wall should be built on the rock and extended into the impervious trench filling sufficiently to form a water-tight bond. The impervious part of the embank-

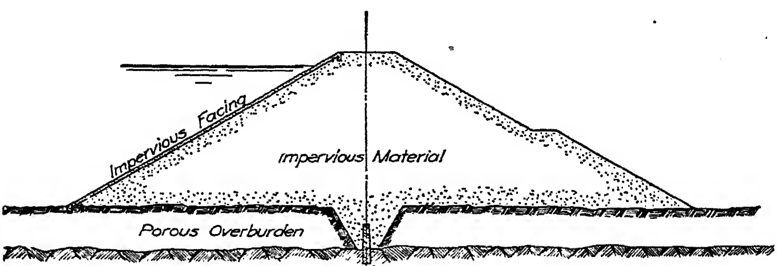


Fig. 60.—Earth dam, Type *C*.

ment would be made to cover the entire upper two-thirds of the dam and the downstream third would be constructed of coarse material (see Fig. 59).

If coarse pervious material is not available at the dam site, and if the available materials are such as to make the entire embankment of relatively impervious homogeneous material, it is desirable to select the Type *C* dam. In this

case, the cutoff trench would be carried down to an impervious stratum. If this impervious stratum consists of rock near the surface or of rock overlain with relatively pervious strata, a concrete cutoff wall would be built on the rock and extended into the impervious trench filling or embankment sufficiently to secure a water-tight bond. The embankment would, of course, be constructed of the homogeneous impervious material available (see Fig. 60).

20-28. Rolled-fill Process.—In order that the dam may be properly designed, it is necessary to determine the process of construction that is to be used in placing the materials in the embankment. In the rolled-fill type, the watertightness and stability of the design depend upon the dam being constructed in thin layers properly watered, consolidated, and joined together. The materials in each layer should be carefully spread and thoroughly mixed for the purpose of attaining uniformity. If the mixing is not carefully done, the clay may form planes of slippage, or the gravel, sand, or shale may form planes of leakage. The compacted layers should vary from 4 to 8 in. in thickness, depending upon the method of compaction used, and they should slope toward the center of the embankment so that any tendency to slide may be towards the center rather than towards the outer edges of the fill.

The materials placed in the layers should be thoroughly and uniformly consolidated, artificially, over the entire surface of the embankment. Unless this is done, the structure will be loose and subject to percolation of rain and reservoir water. Each layer should be left in a roughened condition so far as practicable, to prevent separation and so that seepage between succeeding layers would have to follow a tortuous path. Artificial consolidation and surface roughening are of great importance and the more thoroughly they are done the better and more reliable the completed dam will be. The final consolidation due to settlement takes time, and for this reason the dam should stand without use as long as possible before the reservoir is filled.

In order to secure water-tightness and stability of the dam, the amount of water used should be just sufficient to unite each layer with the previously placed one and produce a solid mass free from stratification.

20-29. Hydraulic-fill Process.—During the early mining days in the western part of the United States, it was found that earth material could be moved in large quantities at a low cost by the use of the hydraulic process. However, the hydraulic process of mining filled the streams with débris and backed the water over agricultural lands, to such an extent that the miners resorted eventually to the building of dams with the sluiced materials to prevent carrying the débris into the streams, and to impound water for mining purposes. While these dams were small, their success led to the use of the process for the construction of dams of much greater magnitude. The power used in the hydraulic-mining process was that of water flowing under artificial pressure or gravity. From the foregoing, it is evident that a hydraulic-fill dam may be defined as a dam constructed of earth by means of excavating, transporting and depositing the materials in it through the use of water flowing under pressure or gravity. Abundance of water and proper materials at the right elevation are necessities for the hydraulic-process dam construction.

The general principles under which a hydraulic-fill dam is constructed are that (*a*) the inner third of the dam shall be composed of an impervious mass of consolidated material; and (*b*) the outer thirds of the dam shall be constructed of a mixture of coarse and fine material, the inner halves of which shall act as a filter to retain the fine particles of the middle third, and the outer halves of which shall act as drains. The principles require a careful sorting of the materials in the process of deposition. It has been found from experience that the materials best suited for use in the construction of a hydraulic-fill dam consist of a mixture of gravel, sand, and clay, or silt, with about 20 per cent of clay. The purpose of the clay is to form a good binder between the sand and the gravel. These ingredients,

when deposited under water in proper proportion, form a compact fill and at the same time permit of good drainage during construction. A mixture of sand and gravel without clay can be used in the hydraulic process with quite satisfactory results. A mixture of broken rock and poor clay, on the other hand, is to be avoided because the broken rock will not retain the clay particles and for this reason imperviousness cannot be secured.

The use of clay or material with a large percentage of clay in it in the construction of a hydraulic-fill dam is dangerous unless the dam is constructed very slowly and with an adequate drainage system. Even in such cases the weight in the center may cause the sides of the embankment to give way with the resulting destruction of the dam. Where materials such as rock and clay only are available, or where the materials available contain a large percentage of clay, the hydraulic process should be avoided.

COLD SPRINGS DAM

20-30. General Statement.—Cold Springs Dam is located on Cold Springs Creek, a branch of the Columbia River near Cold Springs, Ore., and it has for its purpose the storage of about 50,000 acre-ft. of irrigation water for the Umatilla Project of the United States Reclamation Bureau. The dam was constructed during 1907 and 1908. The Cold Springs Dam is of the rolled-fill type, is 87½ ft. high above the valley floor of Cold Springs Creek, 98½ ft. above the lowest point in the creek channel, and 110 ft. above the bottom of the cutoff trench. The dam is 3,800 ft. long at the crest and has a top width of 20 ft., an upstream slope of 3:1, a downstream slope of 2:1, and a total volume of about 673,000 cu. yds.

20-31. Foundation Materials.—The dam is located at the upper end of a canyon formed of non-columnar, hard, sound, heavy, igneous rock of the Columbia River lava flows. The rock at the dam site is overlain with an earth covering of fine loamy and sandy material varying in depth

from 2 to 10 ft., under which are found strata of fine sand, silt, coarse sand, and gravel. Some of the strata are indurated and more or less irregular in formation. The underlying rock forming the canyon drops to lower elevations on both rims as it recedes from the canyon so that the dam could readily have been connected to the rock across the canyon itself, but it could not have been so connected towards the ends of the dam. The volcanic nature of the rock makes water-tightness of the foundation uncertain, as do the sandy layers in the overlying soil. To prevent the passage of water through the foundation and along the base of the dam, a cutoff trench 30 ft. wide and a maximum of 20 ft. deep was excavated under the upstream third and refilled with relatively impervious material. Four other shallow trenches were cut in the ground surface, upstream from the axis of the dam.

20-32. Embankment Materials.—The materials available in suitable quantities for the construction of the dam were fine sandy loam, gravel, and basaltic rock. Experiments showed that variable mixtures of the fine sandy loam and gravel could be successfully used for the embankment, while the basaltic rock formed excellent material for slope protection.

The embankment was constructed of a mixture of gravel and fine sandy loam. The maximum water-tightness in the embankment was given to the upstream part which contains the greatest percentage of fine sandy loam, and the minimum water-tightness to the downstream part which contains the least percentage of fine sandy loam.

The gravel was delivered on to the dam from trains and dumped from a trestle. The fine sandy loam was also partly delivered by trains and partly by wheel scrapers, and spread with a road scraper in layers of 4 to 5 in., after which the gravel was spread over it. The gravel and the loam were then thoroughly mixed with disk harrows and cultivators, following which the mixture was watered and rolled into layers of from 4 to 6 in. in thickness. The gravel portion of the dam was not rolled extensively, as compacting

was secured from the trestle, but the upstream parts of the dam were thoroughly rolled.

20-33. Slope Protection.—The water slope of the dam was covered with 12 in. of pit-run gravel and a layer of rock riprap of 24-in. thickness; the land slope was covered with 12 in. of rock riprap.

20-34. Drainage System.—When the Cold Springs Dam was constructed, it was expected that very little seepage would take place through the embankment itself, but that some seepage would take place through the foundation. A drainage trench was dug under the downstream third of the dam near its upstream edge and another near the downstream edge, and an 8-in. drain tile was installed in this latter trench. The area below the dam was also provided with a drainage system to prevent water logging.

20-35. Outlet Tower and Spillway.—The outlet tower is founded on bedrock on the south side of the canyon approximately 17 ft. above its bottom and is connected with a reinforced-concrete conduit provided with cutoff collars at intervals of 30 ft. The outlet is controlled by a sluice gate at the entrance from the reservoir into the tower and a similar one from the tower into the conduit.

The spillway is located on the north side of the canyon. It has a crest length of 330 ft. and is 8 ft. below the top of the dam. Abutment walls are provided at the ends of the spillway and cutoff walls are extended into the body of the dam. A concrete-lined channel extends from the spillway crest to a level 40 ft. above the canyon bed below the dam. The lined channel rests on a hard-clay and a sand formation, and terminates in rock at the lower end. The channel is provided with cutoff walls at 10-ft. intervals and in the sand formation the concrete is reinforced.

UPPER SAN LEANDRO DAM

20-36. General Statement.—The Upper San Leandro Dam was constructed by the East Bay Water Company on San Leandro Creek near Oakland, Calif., during 1924, 1925, and 1926, and is now owned by the East Bay Munici-

pal Utility District. This dam controls a reservoir of about 50,000 acre-ft. capacity, the water supply from which is used for municipal purposes in connection with the district comprising nine cities on the east shore of San Francisco Bay.

20-37. Foundation Materials.—The general geological formation at the dam site consists of the Chico formation, has a thickness of about ¾ mile, and a lateral extent of about 2 miles. The foundation materials of the dam consist of marine deposits laid down during the cretaceous period, and are composed of massive sandstone, shale, and some conglomerate. The sandstone predominates and, on weathering, yields a sandy soil, the shale reduces to clay, and the conglomerate to its constituents. While the conglomerate is of small extent, a zone of it crosses the dam site and plays an important part in the construction of the dam. San Leandro Creek has cut a channel through a ridge of this material of highly tilted, badly fractured, somewhat faulted, monoclinal structure.

20-38. Control of Stream during Construction.—Preparatory to the construction of the dam, the entire foundation was stripped of surface soil, a diversion dam with a height of 55 ft., a top width of 25 ft., a 3:1 upstream slope and 2:1 downstream slope was constructed at the heel of the main dam. The diversion dam was located so as to become an integral unit of the main dam. A diversion tunnel was constructed around the right end of the dam to divert the flood water during construction. A cutoff trench along the axis of the main dam and another along the axis of the diversion dam were carried down to bedrock across the full length of the dam and were carefully back-filled with selected clay. The maximum width of the cutoff trenches was about 10 ft. and the depth about 15 ft.

20-39. Embankment Materials.—The dam was constructed by the hydraulic-fill process from the sandstone, shale, and conglomerate materials adjacent thereto. The materials were classified as toe material and core material. Careful tests were made during the construction of the

dam on the weight and coefficient of friction of the material. A large number of tests show that the maximum dry weight of the toe material was 128 lb. per cubic foot, the minimum 74 lb., and the average 104 lb. The coefficient of friction of this material showed a maximum value of 0.91, a minimum of 0.35, and an average of 0.46. Screen analyses of the toe material indicated 17 per cent coarser than 2 in., and 27 per cent finer than ¼ in., the 17 per cent ranging in size from 2 to 23 in. The dam was designed on the basis of a weight of 100 lb. per cubic foot and a sliding coefficient of 0.2 for the toe material.

The moisture content of the core material showed a maximum of 43.3 per cent, a minimum of 22 per cent, and an average of 35.1 per cent. Penetration tests on the core material were made during the construction of the dam. Mechanical analyses of the core material indicated that about 11.9 per cent were clay, 27.7 silt, 40 very fine sand, 17 fine sand, 2 medium sand, 0.7 coarse sand, and 0.1 per cent fine gravel. About half of the 11.9 per cent of clay consisted of colloidal clay with a diameter of 0.0015 mm. or less.

20-40. Dimensions of Dam.—The dam at the maximum section is 75 ft. wide at the crest, has an upstream slope of 3:1, and a downstream slope varying from 3:1 at the base to 2:1 at the top. The downstream slope has berms 20 ft. wide at intervals of 40 ft., the slope from the base to the first berm being 3:1; from the first berm to third, 2½:1; and from the third to the top of the dam, 2:1. The puddle core extends to within 15 ft. of the top of the dam, has a top width of 15 ft., has the central point of its top 50 ft. downstream from the upstream edge of the crest, and slopes on both faces 1:10 theoretically. Daily records of the ratio of the width of the core to the width of the dam expressed as a percentage was a maximum of 28 per cent, a minimum of 10 per cent, and an average of 21 per cent. It is of interest to note that the dam has settled less than 0.5 ft. since its completion. There are no slope protections, as the effective reach of the reservoir is small and the coarse

particles in the toe material appear to offer sufficient protection on both slopes. The dam contains 1,200,000 cu. yd. of material of which approximately 97 per cent were placed by the hydraulic-fill method.

20-41. Method of Placing Materials.—The embankment materials were obtained from the side-hill elevations above the ends of the dam. Levees were maintained at the slopes with Fresno scrapers and a steep gradient was maintained towards the puddle core to prevent fine material from escaping to the sides. The core was deposited in a pool maintained along the axis of the dam and was drained with two 30-in. overflow pipes.

20-42. Diversion Tunnel.—The diversion tunnel used during construction as a by-pass is now used as an outlet into San Leandro Reservoir on the same creek. This tunnel is 1,100 ft. long, of circular section 9 ft. in diameter, and is lined with concrete of an average thickness of 18 in. Two concrete cutoff rings, 3 ft. thick, are placed around it at points upstream from the central cutoff trench. The tunnel was pressure grouted to prevent seepage behind the lining. A funnel-shaped shaft with the spillway weir, 30 ft. below the crest of the dam, connects with the tunnel near the heel of the dam. The connection is made by means of three 24-in.-diameter, cast-iron pipes imbedded in concrete at the base of the shaft, each controlled with a 24-in. gate valve.

20-43. Spillway.—An open spillway of 6,000 sec.-ft. capacity, built on the original ground surface, is provided around the right end of the dam. The crest of the weir is 43.8 ft. long and is at an elevation 15 ft. below the top of the dam. Provisions are made for flashboards to raise the water to within 5 ft. of the top of the dam after the flood season is over. The spillway channel is 650 ft. long, and is lined with 6 to 12 in. of reinforced concrete.

20-44. Outlet Tower and Tunnel.—Water from the reservoir is taken out through a horseshoe-shaped, concrete-lined tunnel, 7,200 ft. long and 6½ ft. in diameter. The outlet tower is located 80 ft. from the intake end of the

tunnel. Forty feet of the tower are in shaft and 50 ft. are above ground. The tower is provided with nine inlets.

20-45. Lists of Earth Dams.—In the following tables are listed some of the important rolled-fill and hydraulic-fill dams of the United States. The lists are not intended to be complete, but are only to show the trend of practice in the design of these types of dams.

TABLE XXIV.—IMPORTANT HYDRAULIC-FILL EARTH DAMS IN THE UNITED STATES

Name of dam	Location	River	Year completed	Height above foundation, ft.	Crest length, ft.	Top width, ft.	Up-stream slopes	Down-stream slopes	Volume of dam, cu. yd.
Fort Peck	Montana	Missouri	*	242	9,000	100	4:1	8½:1	100,000,000
Upper San Leandro	California	San Leandro Creek	1926	215	683	75	3:1	2:1, 3:1	1,250,000
San Pablo	California	San Pablo	1921	220	1,230	50	3:1	3½:1	2,250,000
Kingsley	Nebraska	North Platte	*	162	11,000	....	....	....	26,000,000
Ochoco	Oregon	....	1920	127	1,050	18	2½:1, 1¾:1	2:1, 1¾:1	541,000
Englewood	Ohio	....	1922	124	4,700	30	4:1, 1¾:1	4:1, 1¾:1	3,500,000
Germantown	Ohio	....	1921	124	1,210	25	4:1	4:1	798,000
Buell	California	Small Arroyo	1927	...	550	15	2½:1	2½:1	137,000
Horsley	Utah	Price	1926	...	.....	..	....	....	130,000
Shasta River	California	Shasta	1928	105	1,250	25	2½:1	2:1	700,000
Santa Maria	Colorado	....	1913	...	1,300	20	3:1, 1¾:1	2:1, 1¾:1	310,000
Lockington	Ohio	....	1921	78	6,400	30	4:1	4:1	

* Under construction in 1938.

TABLE XXV.—IMPORTANT ROLLED-FILL EARTH DAMS IN THE UNITED STATES

Name of dam	Location	River	Date completed	Height above foundation, ft.	Crest length, ft.	Top width, ft.	Type of core	Upstream slopes	Downstream slopes	Volume of dam, cu. yd.
Puddingstone No. 1, 2, and 3*	California	San Dimas	1927	166	.....	25	Concrete	2½:1, 3½:1	2¼:1	1,600,000
Swift	Montana	.....	1914	165	.....	..	Reinforced concrete	1¼:1		
Goose Creek	Idaho	Goose Creek	1913	145	.....	16	Reinforced concrete	3:1	2:1	
Idaho Irrigating Co.	Idaho	.....	....	135	.....	40	.....	3:1	2½:1	
Encino	California	.....	1924	127	1,027	20	.....	2:1	2:1	501,400
McKay	Oregon	McKay Creek	1927	125	.....	..	.....	1¾:1	2:1	2,330,000
Lahontan	Nevada	Truckee	....	124	1,400	20	.....	3:1	2:1	700,000
Belle Fourche	South Dakota	Owl Creek	1911	122	6,493	19	Timber	2:1, 5:1	2:1	1,600,000
Tabeaud	California	South Fork Jackson Creek	1902	120	636	20	.....	2½:1, 3:1	2½:1	370,350
Apishapa	Colorado	.....	....	120	.....	19	.....	2:1, 3:1	2:1	
Druid Lake	Maryland	.....	1864	119	.....	60	Puddle	4:1	2:1	
Standley Lake	Colorado	South Platte	1911	113	.....	20	Puddle	2:1, 3:1	2:1	
Ashokan Dikes	New York	.....	1917	110	.....	34	Concrete	2:1, 2¾:1	2:1, 2¾:1	
Beaver Park	Colorado	.....	....	108	.....	16	Reinforced concrete	2:1	1½:1	
Gordon Valley	California	.....	1925	104	.....	15	.....	2½:1	2:1	
Titicus	New York	Titicus	1895	102	1,519	30	Rubble	2½:1	2½:1	
Wanaque	New Jersey	.....	1925	100	.....	16	Concrete	3:1	2:1 2½:1	

* Part rolled-fill and part hydraulic-fill.

CHAPTER XXI

ACCESSORIES FOR DAMS

INTRODUCTION

21-1. General Statement.—In the design of an important dam, there are always accessories that must be provided for. The principal of these accessories are spillways, outlets, logways, and fish ladders. Very often the design of these accessory parts is the most difficult problem confronting the designing engineer.

INDEPENDENT SPILLWAYS

21-2. Location of Spillway.—Conditions at the dam and reservoir site determine whether an overflow or a non-overflow dam shall be used. If the dam is very high or the possibility of destruction by overflow is too great on account of the geological formations below the dam, or if a suitable and more economical location is available elsewhere in the rim of the reservoir, an independent spillway is provided.

If at all possible, it is desirable to build the spillway at a low place in the rim of the reservoir at some distance from the dam or in the natural material at one or both ends of the dam. Placing the spillway over the top of an earth dam should be avoided if possible. If solid rock exists at either end of the dam or at a low place in the reservoir rim, it makes a suitable site for the spillway, but it is often necessary to build it on an earth foundation and to protect the foundation with some kind of impervious lining.

21-3. Capacity of Spillway Required.—Where the reservoir is supplied from a pump or a canal of controlled and known capacity, and where there is no drainage area of consequence leading into the reservoir, only a very small spillway is needed and in some cases it may be omitted

altogether. Where, however, the reservoir is built on a stream with a drainage area above it subject to precipitation, great care must be taken to provide a spillway of ample size, as many failures of dams are traceable directly to defective or inadequate spillways.

It is essential that the spillway shall be able to discharge safely the maximum flood from the drainage area; and, inasmuch as formulas for maximum runoff cannot be relied upon for abnormal conditions, it is necessary to make an allowance in excess of the maximum known flood or the maximum computed flood. Where flood observations have been recorded for a period of 25 years or more, an allowance of 25 to 50 per cent over the highest known or computed flood should be made to take care of abnormal floods. Where the record is shorter than 25 years, the allowance should be greater, probably 50 to 100 per cent or more. The damage due to failure should be taken into consideration, and where this would be relatively great, the allowance should be very large.

21-4. Parts of a Spillway.—A spillway consists essentially of four parts: (*a*) the approach channel; (*b*) the crest of the overflow section; (*c*) the tail channel; (*d*) the stilling device.

21-5. Design of Approach Channel and Spillway Crest.—The approach channel to the spillway crest is usually excavated either to crest level or slightly below it. The channel should have a clear unobstructed course of normal full width of the weir length, and if curved, the curvature should be gradual.

The design of the crest is identical with the design of an overflow dam and needs no further elaboration than given in Chap. IX.

21-6. Design of the Tail Channel.—The tail channel must also have a clear and unobstructed course of full normal width at the crest section, and, while it may be narrowed below this section, the narrowing should never be sufficient to back the water up and drown the weir. Unless rock exists at the bottom of the tail channel, protection may be required to prevent back-cutting. In the

design of the tail channel, reference should be made to Hydraulics of Channels in Chap. XXII.

21-7. Design of the Stilling Device.—The stilling device should be so constructed as to reduce the velocity below the toe of the tail channel to a non-destructive one. The principles of design involved are similar to those involved in the provisions for destroying velocity below an overflow dam. As in the overflow dam a hydraulic jump will probably be formed below the toe of the tail channel, and the area between the toe and the downstream end of the standing wave must not be subject to erosion. By depressing the paved area adjacent to the toe of the tail channel and giving it a gradual rise toward the downstream end of the apron, or by obstructing the free progress of the water by a low dam or by baffle piers, the hydraulic jump may be brought close to the toe of the tail channel. In many cases it is more desirable to construct a stilling pool entirely depressed below the channel bed into which the tail channel discharges. By making this pool sufficiently long, wide, and deep, the outlet velocity may be made any desired amount. The dimensions of the pool should be just enough to reduce the velocity of the water issuing from the pool to that non-destructive to the channel below it.

21-8. Spillway Controls.—Many overflow dams and independent spillways for non-overflow dams are provided with spillway controls that can be removed at flood stages of the reservoir and put into service when the high stages have passed. By the use of such control the discharge capacity of an overflow dam or spillway for a given elevation of water in the reservoir is increased during flood stages and the storage capacity increased during low stages. Overflowing of lands by the reservoir is also considerably restricted by reducing the high water level in the reservoir. The facilities used for these purposes consist of ordinary flashboards, sliding gates, Taintor gates, Stoney gates, or roller gates. These types of control will be discussed under the subject of gates for spillways and outlets. Siphon spillways are also used to control the water level in

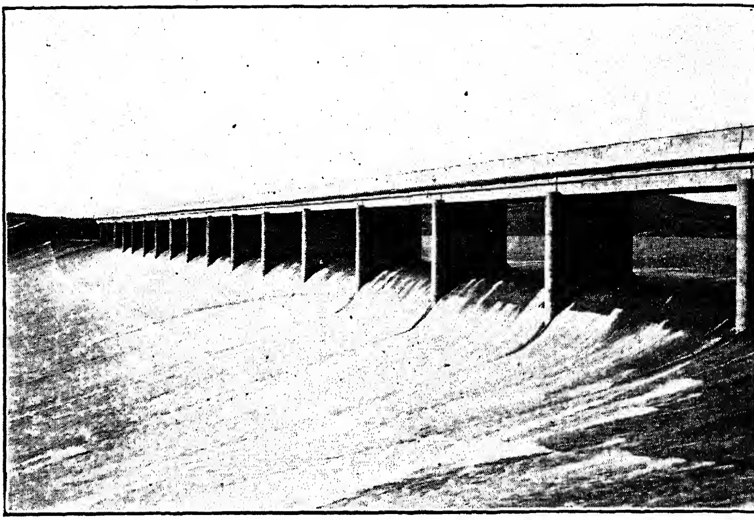


FIG. 61.—South spillway of Pardee Reservoir. (*East Bay Municipal Utility District.*)

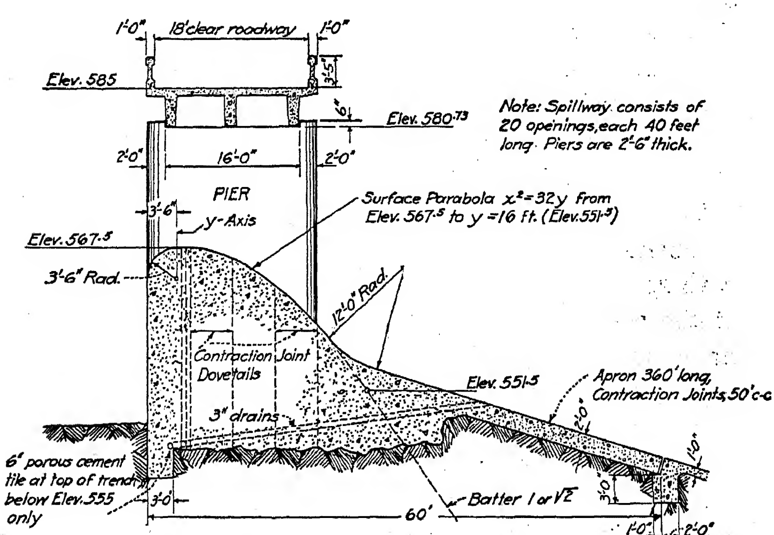


FIG. 62.—Section of south spillway of Pardee Reservoir. (*East Bay Municipal Utility District.*)

reservoirs, but they are more properly considered permanent parts of overflow dams or spillways and are discussed in Chap. IX on overflow solid gravity dams.

21-9. Description of South Spillway.—Figure 61 shows a photograph of South Spillway for Pardee Reservoir in California. This spillway has a discharge capacity of 120,000 sec.-ft., is 847½ ft. long, and is founded on hard seamy diabase rock. About 100 ft. below the toe of the apron the water enters a gulch of upturned shale and slate, and flows back into the river. Figure 62 shows a section of the structure at its center.

RESERVOIR OUTLETS

21-10. Location of Reservoir Outlets.—In addition to a spillway provided at the fixed maximum storage levels of reservoirs to prevent overtopping of the dam, it is in many cases necessary, and in practically all cases desirable, to have outlets at lower levels of the reservoir. Usually an outlet is placed near the reservoir bottom to drain the reservoir. In the case of an earth dam the outlet should be built through cut under the dam or carried through a tunnel around one end of it. In a masonry dam the outlet may be carried through the dam, although in this case it is often also desirable to carry the water around one end of the dam through a tunnel. Such a tunnel, in case of either an earth, rock-fill, or masonry dam, is also of value for diverting the stream flow during the construction of the dam. Where a dam is very high and water is to be let out of the reservoir, it is customary to provide outlets at various levels differing not more than 120 ft. in elevation. The essential features to be considered in the construction of the outlets are capacity, safety, durability, and efficiency.

21-11. Capacity Required for Reservoir Outlets.—The capacity of the reservoir being known, the size of the outlet or outlets can be computed by well-known principles of hydraulics. The inflow into the reservoir during the discharge periods, the discharge quantity desired, the varying head of water in the reservoir, and the minimum head that

will produce the required discharge must be considered in fixing the sizes of the outlets. This minimum head should be as high as possible without undue loss of storage capacity; and the discharge velocities should be held down to reasonable values. Although velocities as high as 100 ft. per second have been used, velocities of 25 ft. per second are about the maximum desirable for use in design.

21-12. Safety of Reservoir Outlets.—Safety in outlets really often means safety of the dam. The principal means by which safety may be secured are the location of the outlet on firm, solid material that will not settle under the load of the dam, and devices to prevent the passage of water along the outer wall of the outlet structure. For construction through earth, collars should be placed around the outlet conduit, the extent of these collars into the materials depending upon the size of the conduit, the head of water to be resisted, and the nature of the material through which the conduit passes. Durability of the material used in the conduit adds greatly to the safety of the dam itself, and consequently materials of short life are not used. Cast-iron pipe and reinforced-concrete pipe are generally most suitable for the purpose.

21-13. Trash Racks.—The upstream ends of conduits are usually protected with trash racks to exclude débris of too large size from the conduit. Trash racks are usually made either of cast-iron or of steel bars properly supported. The distance placed between the bars should depend upon the size of the outlets and the nature of the trash to be excluded from it. Generally, spacing of from 6 in. to 1 ft. on centers is the practice for outlets of large capacity. Closer spacing is, however, required for outlets of small diameter. The correct loading for use in the design of a trash rack is difficult to determine, and must be carefully considered. The design will be safe if the trash rack has sufficient strength to support the entire water load above it when the reservoir is full, but for deep water such a trash rack would be exceedingly expensive. Such substantial construction is not warranted because complete stoppage of the trash-

rack openings is not likely to occur. Trash-rack bars designed to be stressed to their ultimate strength under a head equal to about half the depth of water above them are usually sufficient. No head less than 25 ft. nor more than 50 ft. need be used, however, if methods of clearing the racks are provided. The supports of the bars should have a factor of safety of 2 under these heads. The clear areaway in trash racks should be as large as possible to prevent choking, and a velocity as low as 2 ft. per second or less, if possible, is desirable. In no case should the velocity through the trash rack exceed 2 ft. per second.

21-14. Gates for Reservoir Outlets.—One of the problems in the design of outlets through dams is that of securing adequate and safe discharge control. This is done by means of gates and valves set in the conduit. Unless the structure is a minor one, duplicate control is generally provided, and in the case of very important structures triple control is sometimes installed. There should in all cases be secondary control provided at the upper end of the outlet conduit, so that repairs can be made to the conduit when required. The primary control may be placed at the outlet end of the conduit if the conduit is designed to withstand full reservoir pressure, but if not, it must be placed at the inlet. Gates and gate valves of various kinds are used for control of reservoir outlets, depending upon the quantity of discharge to be handled and the head under which they must be operated.

NEEDLE VALVES

21-15. Description of Needle Valves.—In general, needle valves are of two classes, those of the exterior-flow type and those of the interior-flow type. Figure 63 shows a cross-section of the former type and Fig. 64 one of the latter. It will be noted from these figures that the water passage of the exterior-flow type is not a part of the valve proper, while this passage in the interior-flow type is a part of the valve. The fundamental feature of both types is a piston inclosed in a cylinder and having a needle-like point

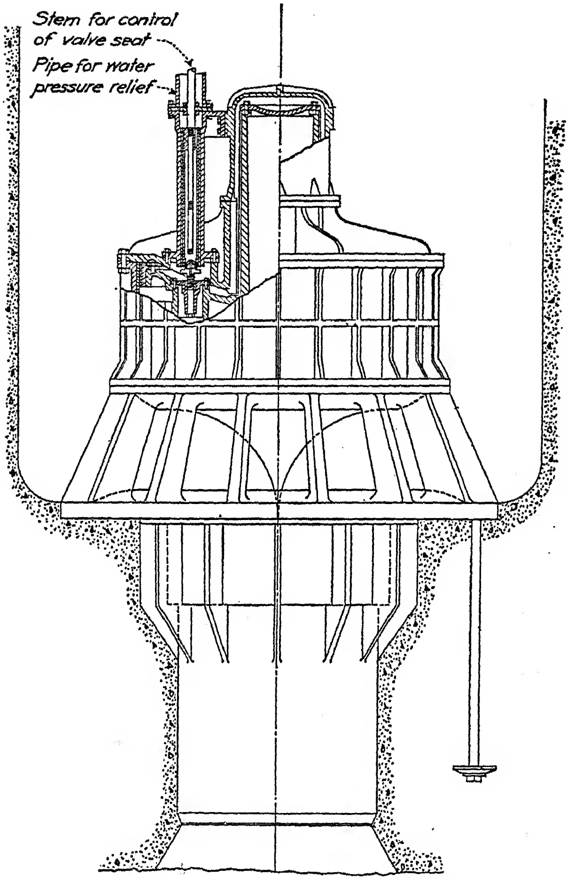


Fig. 63.—Exterior-flow type needle valve. (*U. S. Reclamation Bureau.*)

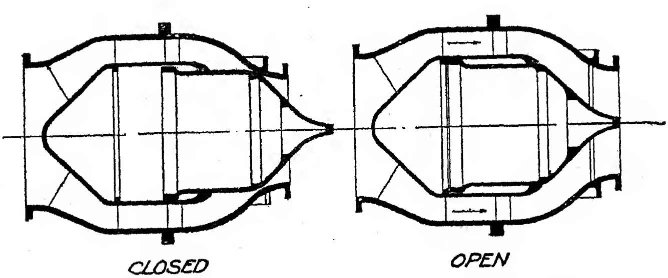


Fig. 64.—Interior-flow type needle valve. (*U. S. Reclamation Bureau.*)

to direct and control the flow of water through a circular discharge opening. The shape of the valve produces a cylindrical jet, which tends to balance the water pressure on the piston and to prevent eddies in the flow. The needle valve may be operated by mechanical means or by control of the water pressure on the piston. The former are called unbalanced needle valves, and the latter, balanced needle valves.

21-16. Design of Needle Valves.—The first principle to be observed in the design of a needle valve is to avoid abrupt changes in direction of flow and velocity of the water through the valve. Consequently, the water passage through or past the valve should be of a nearly uniform or decreasing area, the entering solid water-stream area being diverged gradually around the valve or the interior parts thereof, and again gradually converged back into a solid stream at the throat of the outlet. Beyond the throat, the outlet should flare or diverge so as to permit free access of air to the issuing jet. In the interior-flow type the ribs extending across the water passage should be tapered at the upstream and downstream edges. The aim should be to force the moving water solidly against the bounding surfaces of the water passage and thus prevent the formation of a vacuum. After the water reaches the valve, it should be kept moving at a uniform or increasing velocity to the discharge throat.

21-17. Materials Used in Needle Valves.—On account of the tremendous forces involved in needle-valve operation under high heads, the best of construction materials are used. The seat rings and nozzle tips are usually made of manganese bronze, the supporting guides of the internal cylinder and the screws, bolts, and nuts of phosphor bronze, the body of the valve of steel or cast steel, and the interior cylinder of cast iron or cast steel.

21-18. Unbalanced Needle Valve Operation.—Unbalanced needle valves require either manual or machine operation. In the smaller sizes of valves, power is applied with an ordinary hand wheel or a crank and gear device

exterior to the valve body, and utilizes either a screw and nut or pinion and geared-rack connection with the piston. For the larger sizes, an electric motor, equipped with limit switches, drives a geared device. Operation is sometimes also secured through the use of a hydraulic cylinder. The operation of the valve may be assisted by taking advantage of partially balanced pressure on the piston.

21-19. Balanced Needle Valves.—The most recently developed balanced needle valve consists of a circular body surrounding an internal cylinder closed at one end and connected to the body by radial ribs in which a pointed piston or needle operates and seats on the valve body. Pressure on the inclosed end of the piston is regulated by a drain or control pipe of larger clearance than that between the piston and the interior cylinder. There have been several types of these valves invented and used, but the most improved types adaptable to sluiceway discharge for dams are those developed by the United States Bureau of Reclamation and those developed and patented by manufacturing concerns. A discussion of the Bureau of Reclamation type and the Larner-Johnson valve as the patented type will be given in the following paragraphs.

21-20. Description of Bureau of Reclamation Type.—Figure 65 shows the most improved Reclamation-Bureau type. This valve is part of the outlet control of the Pathfinder Reservoir on the North Platte River in Wyoming. It is comprised of an exterior body A; and an interior cylinder B, forming the water passage channel; a water-balanced piston C with a needle point D; a pressure line E for admitting water into the chamber F, back of the piston; a ring G on the piston; a control pipe H with a sleeve I on the axis J of the cylinder for controlling the flow of the water from behind the piston; a drainage line K for draining the body of the valve when inspection or repairs are required; and an indicator L for showing the position of the piston.

21-21. Operation of Reclamation Bureau Type.—It will be seen from Fig. 65 that the ring has a diameter greater

than that of the main body of the piston. This ring fits the interior cylinder closely, but the clearance around it, together with that especially provided by means of a hole in the inner shell, is enough to equalize the water pressure inside and outside the cylinder when the control pipe is closed, and the control pipe is large enough to reduce the pressure back of the piston to atmospheric when it is opened. In order to explain the principle of operation, let the valve be assumed to be closed by having reservoir

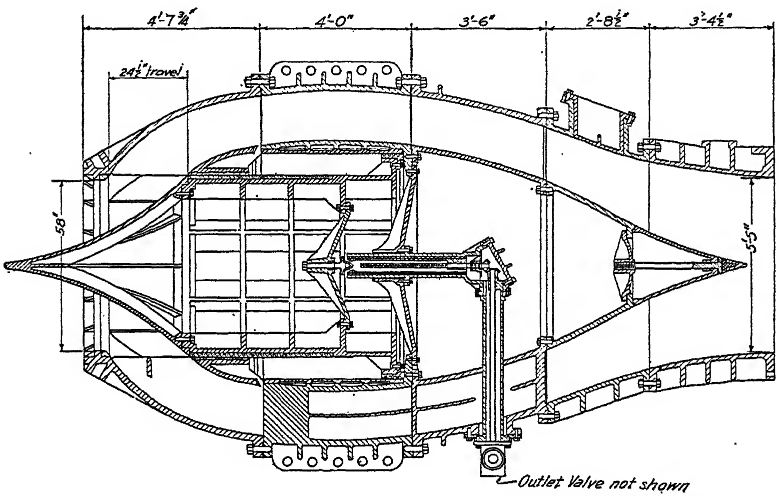


FIG. 65.—Reclamation-Bureau type needle valve. (*U. S. Reclamation Bureau.*)

pressure back of the piston. This pressure will be greater than the opposite water pressure on the ring and the atmospheric pressure on the pointed end of the piston, whence the piston will be held against its seat. In order to open the valve, the pressure in the cylinder behind the piston is reduced by opening the control pipe valve. The atmospheric pressure on the pointed end will then be balanced by the pressure on the corresponding area behind the piston, and the annular space between piston and ring on the discharge side of the valve will be subjected to reservoir pressure, which, being greater than atmospheric pressure behind the piston on the same area, will result in an opening

force on the piston that will move the valve as fast as the water can be forced out of the cylinder back of the piston. In order to close the valve, the control pipe is closed and reservoir pressure accumulates in the cylinder back of the piston through the clearance space around the ring. The valve closes because the pressure in the cylinder is the same as that on the annular ring, and that on the pointed end of the piston is somewhat less than that on the same area on the other end of the piston on account of the velocity head of the moving water. For the purposes of regulation, it is necessary to hold the piston of the valve in a predetermined position that will give the proper discharge. This requires the pressures on the piston to be approximately balanced. The balancing of pressures is accomplished by means of a circular valve that is mounted on the piston and controls the discharge of water through the outlet pipe. The control is obtained with a movable seat located in the outlet pipe and operated with a stem also inside of this pipe and leading to a hand control on the outside of the main valve. The relative position of the movable seat and the circular valve controls the outflow of water, back of the piston, and, therefore, controls the pressure in the chamber. Thus, pressures back of and in front of the piston are balanced. A slight movement of the piston toward closure increases the opening of the control valve, releases a greater flow of water, relieves the pressure in the cylinder, and causes the piston to move back again. A slight movement of the piston towards open position decreases the opening of the control, which lowers the rate of outflow back of the piston, increases the pressure in the cylinder, and causes the piston to move back again. Thus the piston is held in a definite position, or moves slowly to and fro over a short range of motion.

21-22. Description of Larner-Johnson Type Needle Valve.—There are several types of the Larner-Johnson valve, but that of most interest in dam design is the Type *N* hydraulic discharge regulator. This valve is of the interior-flow, hydraulically balanced type. Figure 66

shows a cross-section of the 72-in. Larner-Johnson valve installed in one of the sluiceways through the Pardee Dam. This valve consists, in general, of a circular body *A* surrounding an interior cylinder *B*, closed at one end and connected to the body by radial ribs, in which a pointed piston *C* operates, making contact with the seat in the discharge opening of the body to close the valve. It is

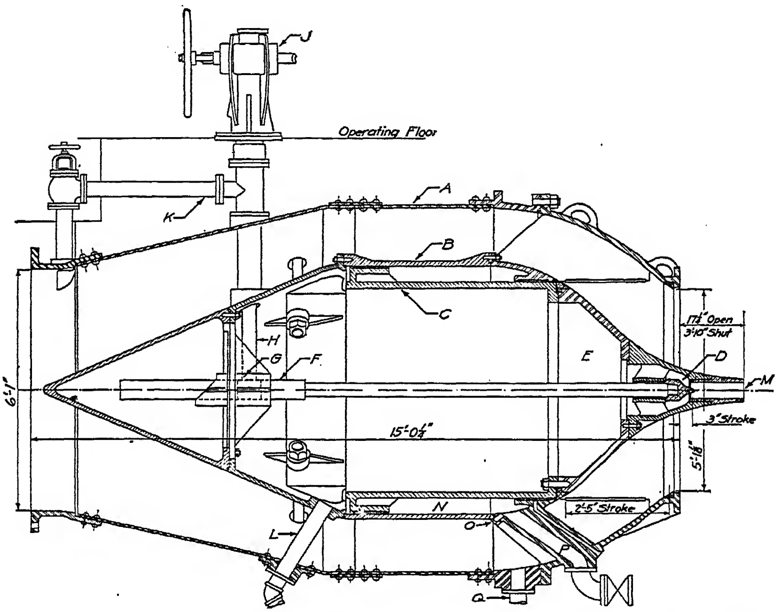


FIG. 66.—Larner-Johnson 72-in. Type *N* discharge regulator. (*Pelton Water Wheel Company.*)

provided with a pilot valve *D*, controlling the outflow from chamber *E* to the point of the piston, and connected through a rack *F* with a pinion *G*, operated by means of the shaft *H*, connected outside of the valve body with a worm-gear drive *J*. Reservoir pressure is admitted to the chamber *E* through the pressure pipe *K*, and is relieved through the drain pipe *L* or the port *M*. Reservoir pressure is admitted to chamber *N* through drilled holes *O*, and is relieved through the drain pipe *P*. Draining of the body is provided through the drain pipe *Q*.

21-23. Operation of Larner-Johnson Type.—The valve is balanced and unbalanced hydraulically by means of the pressure pipe between the reservoir and the chamber E, back of the piston, and the exhaust from the pipe from chamber N. In case it is desired to seat the valve, chamber N is drained by opening the drain pipe P to air pressure below the dam, and reservoir pressure is admitted to chamber E through the pressure pipe K and the closing of the pilot valve D. Since the pressure on the rear of the piston is that due to reservoir pressure, the back pressure due to moving water on the pointed end of the piston will be less than that on the rear end and the valve will close under the action of the hydraulic forces. In opening the valve, the chamber E is drained through the drain pipe L and the opening of the pilot valve, and the pressure on the excess area of the piston in the chamber N will be sufficient to unseat the valve and to carry it to an open position. In order to regulate the flow through the valve, the pilot valve is set at a position to secure the desired opening. If the piston moves away from the pilot valve as set, the port opening will be increased, the water drained from chamber E, the pressure released, and the piston forced back to the desired position. If the piston moves toward the pilot valve, the pilot valve will close the port, shut off the leakage from chamber E, increase the pressure back of the piston, and move the piston back to the desired position. Thus the valve is held in the position determined by the pilot valve.

GATE VALVES

21-24. Uses of Gate Valves.—In dam design, gate valves are often used in connection with cast-iron, cast-steel, or structural-steel sluiceways or penstocks through the dam. The gate valve may be placed in the outlet near the upstream face of the dam in which case the operating mechanism is either placed on top of the dam or in an operating chamber within the dam itself. In some instances these controls are placed at the lower end of the

outlet at the downstream face of the dam. Unless the outlet pipe is made water-tight, this may, however, throw full reservoir pressure on to interior parts of the dam. Where the control is placed at the downstream face of the dam, provision should be made for an emergency closure at the upstream end of the outlet, especially if the reservoir pressure is great. Under very high heads, gate valves should be operated under full-open or full-closed positions only, but under low heads they may be used for regulating purposes.

21-25. Classification of Gate Valves.—Gate valves are classified according to the movement of the stems, materials of manufacture, kinds of seats, amounts of pressure, and methods of operation.

21-26. Types of Stems for Gate Valves.—Gate valves may be either of the non-rising-stem or the rising-stem type. In the non-rising-stem type, the screw hoist operates in threads in the gate disk itself and the gate moves up or down on the stem in accordance with the direction of motion of the operating mechanism. In the rising-stem type the gate is fixed to the lower end of the stem and the stem rises and falls with the gate according to the direction of motion of the operating mechanism.

21-27. Materials Used for Gate Valves.—Generally the body of a gate valve is made of cast iron for low pressures, and of cast steel for high pressures; and the seat rings, stem, and often also the disks, bolts, and nuts, are made of bronze.

21-28. Seats for Gate Valves.—There are several types of gate seats used in gate valves. However, the two principal types are the wedge seat and the parallel-face seat. In the wedge-seat type, the gate is built thicker at the top than at the bottom, and comes to seat through wedging of bronze-faced parts on the gate with bronze-faced parts on the valve body. In the parallel-seat type, there are two parallel disks used to make the gate. These disks are arranged to close with bronze seats on the disks meeting bronze seats on the valve body. As the gate comes to its

seat, the stem acting on wedges on the back of the disks crowds the seats tightly together.

21-29. Pressures for Gate Valves.—The valve body, disks, and other parts of the gate valve are designed to withstand specified pressures, and are usually classified as light-pressure valves, standard-pressure valves, heavy-pressure valves, and extra-heavy-pressure valves. The allowable working and test pressures of gate valves usually decrease as the diameter of the valve increases, so the designations may not always represent exactly the same pressures. Moreover, there is not exact uniformity among manufacturers in the use of the designating terms, and it is, therefore, necessary to know the make of the valve in order to determine the pressure that the designation is meant to describe. There are many reliable manufacturers of gate valves, and the problem of the designing engineer is one of proper selection of type and make of valve, rather than of design.

21-30. Operation of Gate Valves.—Gate valves under ordinary pressure and of ordinary size are usually hand operated with the ordinary hand-wheel hoist. As the pressure and size increase, geared hoists are resorted to. For high heads and large valves, hydraulic, gasoline, or electric power is used for operation.

GATE HOISTS AND STEMS

21-31. Types of Gate Hoists.—There are two general types of gate hoists in use, the screw type and the hydraulic type. The essential features of the screw-type hoist are shown in Fig. 67. This type may or may not be provided with ball bearings, but for heavy lifts they are usually provided. When a series of gates is to be handled simultaneously, a shaft with a pinion for each hoist is often utilized and the shaft is operated either by means of electric, fuel, or water motive power, or by hand power. In some instances, electric motors or gas engines are attached directly to the pinion shaft of the hoist. In case of the use

of power machinery, automatic stops are usually provided for both closed and open position of the gate. This automatic feature is accomplished by means of switch or gear releases.

21-32. Mechanical Efficiency of Gate Hoists.—Gate hoists are at best relatively inefficient, as only a small

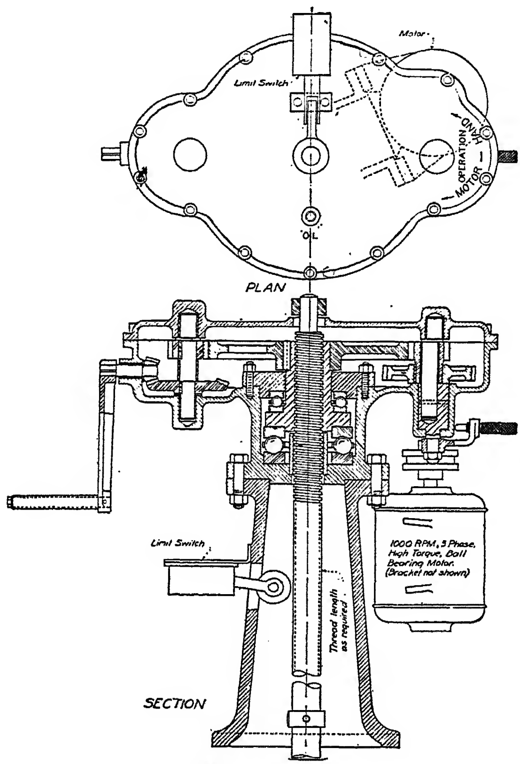


FIG. 67.—Motor-driven screw hoist.

percentage of the applied power is available for moving the gate, owing to the friction of the various parts. The friction varies with the kind of material used in the construction of the hoist. Formulas for efficiency may be found in all good handbooks on mechanical engineering. The principal parts of the hoists reducing their efficiency are the screw and gearing. A formula for computing the

efficiency of screws may be found on page 567 of the tenth edition of Kent's "Mechanical Engineers' Handbook," and tabulations of the efficiencies of gearing may be found on pages 1668 and 1669 of the same handbook.

21-33. Hand-operated Hoists.—In hoists operated by one man the pressure on the crank handle should not exceed 50 lb. for starting the gate from its seat and should not exceed about 25 lb. after starting. In hoists equipped with double-manual operating mechanism, about double these amounts of pressure may be used. The speed of the crank handle in manually operated hoists may be taken as about 50 ft. per minute. Since the actual speed of the gate in operation should not be large it may be possible, where several gates are to be opened and closed, to employ motive power in sufficient quantity to operate all of the gates simultaneously.

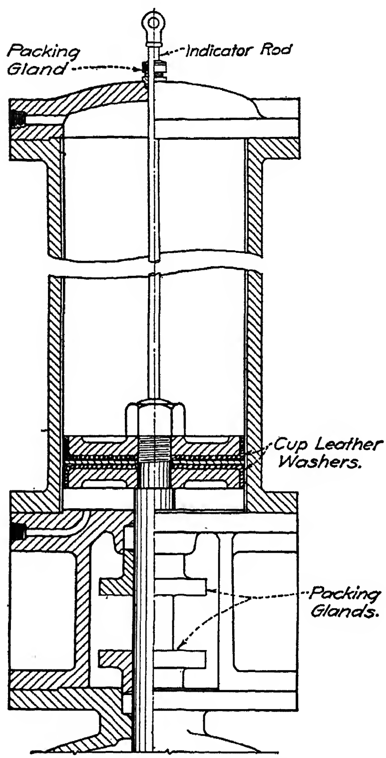


Fig. 68.—Hydraulic hoist.

21-34. Hydraulic Hoists.—Under high heads, hydraulic hoists are very often used, in which water or oil acting under pressure on a piston, moving in a brass- or bronze-lined cylinder, is employed. Rings or packing around the piston and around the gate and piston stems are provided to secure water-tightness. They are usually installed immediately above and close to the gates and, therefore, have short stems, thus avoiding the difficulties accompanying the use of the long stems usually used in screw hoists. Figure 68 shows the design of a hydraulic hoist.

21-35. Design of Hoist Stems.—Where screw hoists are used, they are generally placed on the top of the dam or outlet tower and are connected to the gates with long steel or bronze stems supported at intervals with brackets attached to the concrete of the structure. The forces acting on the stem are that of the hoist pulling the gate up or pushing it down and the resistance of the gate due to water pressure on its face and friction on its seat. The action of the hoist in lifting the gate produces tension and torsion, and in closing it produces compression and torsion.

In closing the gate the combined unit stresses due to compression and torsion may be found from the following formulas:

$$t = \tfrac{1}{2} S_c + \sqrt{S_s^2 + \tfrac{1}{4}S_c^2},$$

and

$$s = \sqrt{S_s^2 + \tfrac{1}{4}S_c^2}$$

in which t is the resulting compression and s the resulting shear due to the combined torsional and compressive stresses $S_s + S_c$, all in pounds per square inch. S_c is obtained by the formula,

$$S_c = \frac{P}{A},$$

in which P is the force in pounds required to move the gate and A is the cross-sectional area of the gate stem in square inches, and S_s is obtained from the following formula:

$$S_s = 103\frac{H}{nd^3},$$

in which H is the foot-pounds per minute applied to the gate stem, n the number of revolutions of the gate stem per minute, and d the diameter of the gate stem in inches. The horsepower applied to the stem is comprised of the horsepower applied to the crank handle or driveshaft reduced by the efficiency factor of the hoist. The same formulas may be used for combined tension and torsion resulting from the lifting of the gate from its seat by considering tension as negative compression.

The allowable compressive working stress in the stem should be found by the long column formula, which is as follows:

$$p = \frac{S_c}{1 + \frac{L^2}{25{,}000\,R^2}}$$

in which p is the allowable unit compressive working stress in pounds per square inch, S_c the allowable unit compressive stress in a short column, L the unsupported length of the stem in inches, and R the least radius of gyration of the stem in inches.

21-36. Design of Hoists.—In the design of hoists it is first necessary to find the amount of the force necessary to move the gate, and this involves knowledge of the proper friction coefficients to apply to the various seat materials used in the gate construction and of the character of the workmanship that is to be applied in the construction. The United States Bureau of Reclamation has had wide experience in this subject and has recommended in its publication entitled "High Pressure Reservoir Outlets" the following coefficients of friction and working stresses.

Coefficient of sliding friction, bronze on cast iron or steel, line-bearing nuts	0.15
Coefficient of sliding friction, bronze on cast iron or steel, flat-bottom nuts and threads	0.20
Coefficient of sliding friction, steel on babbitt	0.125
Coefficient of rolling friction, ball bearings	0.002
Coefficient of sliding friction, gate seat, starting	0.7
Coefficient of sliding friction, gate seat, moving	0.35
Efficiency of spur gear and bearing, cut teeth	0.90
Efficiency of spur gear and bearing, cast teeth	0.85
Efficiency of bevel gear and bearing, cut teeth	0.85
Efficiency of bevel gear and bearing, cast teeth	0.80
Practicable hand pull for starting, . . . pounds	40
Practicable hand pull after starting, . . . pounds	20
Allowable tension or compression in cold-rolled steel gate stems, combined axial stress and torsion, true stress . . . pounds per square inch	12,000
Allowable shear in cold-rolled steel gate stems, combined axial stress and torsion, true stress, . . . pounds per square inch	10,000

Allowable bearing load on threads, . . . pounds per square inch	750
Allowable shear on bronze nuts, . . . pounds per square inch	1,500
Allowable tension in cold-rolled steel shafts combined flexure and torsion, true stress, . . . pounds per square inch	10,000
Allowable shear in cold-rolled steel shafts, combined flexure and torsion, true stress, . . . pounds per square inch	8,000
Direct tension in gate-stand pedestals, . . . pounds per square inch	1,000

(Other stresses as per Table XXIX.)

SLIDE GATES

21-37. Uses of Slide Gates.—Slide gates are used for controlling the flow over spillways where the quantity of water to be handled is relatively small and the range of fluctuation is about 10 ft. or less. They are also used for relatively small discharges through outlets under heads up to about 120 ft. These gates are usually made of cast iron or cast steel in one piece or of rolled-steel plates and sections. The frames of the cast gates are made of the same materials as the gate leaves, and the leaves are reinforced with horizontal and vertical ribs. The gates are operated with hoists.

21-38. Assumption Used in the Design of Slide Gates.—In design, the stresses in the leaf are determined by considering each section of it between the adjacent ribs as a plate supported on four sides, and those in the ribs by considering the horizontal ribs and strips of the leaf adjacent thereto as symmetrical T-beams supported at the ends on the gate seats, the ribs being taken as the stems and the leaf as the flanges. The width of the strip of leaf considered as a flange of the T-beam is taken as the distance to the nearest adjacent horizontal rib with a maximum limit of the depth of the rib at its middle. The entire water load is assumed to be carried by the horizontal T-beams.

21-39. Thickness of the Gate.—The thickness of the plate may be determined from the following formula:

$$t = \frac{c\,L^2b^2p}{(L^2 + b^2)\,f}, \qquad (296)$$

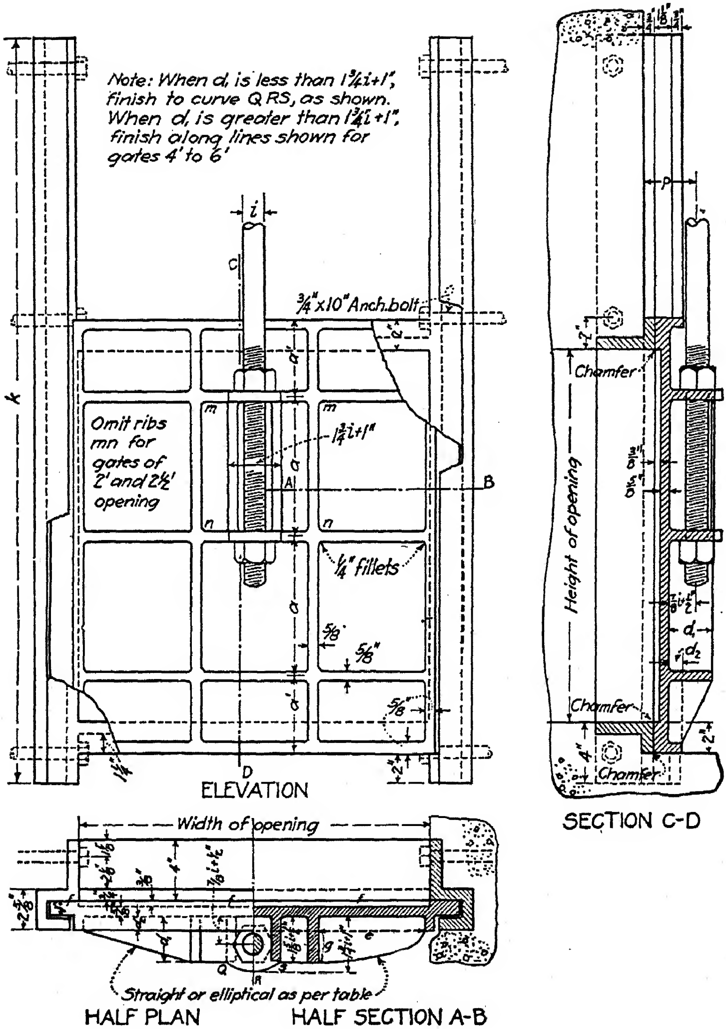
in which t equals the thickness of the section in inches, L the length of the rectangle in inches, b the width of the rectangle in inches, p the pressure in pounds per square inch, f the unit working stresses in pounds per square inch; c, 0.5 for steel, 0.6 for cast iron, and 0.55 for cast steel. The stress in the T-beam may be computed by the following formula:

$$f = \frac{Mc}{I}, \tag{297}$$

in which f equals the unit working stress in pounds per square inch, M the bending moment in inch-pounds, c the distance from the neutral axis to the most distant fiber on either face, and I the moment of inertia of the T-beam about its neutral axis.

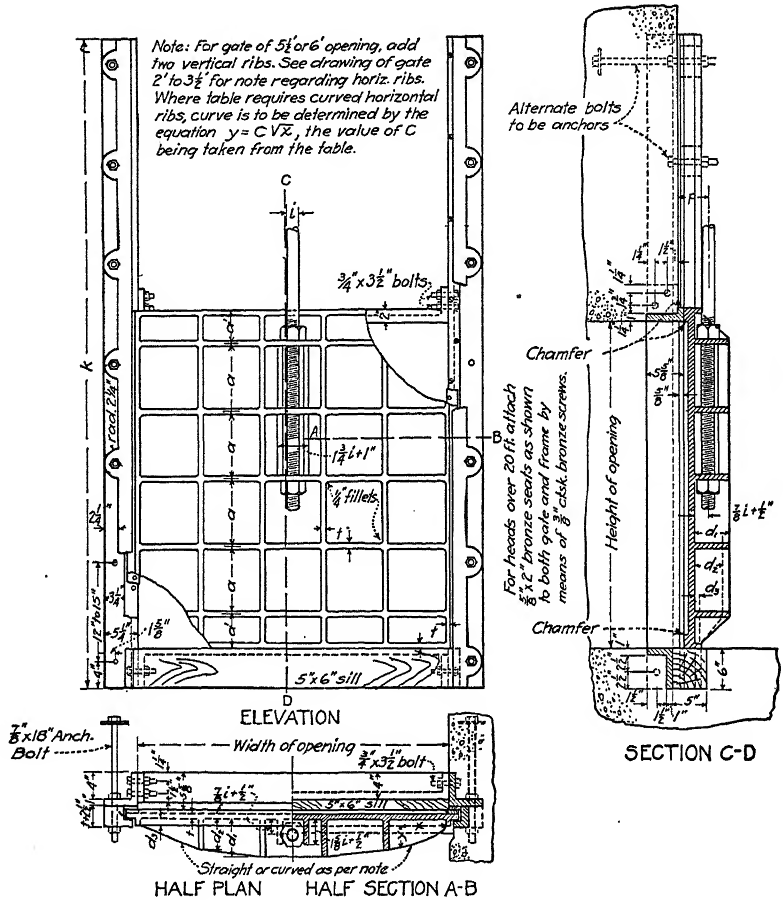
Regardless of the computed thickness required to withstand the computed stresses, the thickness of the ribs and the plate should not be less than about ⅝ in. and the ribs should not be thinner than the plate. Sudden changes in the thickness of the castings should be avoided and all corners should be rounded. Sufficient width should be given to the seats and the bearing area on the gate in order to properly withstand the pressure without scoring the seats. Considerable care must be given to securing proper limits of working pressure on the seats and gate frames.

21-40. Standard Design of Slide Gates.—Figure 69 shows a lap-closure type and Fig. 70 a direct-end closure type of cast-iron gate for low heads. Details of a satisfactory guide for use with these gates are shown in Fig. 71. Tables XXVI, XXVII, and XXVIII give the dimensions, weights, and friction loads for these two types of gates for widths varying from 2 to 6 ft. and heights varying from 2 to 9 ft., under heads varying from 10 to 50 ft. by 10-ft. intervals. The dimensions given in the table may be applied to intervening heads by taking the gate for the head nearest to that desired. For heads of water less than 10 ft., the design for a 10-ft. head should be used. Conditions may make it desirable to use either type of gate, in which case the dimensions opposite the required size are applicable



STANDARD CAST-IRON GATES
OPENING 2 FEET TO 3½ FEET

FIG. 69.—Slide gate, lap-closure type. (*U. S. Reclamation Bureau.*)



CAST-IRON SLIDE GATES
SIZE 4 FEET TO 6 FEET

FIG. 70.—Slide gate, direct-end closure type. (*U. S. Reclamation Bureau.*)

without recomputation, merely by changing the spacing of the horizontal rib to provide for the increase or decrease in height.

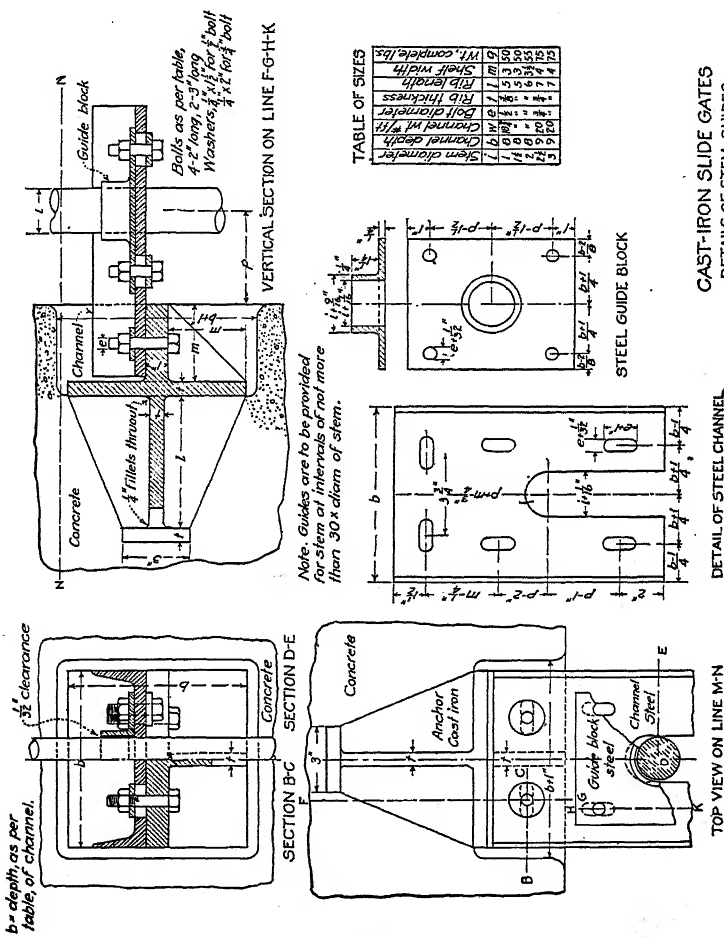


FIG. 71.—Gate stem-guide details. (*U. S. Reclamation Bureau.*)

Weights given in the tables are based on a weight of 450 lb. per cubic foot, and are exclusive of the gate stem and bronze seats. The height of the gate frames above the gate sills may be reduced to 1½ times the gate height where conditions require. The friction loads are based on a coefficient of moving friction of ⅓ for cast iron and 0.23 for bronze.

TABLE XXVI.—SQUARE GATES, TYPE A

(For widths of 3½ ft. or less)

Size of opening, ft.	Head on sill, ft.	Thickness skinplate and ribs, in.	Spacing horizontal ribs, in.	Distance first horizontal rib from edge, in.	Number of vertical ribs	Rib depth, in., Fig. 69	Bottom rib depth, in., Fig. 69	Semi-major axis of elipse, in.	Semi-minor axis of elipse, in.	Stem diameter, in.	Height of gate frame, ft.	Weight of gate frame, lb.	Weight of gate frame, lb.	Moving friction load, lb.
	H	t	a	a'	n_v	d_1	d_2	e	g	i	k	w	w'	F
2 by 2	10	⅝	9	5	4	2	⅞	......	...	1	4	200	230	1,000
2 by 2	20	⅝	9	5	4	2	⅞	......	...	1	4	200	230	2,200
2 by 2	30	⅝	9	5	4	2	⅞	......	...	1	4	200	230	3,300
2 by 2	40	⅝	9	5	4	3	⅞	......	...	1.5	4	220	230	4,400
2 by 2	50	⅝	9	5	4	3	⅞	......	...	1.5	4	220	230	5,600
2½ by 2½	10	⅝	9	3.5	4	2	⅞	......	...	1	5	290	290	1,500
2½ by 2½	20	⅝	9	3.5	4	2	⅞	......	...	1	5	290	290	3,100
2½ by 2½	30	⅝	9	3.5	4	3	⅞	......	...	1.5	5	310	290	4,800
2½ by 2½	40	⅝	9	3.5	4	3.5	⅞	......	...	1.5	5	320	290	6,500
2½ by 2½	50	⅝	9	3.5	4	4	⅞	9–⅜	3	1.5	5	340	290	8,100
3 by 3	10	⅝	10	5	4	2	⅞	.....	...	1	6	380	350	2,000
3 by 3	20	⅝	10	5	4	3	⅞	.....	...	1.5	6	410	350	4,300
3 by 3	30	⅝	10	5	4	3.5	⅞	11–⅜	2.5	1.5	6	440	350	6,600
3 by 3	40	⅝	10	5	4	4	⅞	11–⅜	3	2	6	460	350	8,900
3 by 3	50	⅝	10	5	4	5	⅞	11–⅜	4	2	6	490	350	11,200
3½ by 3½	10	⅝	9	5	4	2.5	⅞	.....	...	1.5	7	520	400	2,500
3½ by 3½	20	⅝	9	5	4	3.5	⅞	.....	...	1.5	7	550	400	5,600
3½ by 3½	30	⅝	9	5	4	4.5	⅞	.....	...	2	7	600	400	8,800
3½ by 3½	40	⅝	9	5	4	5	⅞	13–⅜	4	2	7	650	400	11,700
3½ by 3½	50	⅝	9	5	4	6	⅞	13–⅜	5	2.5	7	670	400	14,800

TABLE XXVII.—SQUARE GATES, TYPE *B*
(For widths of 4 ft. or greater)

Size of opening, ft.	Head on sill, ft.	Thickness skinplate and ribs, in.	Spacing horizontal ribs, in.	Distance first horizontal rib from edge, in.	Number of vertical ribs	Horizontal rib depth, in., Fig. 70	Horizontal rib depth, in., Fig. 70	Horizontal rib depth, in., Fig. 70	Horizontal rib depth, in., Fig. 70	Constant for curved rib end	Stem diameter, in.	Height of gate frame, ft.	Weight of gate, lb.	Weight of gate frame, lb.	Moving friction load, lb.
	H	t	a	a'	n_v	d_1	d_2	d_3	d_4	c	i	k	w	w'	F
4 by 4	10	3/4	10	5	6	3	2	1	..	....	1.5	8	820	970	3,000
4 by 4	20	3/4	10	5	6	3.5	3	1	..	....	2	8	860	970	4,700
4 by 4	30	3/4	10	5	6	4.5	3.5	1	..	....	2	8	900	970	7,300
4 by 4	40	3/4	10	5	6	5.5	4.5	1	..	....	2	8	970	970	9,900
4 by 4	50	3/4	10	5	6	6.5	5	1	..	....	2.5	8	1,020	970	12,500
4½ by 4½	10	7/8	9	5.5	6	3	2.5	1	..	....	1.5	9	1,180	1,090	3,700
4½ by 4½	20	7/8	9	5.5	6	4	3.5	1	..	....	2	9	1,250	1,090	5,800
4½ by 4½	30	7/8	9	5.5	6	5	4	1	..	....	2	9	1,330	1,090	9,000
4½ by 4½	40	7/8	9	5.5	6	6	4.5	1	..	1.11	2.5	9	1,410	1,090	12,300
4½ by 4½	50	7/8	9	5.5	6	7	5	1	..	1.27	2.5	9	1,500	1,090	15,500
5 by 5	10	7/8	9	4	6	3	2.5	1	..	....	1.5	9.5	1,430	1,170	4,300
5 by 5	20	7/8	9	4	6	4.5	3.5	1	..	....	2	9.5	1,550	1,170	7,000
5 by 5	30	7/8	9	4	6	6	4.5	1	..	....	2	9.5	1,700	1,170	10,900
5 by 5	40	7/8	9	4	6	7	5	1	..	1.10	2.5	9.5	1,820	1,170	14,800
5 by 5	50	7/8	9	4	6	8	6	1	..	1.49	2.5	9.5	1,940	1,170	18,800
5½ by 5½	10	1	10	4	8	3	2.5	2	1	....	1.5	10.5	1,930	1,290	5,000
5½ by 5½	20	1	10	4	8	4.5	4	3	1	....	2	10.5	2,150	1,290	8,200
5½ by 5½	30	1	10	4	8	6	5.5	4	1	....	2.5	10.5	2,410	1,290	13,000
5½ by 5½	40	1	10	4	8	7.5	6.5	4.5	1	1.20	2.5	10.5	2,630	1,290	17,700
5½ by 5½	50	1	10	4	8	9	7.5	5.5	1	1.55	3	10.5	2,870	1,290	22,500
6 by 6	10	1	9	5.5	8	3.5	3	2	1	....	2	11.5	2,330	1,410	5,800
6 by 6	20	1	9	5.5	8	5	4.5	3.5	1	....	2	11.5	2,620	1,410	9,600
6 by 6	30	1	9	5.5	8	7	6	4	1	0 99	2.5	11.5	2,910	1,410	15,200
6 by 6	40	1	9	5.5	8	8.5	7.5	5	1	1 31	3	11.5	3,260	1,410	20,800
6 by 6	50	1	9	5.5	8	10	9	6	1	1.64	3	11.5	3,550	1,410	26,400

TABLE XXVIII.—RECTANGULAR GATES

Type	Height of gate opening, ft.	Number of horizontal ribs	Spacing horizontal ribs, in.	Distance, first horizontal rib from edge, in.	Height of gate frame, ft.
	h	n_h	a	a'	k
A, for widths of 3½ ft. or less	2	3	9	5	4
	2.5	4	9	3.5	5
	3	4	10	5	6
	3.5	5	9	5	7
	4	6	9	3.5	8
	4.5	6	10	4	9
	5	7	9	5	9.5
	5.5	7	10	5	10.5
	6	8	9.5	4.75	11.5
B, for widths of 4 ft. or greater	3	4	10	4	6
	3.5	5	9	4	7
	4	5	10	5	8
	4.5	6	9	5.5	9
	5	7	9	4	9.5
	5.5	7	10	4	10.5
	6	8	9	5.5	11.5
	6.5	9	9	4	12.5
	7	9	9.5	5	13.5
	7.5	10	9	5.5	14
	8	11	9	4	15
	8.5	11	9.5	4.5	16
	9	11	10	5	17

MISCELLANEOUS TYPES OF GATES

21-41. Roller Gates.—In order to avoid the excessive lift due to friction in slide gates, gate seats are sometimes provided with rollers. There are four types of these roller gates in use: the common, the Stoney, the Sirnit, and the Broome. In the common type, the rollers are attached either to the gate frame or to the gate itself, but generally the latter. In the Stoney type, the rollers are not rigidly attached either to the gate frame or to the gate, but are enclosed in a frame that moves at a different rate than that

of the gate. In the Sirnit type, the gate is provided at the sides with wheels that work in the guides. As the gate comes to seat, the wheels roll into a depression in the guide, allowing the gate seat to come into contact with the frame seat. When the gates are raised the lifting force is increased sufficiently to pull the wheels out of the depressions which are relatively shallow and make a slight angle with the face of the seat. In the Broome type, the roller train travels in the same manner as the traction tread of the caterpillar tractor. The rollers bear on a track on the gate and a track on the frame. The frame seat is inclined toward the gate near the bottom, and the gate being thus inclined is forced to seat by gravity and vertical water pressure, and the rollers are relieved of the load. The roller type of gate is usually used for heads under about 70 ft., but they are applicable to wider spans than slide gates on account of their lesser frictional resistance to motion. They are readily used for higher heads by using short spans. The principles of design of the gates and frames are similiar to those used for slide gates, except that different means of securing water-tightness are used.

21-42. Butterfly Gates.—The butterfly gate is usually installed inside of a pipe, and consists of a large horizontal axis and of a circular disk exactly fitting into the pipe section, on a seat in the pipe section when the gate is closed. Since the axis is passed through the center of the disk, it is evident that such a gate will be balanced under water pressure when it is entirely closed or entirely open. The gate is unbalanced for intervening positions, however, and is consequently not useful for regulating the flow of water under high heads, but is often used for high heads in full-open or full-closed operation. This type makes an excellent gate for low heads. The stresses in the leaf are those due to water pressure but a large factor of safety must be used to cover impact and vibration. Figure 72 shows a photograph of a large butterfly gate.

21-43. Cylinder Gates.—Cylinder gates are sometimes used for the control of water. In these gates the controlled

opening is in a horizontal plane and is placed directly under the center of the base of the cylinder. The water pressure is theoretically carried by the barrel of the cylinder, but practically it is necessary to stiffen the barrel with ribs. For small gates, one or two stem connections are provided, but for larger gates three connections are usually required

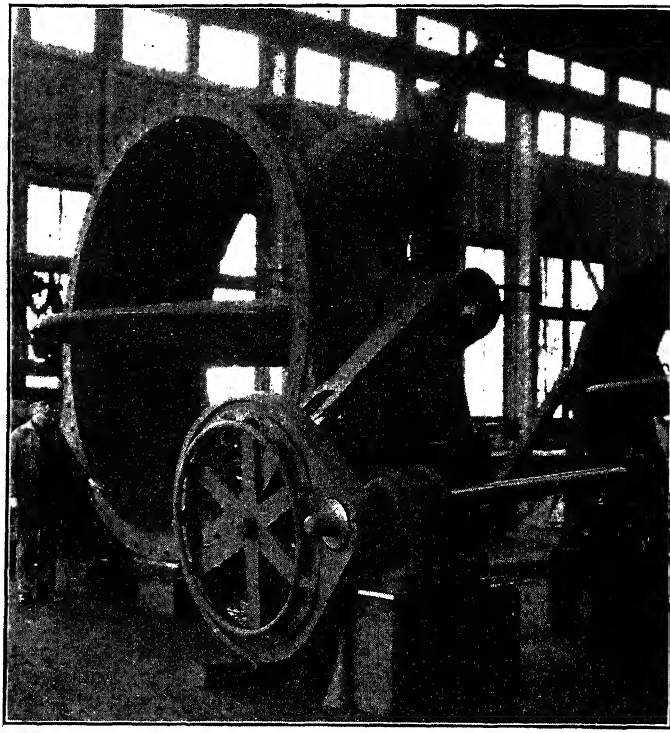


FIG. 72.—Large butterfly valve. (*Pelton Water Wheel Company.*)

to prevent binding on the guides. Three or more guides distributed around the perimeter of the cylinder are required to hold the gate in position, and sliding surfaces are provided on the outer surface of the cylinder to work in these guides. The stresses in cylinder gates are those produced by the external water pressure and can be computed by means of the ordinary thin-cylinder formula. A large factor of safety must be used, however, to care for

unbalanced pressure, impact, and vibration. Figure 73 shows a cylinder gate design.

21-44. Taintor Gates.—The Taintor gate consists of a circular sector, convex upstream for the water face, supported by means of struts leading to a pivot at the center of the sector. These gates are usually made of structural steel throughout, although in some instances the circular sector is composed of steel covered with a creosoted wooden

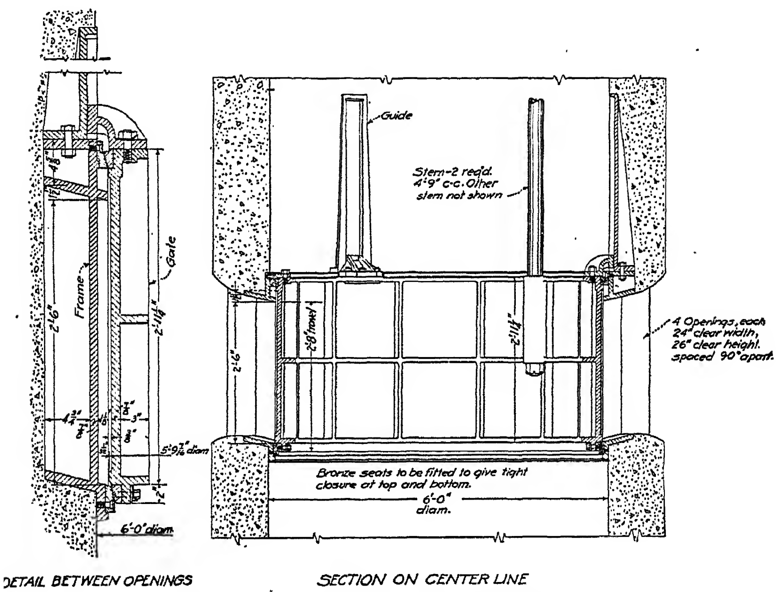


FIG. 73.—Typical cylinder gate. (*U. S. Reclamation Bureau.*)

facing. Counterweights are sometimes provided to reduce the required capacity of the lifting device. The gates close readily because of the balanced water pressure on their faces. Like roller gates, special means must be provided to secure water-tightness at the bottom and the ends of the gate; but they are not especially tight gates. The stresses in a Taintor gate are computed in a manner similar to that already outlined for cast-iron, cast-steel and structural-steel slide and roller gates. The skin plate in the circular sector is considered as supported on, or fixed to, the adjacent

horizontal beams, and the beams are considered to carry the entire water load without assistance from the skin

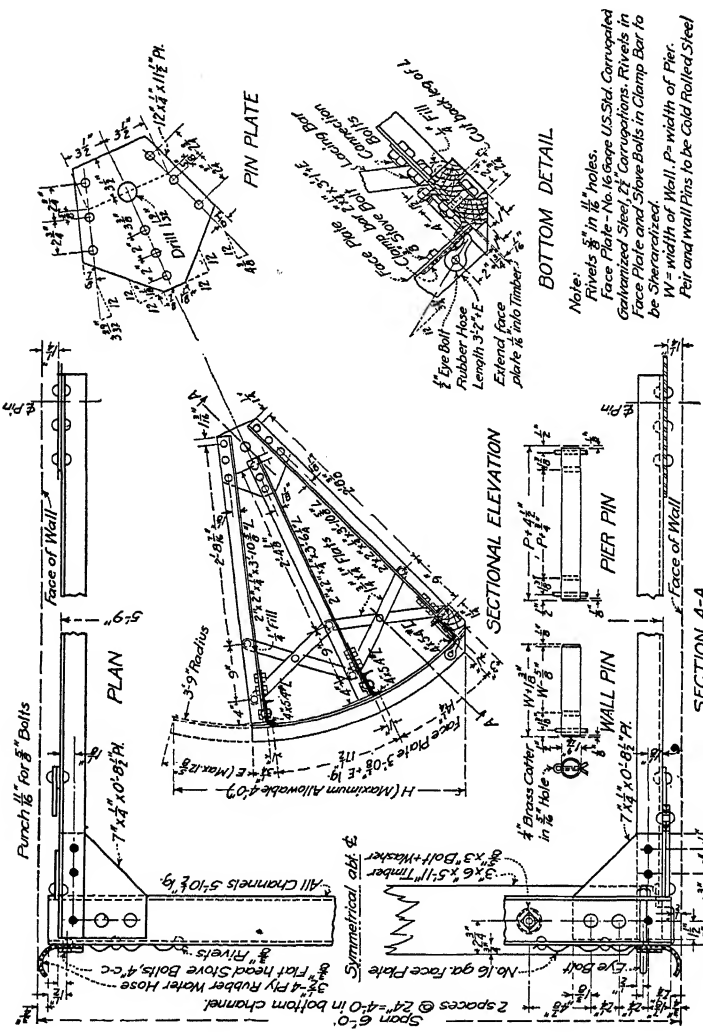


Fig. 74.—Taintor gate,—6-ft. span. (*U. S. Reclamation Bureau.*)

plate. Figure 74 shows the detailed design of a Taintor gate for a 6-ft. span and heads of 3 to 4 ft.; and Fig. 75 shows a similar type of gate for an 18-ft. span and heads

of 7 to 8 ft., both as used by the United States Bureau of Reclamation.

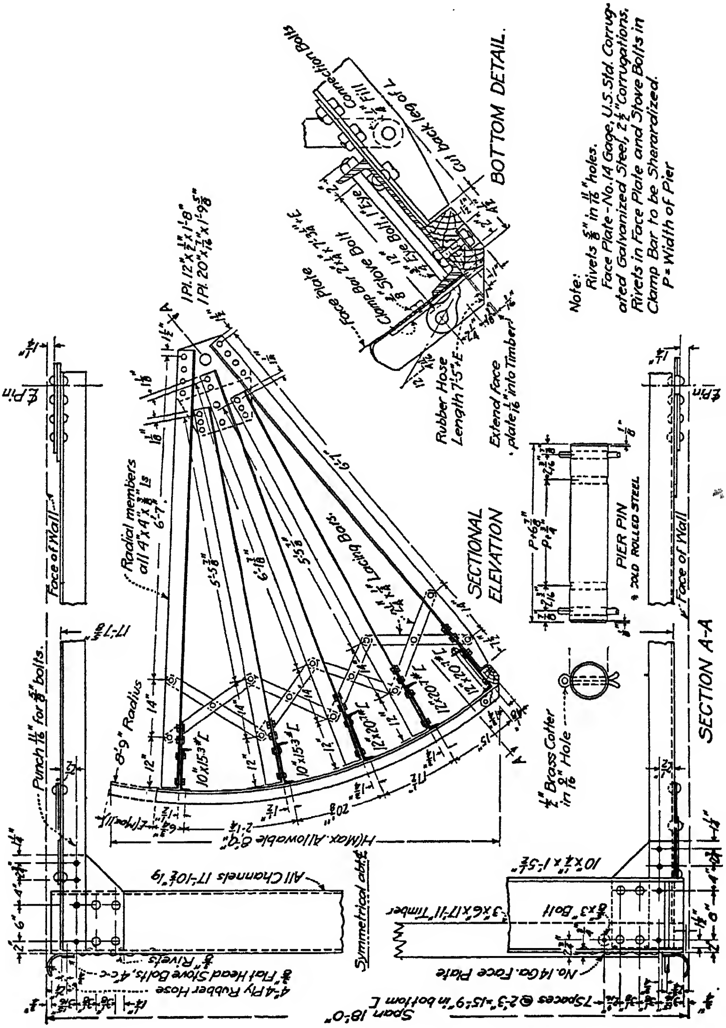


FIG. 75.—Taintor gate, 18-ft. span. (U. S. Reclamation Bureau.)

21-45. Working Stresses for Gates.—Table XXIX gives the working stresses in pounds per square inch for the materials used by the United States Bureau of Reclamation

for hydraulic slide gates. These working stresses may be used in the design of roller and Taintor gates also.

TABLE XXIX.—WORKING STRESSES FOR HYDRAULIC SLIDE GATES LB. PER SQUARE INCH

Material	Tension	Compression	Shearing	Bearing
Cast iron	2,000	12,000	2,000	
Cast steel, hard	15,000	15,000	12,000	
Cast steel, medium	12,500	12,500	10,000	
Cast steel, soft	10,000	10,000	8,000	
Structural steel	12,000	12,000	8,000	
Rolled Tobin bronze	16,000	15,000	10,000	
Rolled manganese bronze	16,000	15,000	10,000	
Cast manganese bronze	16,000	15,000	10,000	
Phosphor bronze	7,500	10,000	7,000	
Gun bronze	7,500	10,000	7,000	
Cast iron on cast iron	......	......	......	300
Cast iron on cast steel	......	......	......	400
Bronze on bronze	......	......	......	500

21-46. Flashboards and Stop Planks.—Flashboards consist of horizontal boards or planks of suitable lengths and thicknesses supported on pins set in sockets on the crest

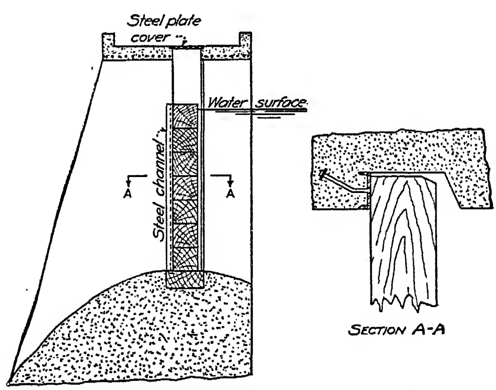


FIG. 76.—Simple stop-plank installation.

of a dam or in slots provided in concrete piers or steel bents attached at suitable intervals to the crest of the dam for the purpose of controlling the water level in the reser-

voir. This latter type of flashboards is sometimes called stop planks. The design of the piers and the steel bents are simple problems in reinforced-concrete and steel construction, the forces to provide against being the water pressure on the flashboards when in position and impact of water, floating ice, or débris when the flashboards are removed. In some cases the supports for the flashboards are designed to be tripped automatically and the flashboards released quickly in the case of sudden floods. In these cases, the flashboards are often carried away and the lost

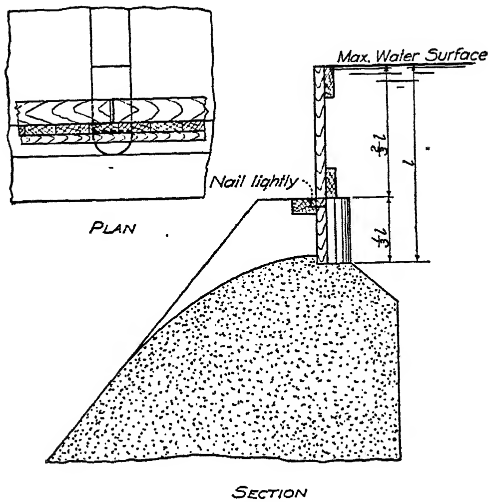


FIG. 77.—Automatic flashboard installation.

boards replaced by new ones. Figure 76 shows a typical stop-plank installation and Fig. 77 an automatic type of flashboard installation.

21-47. Automatic Gates.—Gates of the automatically closing and opening type were formerly used to a considerable extent on the crests of spillways and logways. These gates are generally so designed as to automatically open as the water rises above a certain elevation in the reservoir or canal and then to close again when the water falls to a certain level. There are many ingenious types and designs of these automatic gates, but they are not so widely used

at the present time as formerly. It is not desirable to go into an elaborate description of the many types that have been designed, as their design is subject to variation due to differing local conditions. Probably the most interest-

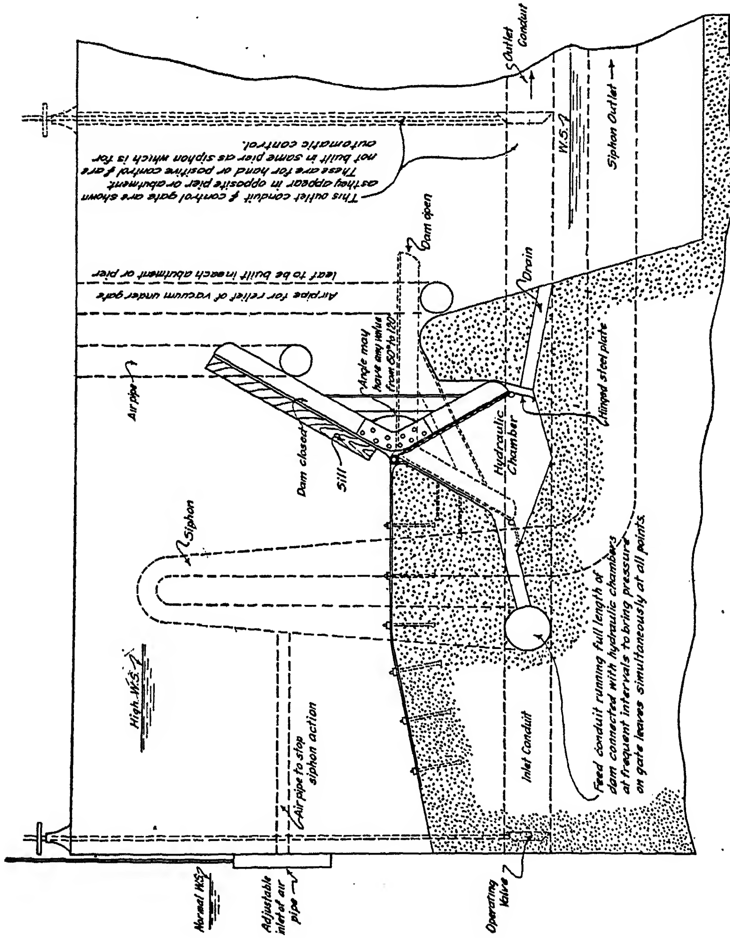


Fig. 78.—Automatic crest gate (*U. S. Reclamation Bureau.*)

ing is one of the United States Reclamation *Bureau* types of gate as shown in Fig. 78. When the gate is to be operated automatically by means of the siphon, all conduits leading to the hydraulic chamber should be closed except such as connect with the siphon. In this case the inlet

conduit leading to the siphon and hydraulic chamber should be adjusted to admit only such flow as necessary to maintain the crest at maximum height to insure effective action of the siphon. The crest may also be raised or lowered by hand operation to any reservoir height.

21-48. Bear-trap Gates.—There have been numerous designs of bear-trap gates, mostly used for logway control. Probably the most interesting and useful of this type is that used on the Chicago Drainage Canal. This gate is of a different design from many of the old bear-trap .types, and consists of two leaves hinged together at the apex, as

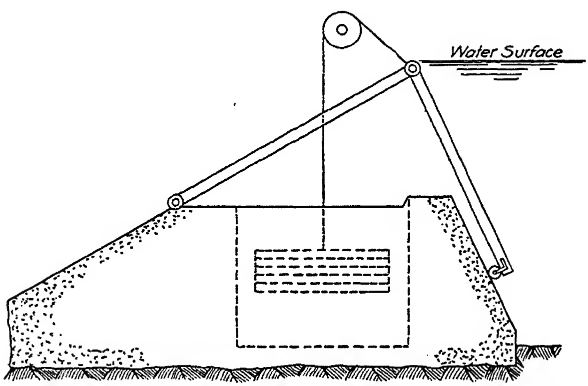


FIG. 79.—Bear-trap gate used on Chicago Drainage Canal.

shown in Fig. 79, the downstream end of the downstream leaf being hinged to the crest of the dam, and the upstream end of the upstream leaf being carried on a roller that moves up and down on the upstream face of the dam. This gate is counterweighted and its movement up and down is controlled by hydraulic cylinders.

LOGWAYS AND FISH LADDERS

21-49. Logways.—On streams where logging operations exist or are likely to come into existence, it is desirable, if not necessary, to provide a logway at the dam at the time of its construction. The logway is constructed on an incline of about 3 or 4 to 1, so that the logs will pass down it without damage to them or the stream bed and bank when they

reach the bottom. Logways are usually made from about 6 to 30 ft. in width, with their crests from 4 to 6 ft. below the crest of the dam and their lower ends from 4 to 6 ft. below water level in the stream bed. The discharge over the crest of the logway is controlled with either gates or stop planks. Figure 80 shows a typical logway.

21-50. Fish Ladders.—Many of the states of the United States require fish ladders to be provided at dams on some of the streams within their boundaries. A fish ladder usually consists of an inclined reinforced-concrete water passage on a slope of about 3 or 4:1 divided into pools about

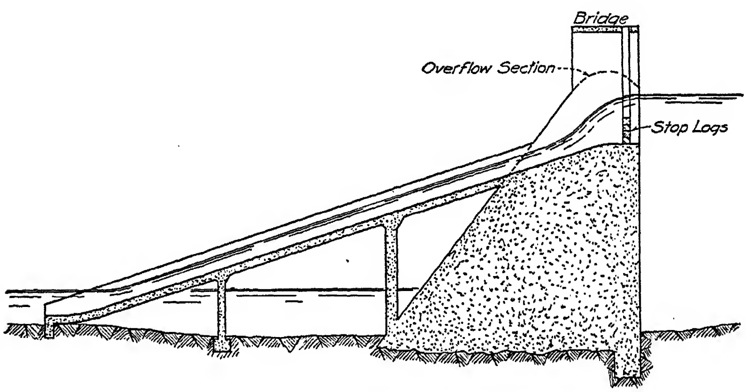


FIG. 80.—Typical logway.

4 ft. wide and 6 ft. long, arranged so that the water level in each successive pool from the bottom is about 1 to 2 ft. higher than the preceding one. There are three different types in use, namely: (*a*) the triangular-weir type in which the water flows from one pool to the other through a triangular notched weir in the partition wall; (*b*) the square-weir type in which the water flows from one pool to the next through a rectangular notch; and (*c*) the submerged-orifice type in which the water flows from one pool to the next through an orifice. The weir and orifice openings are staggered in the adjacent partition walls. The submerged-orifice type is recommended by the United States Commission of Fish and Fisheries. Where a state law provides for the

construction of fish ladders over dams, the state generally furnishes designs for their construction. Figure 81 shows a

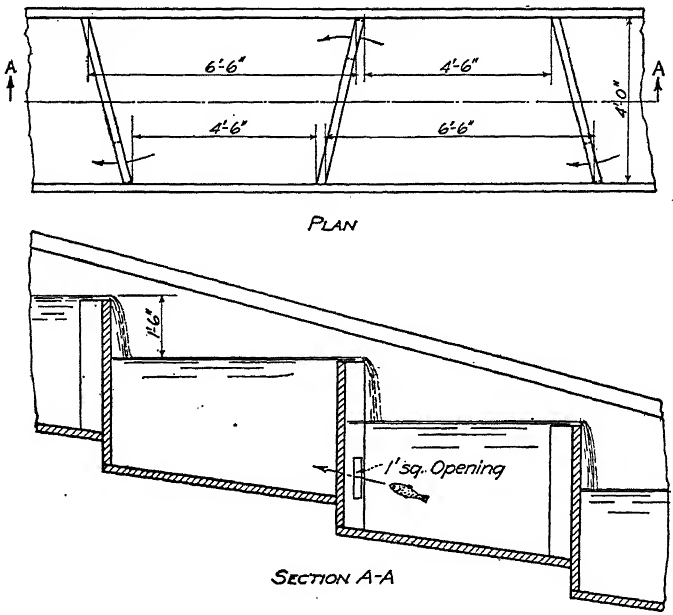


FIG. 81.—Fish ladder. (*U. S. Commission of Fish and Fisheries.*)

fish ladder of the type recommended by the United States Commission of Fish and Fisheries.

CHAPTER XXII

HYDRAULICS

INTRODUCTION

22-1. General Statement.—In connection with the design of dams, many problems of theoretical hydraulics often arise. In order that this treatise may be complete, the more usual of these problems will be discussed in this chapter.

MEASUREMENT OF WATER WITH WEIRS

22-2. Types of Weirs.—A weir may be defined as a notch in the top of a vertical wall through which water flows, and the determination of the quantity of water that will flow through such a notch under specific conditions depends upon experimental data taken in connection with the cross-sectional area of the discharge sheet through the notch. The weirs generally employed for the measurement of water are either of a rectangular shape or a trapezoidal shape. A weir may be either free or submerged. The free weir is one in which the water elevation on the downstream side thereof does not reach up to the elevation of the crest, and a submerged weir is one in which the water elevation on the downstream side rises above the elevation of the crest. A standard rectangular weir may also be suppressed or contracted. The standard suppressed rectangular weir is one with its sides the channel of the stream above it, so that the water filaments pass through the weir without being deflected from their normal course. The standard contracted rectangular weir is one with its sides so far removed from the sides of the channel above it that the filaments of water passing through the weir are fully contracted. A standard Cippoletti weir is a trapezoidal

weir with its crest and sides consisting of a thin plate and with its sides sloping outward as they rise in a ratio of 1:4.

22-3. Requirements for Accurate Measurements.—In order to insure accurate measurements over a weir, the upstream crest and side edges of the weir should be sharp and smooth; the distance of the crest and sides, respectively, from the bottom and sides of the leading channel should not be less than twice the depth of water on the weir and in no case less than 1 ft.; the overflowing sheet should touch only the upstream edge of the crest and sides; air should circulate freely both underneath and on the sides of the overflowing sheet; the upstream face of the weir should be vertical; the crest of the weir should be level from end to end; the sides of the weir should be truly vertical for rectangular weirs and true to the proper slopes for Cippoletti weirs; the measurement of head on the weir should be taken from 4 to 16 ft. upstream from the weir; the cross-sectional area of the leading channel for 20 to 30 ft. upstream from the weir should be at least six times that of the overflowing sheet at the weir crest, and correction should be made for velocity of approach unless this element is negligible.

22-4. Symbols Used.—In the discussion of weirs, the following symbols will be used:

Q = discharge in second-feet, without velocity of approach considered.
Q' = discharge in second-feet with velocity of approach considered.
L = length of weir crest in feet.
H = head on weir in feet, exclusive of the velocity of approach.
h = equivalent head in feet on weir due to velocity of approach.
D = head in feet on weir inclusive of head due to velocity of approach, or $D = H + h$.
P = height in feet of the weir crest above the bed of the leading channel.
g = acceleration of gravity in feet per second per second.
v = velocity of approach in feet per second.
A = area of leading channel in square feet.

22-5. Francis Formula.—The Francis and the Bazin formulas are most widely used for computing discharges over standard rectangular weirs. The Francis formula for

the standard *contracted* rectangular type of weir without velocity of approach is,

$$Q = 3.33\, H^{\frac{3}{2}}\,(L - 0.2\,h). \tag{298}$$

With velocity of approach the formula is,

$$Q' = 3.33\left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right](L - 0.2\,H). \tag{299}$$

For the standard *suppressed* rectangular type of weir without velocity of approach, the Francis formula is,

$$Q = 3.33\, LH^{\frac{3}{2}}. \tag{300}$$

With velocity of approach the formula becomes,

$$Q' = 3.33\, L\left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right]. \tag{301}$$

22-6. Bazin Formula.—The Bazin formula was developed by him for standard suppressed rectangular weirs, but it takes into consideration the effect of the height of the weir crest above the bed of the leading channel which the Francis formula does not do. On account of this inclusion, the Bazin formula is more applicable to the discharge over crests of dams than is the Francis formula.

The Bazin formula for the standard suppressed rectangular weir with velocity of approach is,

$$Q' = \left(0.405 + \frac{0.00984}{D}\right)\left(1 + 0.55\,\frac{D^2}{(P + D)^2}\right)LD\sqrt{2\,gD}. \tag{302}$$

22-7. Cippoletti Formula.—The Cippoletti formula for his type of weir without velocity of approach should theoretically be the same as the Francis formula for the standard suppressed rectangular weir, but Cippoletti increased the Francis coefficient by about 1 per cent so that his formula without velocity of approach is,

$$Q = 3.367\, LH^{\frac{3}{2}}. \tag{303}$$

With velocity of approach the formula becomes,

$$Q' = 3.367\, L(H + 1.5\,h)^{\frac{3}{2}}. \tag{304}$$

22-8. Velocity of Approach.—The velocity of approach may be computed by the formula,

$$v = Q' \div A. \tag{305}$$

The discharge may be computed by the appropriate formula without velocity of approach with sufficient accuracy for determining v for ordinary purposes, but successive approximations may be used for special cases requiring extreme accuracy. Having obtained the value of v, the velocity of approach head may be computed from the following formula:

$$h = v^2 \div 2g = 0.0155\, v^2. \tag{306}$$

22-9. Flow over Submerged Weirs.—The measurement of water flowing over submerged weirs needs special consideration. Herschel, from a discussion of the older experiments on submerged weirs, which were mainly of the suppressed rectangular type, derived the following formula for computing the discharge:

$$Q_1 = 3.33\, L(nH)^{\frac{3}{2}}, \tag{307}$$

in which L and H have the designations assigned them for free suppressed rectangular weirs; Q_1, the discharge in second feet; and n, a factor of correction depending on the amount of submergence. Other experiments were later made by Stevens on both contracted and suppressed submerged weirs. The results obtained by him differ so slightly from those obtained by Herschel that it may be considered that the Herschel formula applies with but slight error to contracted as well as suppressed weirs. A comparison of the Francis formula for free weirs with the Herschel formula for submerged weirs shows that the discharge over the latter can be obtained from those for the former by using a multiplier depending upon the amount of submergence. In Table XXX are given values of the coefficient desirable to apply which is based on the ratio of the head of water on the downstream side divided by the head of water on the upstream side.

TABLE XXX.—HERSCHEL'S MULTIPLIERS FOR SUBMERGED WEIRS
(d equals downstream head on weir; D equals upstream head on weir)

d/D	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.000	1.004	1.006	1.006	1.007	1.007	1.007	1.006	1.006	1.005
0.1	1.005	1.003	1.002	1.000	0.998	0.996	0.994	0.992	0.989	0.987
0.2	0.985	0.982	0.980	0.977	0.975	0.972	0.970	0.967	0.964	0.961
0.3	0.959	0.956	0.953	0.950	0.947	0.944	0.941	0.938	0.935	0.932
0.4	0.929	0.926	0.922	0.919	0.915	0.912	0.908	0.904	0.900	0.896
0.5	0.892	0.888	0.884	0.880	0.875	0.871	0.866	0.861	0.856	0.851
0.6	0.846	0.841	0.836	0.830	0.824	0.818	0.813	0.806	0.800	0.794
0.7	0.787	0.780	0.773	0.766	0.758	0.750	0.742	0.732	0.723	0.714
0.8	0.703	0.692	0.681	0.669	0.656	0.644	0.631	0.618	0.604	0.590
0.9	0.574	0.557	0.539	0.520	0.498	0.471	0.441	0.402	0.352	0.275

22-10. Flow over Spillways.—In the weir formulas thus far discussed, only standard forms have been considered, but the engineer designing dams has more need for determining flows over the forms of weirs found in spillways of dams. The overflow for such weirs is often and best determined by first computing the discharge either with the Francis or Bazin formula and then applying multipliers

TABLE XXXI.—MULTIPLIERS FOR BROAD-CRESTED WEIRS OF RECTANGULAR CROSS-SECTION

(P = height of weir; c = width of crest; H = observed head; all in feet)
Type *a*

P c	4.6 2.6	4.6 6.6	11.25 0.48	11.25 0.93	11.25 1.65	11.25 3.17	11.25 5.88	11.25 8.98	11.25 12.24	11.25 16.30
H										
0.5	.....	.....	0.821	0.792	0.806	0.792	0.799	0.801	0.786	0.790
1.0	0.765	0.708	0.997	0.899	0.808	0.795	0.791	0.794	0.815	0.790
1.5	0.789	0.709	1.00	0.982	0.878	0.796	0.796	0.793	0.814	0.792
2.0	0.814	0.710	1.00	1.00	0.906	0.815	0.797	0.792	0.797	0.793
2.5	0.835	0.711	1.00	1.00	0.985	0.844	0.797	0.790	0.796	0.793
3.0	0.857	0.711	1.00	1.00	1.00	0.870	0.797	0.788	0.794	0.791
3.5	0.878	0.712	1.00	1.00	1.00	0.90	0.812	0.787	0.794	0.791
4.0	0.899	0.714	1.00	1.00	1.00	0.93	0.834	0.786	0.792	0.789
5.0	0.940	0.716	1.00	1.00	1.00	0.97	*	0.78	0.79	0.78
6.0	0.986	0.718	1.00	1.00	1.00	0.98	*	0.78	0.78	0.78

* Value doubtful.

to the discharges thus found. Figure 82 gives a few of the types of irregular-crested weirs met with in the design of dams, and there are given, in the following Tables XXXI, XXXII, and XXXIII, the multipliers that must be applied

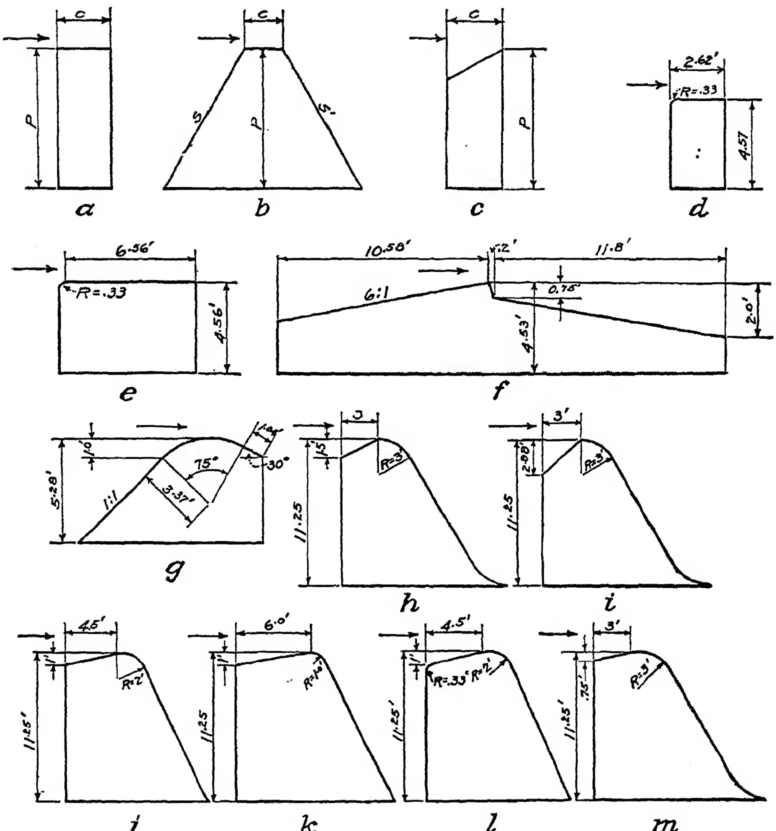


FIG. 82.—Types of weir crests.

to the different types. The effect of rounding the upstream crest of a weir is to increase its discharge by about 4 per cent. If the shape of the weir is such as to permit the formation of a partial vacuum adjacent to the crest, experiments indicate an increase in discharge of about 5 per cent at heads of 4 ft. or more.

TABLE XXXII.—MULTIPLIERS FOR WEIRS OF TRAPEZOIDAL CROSS-SECTION

(P = height of weir, in feet; c = width of crest, in feet; s = upstream slope; s' = downstream slope; H = observed head, in feet)

	Type b							Type c	
P	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.65	11.25
c	0.33	0.66	0.66	0.66	0.66	0.33	0.66	7.00	6.00
s	2:1	2:1	3:1	4:1	5:1	2:1	2:1	4.67:1	6:1
s'	0	0	0	0	0	5:1	2:1	......	......
H									
1.0	1.137	1.048	1.066	1.039	1.009	1.095	1.071	1.042	1.060
1.5	1.131	1.068	1.066	1.039	1.009	1.071	1.066	1.033	1.069
2.0	1.120	1.080	1.061	1.033	1.005	1.044	1.053	1.024	1.054
2.5	1.106	1.085	1.052	1.026	0.997	1.024	1.047	1.012	1.012
3.0	1.094	1.088	1.047	1.020	0.991	1.009	1.047	0.995	0.985
3.5	1.085	1.087	1.043	1.017	0.988	1.003	1.050	0.983	0.979
4.0	1.072	1.084	1.038	1.012	0.984	1.014	1.052	0.977	0.976
4.5	1.064	1.081	1.035	1.009	0.980	1.023	1.055	0.974	0.973
5.0	......	......	......	......	......	......	......	0.97	0.97
6.0	......	......	......	......	......	......	......	0.97	0.96

TABLE XXXIII.—MULTIPLIERS FOR COMPOUND WEIRS

(P = height of weir, in feet; H = observed head, in feet)

P	4.57	4.56	4.53	5.28	11.25	11.25	11.25	11.25	11.25	11.25
Type:	d	e	f	g	h	i	j	k	l	m
H										
0.5	.....	.....	.....	.....	0.941	0.924	0.933	0.962	0.971	0.947
1.0	0.842	0.836	0.929	0.976	1.039	1.033	0.988	1.045	1.033	1.000
1.5	0.866	0.834	0.950	0.979	1.087	1.093	1.018	1.066	1.042	1.036
2.0	0.888	0.831	0.953	0.988	1.109	1.133	1.033	1.063	1.035	1.063
2.5	0.906	0.826	0.947	1.000	1.118	1.153	1.045	1.020	1.033	1.085
3.0	0.927	0.822	0.942	1.016	1.120	1.163	1.054	0.997	1.045	1.096
3.5	0.945	0.817	0.936	1.032	1.127	1.169	1.060	0.994	1.054	1.108
4.0	0.965	0.812	0.931	1.044	1.123	1.165	1.060	0.991	0.057	1.110
5.0	1.00	0.80	0.92	1.05	1.11	1.16	1.05	0.98	1.05	1.10
6.0	.....	.....	.....	.....	1.11	1.15	1.04	0.98	1.04	1.10

MEASUREMENT OF WATER WITH ORIFICES

22-11. Types of Orifices.—An orifice may be defined as an opening placed in a wall of a channel, vessel, or reservoir carrying or holding water and lying completely below the surface of the water on the upstream side thereof. If the water on the downstream side of the wall discharges into air, the orifice is said to be free, and if it discharges under water it is said to be submerged. The wall may have any angular position from horizontal to vertical; the opening may have any geometrical shape; and the issuing stream may be suppressed or contracted. A contracted orifice is one with its perimeter so far removed from the bounding surfaces of the water prism in the channel, vessel or reservoir, or other surfaces of a disturbing nature, that the filaments of water are fully contracted as they pass through the orifice. A suppressed orifice is one with its perimeter coincident with the sides of the channel of approach or with other surfaces eliminating contraction. An orifice may be contracted or suppressed on any part or all of its perimeter, or it may be imperfectly contracted or suppressed on any part or all of its perimeter. If the opening is not sharp edged or if the wall in which it is cut or formed has material thickness, or if a discharge tube is attached, then the opening becomes a submerged tube. This condition may exist all around the opening or partially so, or it may be caused by placing too close to the opening the bounding surfaces of the water prism in the channel of approach. For these different conditions different coefficients of discharge apply.

22-12. Standard Orifice.—The standard orifice is an orifice with its sides consisting of a thin-edged plate so far removed from the adjacent sides, bottom, or top of the water prism in the leading channel, as to cause the filaments of water to be fully deflected from their normal courses as they pass through the orifice. The conditions that are considered necessary to secure perfect contraction and accuracy of measurement in any orifice are as follows:

a. The upstream edge of the orifice should be sharp and smooth and the distance from the bounding surfaces of the channel both on the upstream and on the downstream side should preferably be not less than twice the least dimension of the orifice.

b. The upstream face of the orifice wall should be vertical.

c. The top and bottom edges of a rectangular orifice should be level from end to end.

d. The sides of a rectangular orifice should be truly vertical.

e. The head on the orifice should be the actual measured difference in elevation between the water surface on the upstream side of the orifice and the center of the opening for a free orifice, and the water surface on the downstream side thereof for a submerged orifice.

f. The cross-sectional area of the water prism for twenty to thirty times the largest dimension of the orifice on the upstream side thereof, for either a free or a submerged orifice, and also on the downstream side thereof for a submerged orifice, should be at least six times the cross-sectional area of the orifice.

g. Corrections for velocity of approach should be made where an appreciable error is caused by neglecting the head due to it.

22-13. Discharge through a Standard Orifice.—The formula for computing the discharge through a free orifice with the head greater than twice the height, or any submerged orifice is

$$Q = cA\sqrt{2gH}. \tag{308}$$

For ordinary cases, Eq. (308) will be sufficiently accurate for general use; but for a free orifice under a head less than twice its height, it is necessary to take into consideration the effect of the difference in the head on the top and the bottom of the orifice, which changes the formula considerably. For circular orifices with free discharge, the following formula may be used:

$$Q = cA\left(1 - \frac{1}{128}\frac{d^2}{H^2} - \frac{5}{16{,}384}\frac{d^4}{H^4}\right)\sqrt{2gH}. \tag{309}$$

For horizontal rectangular openings with free discharge the formula becomes

$$Q = cA\left(1 - \frac{1}{96}\frac{d^2}{H^2} - \frac{1}{2{,}048}\frac{d^4}{H^4}\right)\sqrt{2gH}. \qquad (310)$$

In all of these formulas, c is a coefficient depending on the conditions at the orifice; Q, the discharge in second-feet through the orifice; H, the measured head on the orifice in feet; g, the acceleration of gravity in feet per second per second; A, the area of the orifice in square feet; and d, the height of the orifice. In general, c may be taken to have the following values:

$c = 0.61$ for the standard free or the standard submerged orifice.
$c = 0.61\ (1 + 0.15r)$ for the standard free or the standard submerged orifice where r is taken as the ratio of the suppressed portion of the perimeter of the orifice to the whole perimeter.
$c = 0.65$ for sluiceways without side walls discharging through a dam into a large basin or outlet conduit.
$c = 0.66$ for large siphon spillways.
$c = 0.85$ for moderate-sized sluiceway openings at the bottom of the reservoir.
$c = 0.95$ for large-sized sluiceway openings with side walls.

22-14. Effect of Velocity of Approach.—The velocity of approach may be computed from Eq. (305), after which the head due to the velocity of approach may be computed from Eq. (306). This velocity head, designated as h, should then be added to the measured head on the orifice before computing the discharge by any of the Eqs. (308), (309), or (310). Attention is directed to the fact that neglecting moderate velocities of approach with low heads on the orifice produces relatively large errors, whereas neglecting comparatively high velocities of approach with large heads on the orifice produces relatively small errors.

MEASUREMENT OF WATER IN OPEN CHANNELS

22-15. Kutter-Chezy Formulas.—The flow of water in open channels is usually measured either by means of a slope formula or a current meter-gaging station. The most

TABLE XXXIV.—VALUES OF c FOR USE IN THE CHEZY FORMULA $v = c\sqrt{rs}$

r \ n	0.009	0.010	0.011	0.012	0.013	0.014	0.015	0.017	0.020	0.0225	0.025	0.030	0.035	0.040
Slope s = 0.00005 = 1 in 20,000 = 0.264 ft. per mile														
0.1	78	67	59	52	47	43	39	33	26	22	20	16	13	11
0.2	100	87	77	68	62	56	51	44	35	30	26	21	18	15
0.3	114	99	88	79	71	65	59	50	41	36	31	25	21	18
0.4	124	109	97	88	79	72	66	57	46	40	35	28	24	20
0.6	139	122	109	98	90	82	76	65	53	46	41	33	28	24
0.8	150	133	119	107	98	90	83	71	59	52	46	37	31	27
1.0	158	140	126	114	104	96	89	77	64	56	49	40	34	29
1.5	173	154	139	126	116	107	99	87	72	64	57	47	40	34
2	184	164	148	135	124	115	107	94	79	70	62	51	44	38
3	198	178	161	148	136	127	118	104	88	79	71	59	50	44
3.28*	201	181	164	151	139	129	121	106	91	81	72	60	52	46
4	207	187	170	156	145	135	126	111	95	85	77	64	56	49
6	220	199	182	168	156	146	137	122	105	94	85	72	63	56
10	234	212	195	181	169	158	149	134	116	105	96	82	72	64
20	250	228	211	196	184	174	165	149	131	120	110	96	85	77
50	266	245	228	213	201	190	181	165	148	136	127	112	101	93
100	275	254	237	222	210	200	190	175	158	146	137	123	112	104
Slope s = 0.0001 = 1 in 10,000 = 0.528 ft. per mile														
0.1	90	78	68	60	54	49	44	37	30	25	22	17	14	12
0.2	112	98	86	76	69	63	57	48	39	33	29	23	19	16
0.3	125	109	97	87	78	72	65	56	45	39	34	27	22	19
0.4	136	119	106	95	86	79	72	62	50	43	38	31	25	22
0.6	149	131	118	105	96	88	81	70	57	50	44	35	30	25
0.8	158	140	126	114	103	95	88	76	63	55	48	39	33	28
1.0	166	147	132	120	109	101	93	81	67	59	52	42	35	31
1.5	178	159	144	130	120	111	103	89	75	66	59	48	41	35
2	187	168	151	138	127	118	109	96	81	71	64	53	45	39
3	198	178	162	149	137	127	119	104	89	79	71	59	51	45
4	206	186	169	155	143	134	125	111	94	84	76	64	55	49
6	215	195	178	164	152	142	134	119	102	92	84	71	61	54
10	226	205	188	174	162	152	143	128	111	100	92	78	69	62
20	237	216	200	185	173	163	154	139	122	111	102	89	79	71
50	249	227	211	197	185	175	166	151	134	123	114	100	91	83
100	255	234	218	204	191	181	172	158	140	130	121	108	98	91
Slope s = 0.0002 = 1 in 5,000 = 1.056 ft. per mile														
0.1	99	85	74	65	59	53	48	41	32	27	24	18	15	12
0.2	121	105	93	83	74	67	61	52	42	36	31	25	21	17

0.3	133	116	103	92	83	76	69	59	48	42	36	29	24	20
0.4	143	125	112	100	91	83	76	65	53	46	40	32	27	23
0.6	155	138	122	111	100	92	85	73	60	52	46	37	31	26
0.8	164	145	131	118	107	99	91	79	65	57	50	41	34	29
1.0	170	151	136	123	113	104	96	83	69	60	54	44	37	32
1.5	181	162	146	133	122	113	105	91	77	67	60	49	42	36
2	188	170	154	140	129	119	111	97	82	72	64	54	45	40
3	200	179	163	149	137	128	119	105	89	79	72	59	51	45
4	205	185	168	155	143	133	125	111	94	84	76	63	55	48
6	213	193	176	162	150	140	132	117	100	90	82	69	60	53
10	222	201	185	170	158	148	140	125	108	98	89	76	67	60
20	231	210	194	180	168	158	149	134	117	106	98	85	76	68
50	240	220	203	189	177	167	158	143	126	116	108	94	85	78
100	245	224	208	194	182	172	163	148	131	121	113	99	90	83
Slope $s = 0.0004 = 1$ in 2,500 = 2.112 ft. per mile														
0.1	104	89	78	69	62	56	50	43	34	29	25	19	16	13
0.2	126	110	97	87	78	71	65	54	44	37	32	25	21	18
0.3	138	120	107	96	87	79	73	62	50	43	37	30	24	21
0.4	148	129	115	104	94	86	79	68	55	47	42	33	27	23
0.6	157	140	126	113	103	95	87	75	62	54	47	38	31	27
0.8	166	148	133	121	110	101	93	81	67	58	51	42	35	30
1.0	172	154	138	125	115	106	98	85	70	62	55	45	37	32
1.5	183	164	148	135	124	114	106	93	78	68	61	50	42	37
2	190	170	154	141	130	120	112	98	83	73	65	54	45	40
3	199	179	162	149	138	128	119	105	89	79	71	59	51	45
4	204	184	168	154	142	133	124	110	94	84	76	63	55	48
6	211	191	175	161	149	139	130	116	99	89	81	69	60	53
10	219	199	183	168	157	146	138	123	107	96	88	75	66	59
20	227	207	190	176	164	154	146	131	115	104	96	83	73	66
50	235	215	198	184	173	162	154	139	123	112	104	91	82	75
100	239	219	203	189	177	167	158	143	127	116	108	96	87	80
Slope $s = 0.001 = 1$ in 1,000 = 5.28 ft. per mile														
0.1	110	94	83	73	65	59	54	45	36	30	27	21	17	14
0.2	129	113	99	89	81	73	66	57	45	39	34	27	22	18
0.3	141	124	109	98	89	81	74	63	51	44	39	30	25	21
0.4	150	131	117	105	96	88	80	69	56	48	43	34	28	24
0.6	161	142	127	115	104	96	88	76	63	55	48	39	32	27
0.8	169	150	134	122	111	102	94	82	68	59	52	42	35	30
1.0	175	155	139	127	116	107	99	86	71	62	56	45	38	33
1.5	184	165	149	136	124	115	108	93	78	69	62	50	43	37
2	191	171	155	142	130	121	112	98	83	73	66	54	46	40
3	199	179	163	149	138	128	119	105	89	79	71	59	51	45

* Values of c are the same for all slopes when r = 3.28 ft.

TABLE XXXIV.—VALUES OF c FOR USE IN THE CHEZY FORMULA $v = c\sqrt{rs}$.—(*Continued*)

r \ n	0.009	0.010	0.011	0.012	0.013	0.014	0.015	0.017	0.020	0.0225	0.025	0.030	0.035	0.040
4	204	184	168	154	142	133	124	110	93	83	75	63	54	48
6	211	190	174	160	149	139	130	116	99	89	81	68	59	52
10	218	197	181	167	155	145	136	122	105	95	87	74	65	58
20	225	205	188	175	163	153	144	129	113	102	94	81	72	65
50	232	212	196	182	170	160	151	137	120	110	101	89	79	72
100	236	216	200	186	174	164	155	141	124	114	105	94	85	77
Slope s = 0.01 = 1 in 100 = 52.8 ft. per mile														
0.1	110	95	83	74	66	60	54	46	36	31	27	21	17	14
0.2	130	114	100	90	81	74	67	57	46	39	34	27	22	19
0.3	143	125	111	100	90	83	76	64	52	45	39	31	25	22
0.4	151	133	119	107	98	89	82	70	57	49	44	35	29	24
0.6	162	143	129	116	106	98	90	77	64	55	49	39	33	28
0.8	170	151	135	123	112	103	95	82	68	60	53	43	35	31
1.0	175	156	141	128	117	108	99	87	72	63	56	45	38	33
1.5	185	165	149	136	125	116	107	94	79	69	62	51	43	37
2	191	171	155	142	130	121	112	99	83	74	66	55	46	40
3	199	179	162	149	138	128	119	105	89	79	71	59	51	45
4	204	184	167	154	142	132	123	109	93	83	76	63	55	48
6	210	190	173	160	148	138	129	115	99	88	81	68	59	52
10	217	196	180	166	154	145	136	121	105	94	86	74	65	58
20	225	204	187	173	161	152	143	128	112	101	93	80	71	64
50	231	210	194	181	168	158	150	135	119	108	100	87	78	71
100	235	214	197	184	172	162	153	139	122	112	104	91	82	75

NOTE.—For slopes greater than 0.01, c remains nearly constant.

widely used formula for this purpose is the Chezy formula which is,

$$v = c\sqrt{rs}, \tag{311}$$

in which v is the mean velocity of the water in the channel; c, a function of the roughness of the channel, slope, and the hydraulic radius; r, the hydraulic radius determined by dividing the cross-sectional area of the channel by the wetted perimeter of that cross-section; and s, the slope of the water surface in the channel at the cross-section. The value of c in this formula, as carefully elaborated for various channel conditions by Ganguillet and Kutter, known as the Kutter formula, is as follows:

$$c = \frac{\frac{1.811}{n} + 41.6 + \frac{0.00281}{s}}{1 + \left(41.6 + \frac{0.00281}{s}\right)\frac{n}{\sqrt{r}}}. \tag{312}$$

Values of c for various values of n, s, and r will be found in Table XXXIV. In Eq. (312) the value of n, called the coefficient of roughness, depends on the roughness of the materials forming the bed and banks of the channel caused by irregularities, load of silt, curves, eddies, aquatic plants, and other conditions that tend to produce retardation of the flow.

If A represents the area of the cross-section of the channel at the point of measurement, it is evident that the final formula for computing the discharge is as follows:

$$Q = cA\sqrt{rs}. \tag{313}$$

For accuracy, it is essential to secure the slope s of the water surface when the flow is steady, *i.e.*, when the river is at neither a rapidly rising or rapidly falling stage.

22-16. Values of *n*.—The approximate values of n, based on considerations of the available data, are as follows:

$n = 0.010$ for clean straight channel newly lined with plain boards or neat cement plaster.

$n = 0.011$ for clean channels consisting of long tangents joined by gentle curves newly lined with plain boards, cement plaster or metal.

n = 0.012 for clean straight channels lined with plain boards, metal, steel-troweled concrete, sand-cement plaster, and best brickwork.

n = 0.013 for clean regular channels consisting of tangents and gentle curvatures lined with smooth concrete or metal.

n = 0.014 for clean regular channels consisting of tangents and gentle curves lined with smooth untroweled concrete.

n = 0.015 for clean regular channels consisting of tangents connected by gentle curves, lined with concrete with board-formed surfaces.

n = 0.020 for channels of fine firm gravel or earth in good condition.

n = 0.0225 for uniform channels of earth in fair condition lined with sediment and occasional patches of algae or loose gravel without vegetation.

n = 0.025 for channels of reasonably uniform cross-section, slope, and direction lined with sediment and light algae or with loose coarse gravel.

n = 0.030 for channels in rather poor condition with beds partially covered with débris or aquatic plants.

n = 0.035 for channels in bad order with beds strewn with stones or about one-third full of vegetation.

n = 0.40 to 0.60 for channels in very bad order.

MEASUREMENT OF WATER WITH CURRENT-METER GAGING STATIONS

22-17. Requirements for Accuracy of Measurements.—A current-meter gaging station is generally used for the measurement of water in streams where the quantity of water to be measured is large. Such a station should be located in a straight uniform stretch of channel with smooth banks and bed of permanent nature so far removed from falls, dams, or other channel obstructions that the relation of discharge to gage height will not be disturbed by them. The essential features of a current-meter gaging station are a gage, a bench mark to which the elevations on the gage are referred, fixed measuring points in the channel cross-section, and a stay line to hold the meter in the measuring plane or cross-section when the velocity is high and the water deep. The gage should be of a good design, substantially installed where it will indicate the water elevation at all stages and so graduated as to permit accurate readings

of the water elevations. The bench mark should be conveniently and permanently located and the elevation of the datum of the gage should be carefully referred to it. The measuring points should be permanently fixed at suitable intervals in the cross-section at right angles to the stream flow on a tagged wire stretched across the channel or on a bridge located at the station cross-section. The stay line should be stretched across the channel far enough above the measuring section to hold the current meter in

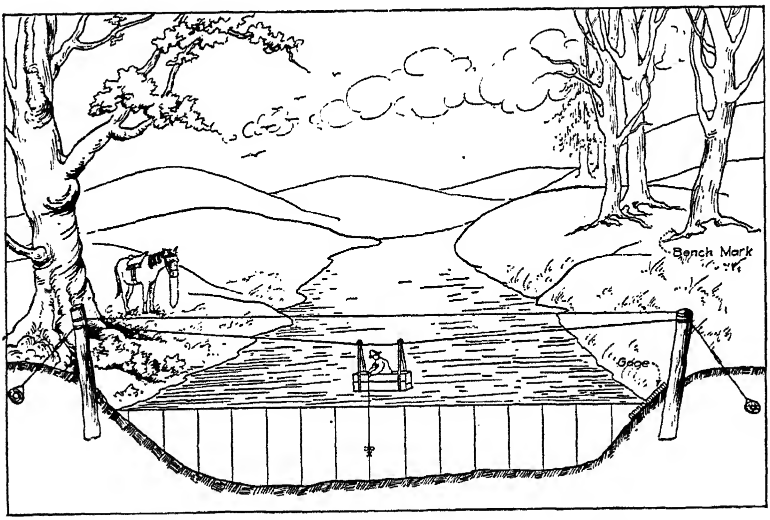


FIG. 83.—Typical current-meter stream-gaging station.

proper position. Figure 83 is a sketch of a typical current-meter stream-gaging station.

22-18. Velocity Measurements.—Soundings either with a current meter or with a special sounding line and weight should be made at the permanent measuring points. The mean velocity of each of these measuring points should be determined by means of the current meter in accordance with one of the approved methods of determining mean velocity. In general there are five methods of determining mean velocities in a vertical line with a current meter: (1) by taking half the sum of the measured velocity

at 0.2 and that at 0.8 of the water depth; (2) by taking the measured velocity at 0.6 of the water depth; (3) by taking the arithmetical mean of the measured velocities obtained at a number of equal vertical intervals; (4) by taking 0.85 to 0.95 of the measured velocity near the water surface; (5) by taking the measured velocity in a vertical line obtained by slowly and uniformly lowering and raising the meter throughout the range of the water depth one or more times.

22-19. Discharge Computations.—The total stream discharge is found from the measurements by computing the discharges of the elementary areas between the measuring points and taking their sum. In computing the discharges of the elementary areas, either the straight-line or parabolic-line method is generally used. In the straight-line method, the discharge is computed for each elementary area on the assumption that both the velocity and the water depth vary uniformly from one measuring point to another. The formula for computing the discharge of the elementary area by this method is as follows:

$$q = \left(\frac{V_a + V_b}{2}\right)\left(\frac{a + b}{2}\right)L, \tag{314}$$

in which a and b are the water depths in feet at two adjacent measuring points, V_a and V_b, the respective mean velocities in feet per second at these points; L, the distance in feet between the points; and q, the discharge in second feet for the elementary area. In the parabolic-line method, the discharge is computed for consecutive pairs of elementary areas on the assumption that the velocity and the water depths for three consecutive measuring points each lie on the arc of a parabola. The formula for computing the discharge for each pair of elementary areas by this method is as follows:

$$q = \left(\frac{V_a + 4V_b + V_c}{6}\right)\left(\frac{a + 4b + c}{6}\right)2L, \tag{315}$$

in which a, b, and c are the water depths in feet at three consecutive measuring points; V_a, V_b, and V_c, the respective

mean velocities in feet per second at these points; L, the distance in feet between the consecutive points; and q, the discharge in second-feet for the pair of elementary areas.

22-20. Records of Stream Discharge.—The discharge measurements at a current-meter gaging station should be taken at sufficient intervals of gage heights to permit of making accurate velocity, area, and discharge curves. In order to determine the quantity of water carried in the channel over a period of time, the gage should be read sufficiently often to give the water stages at close intervals, or an automatic-gage height recorder should be used. The current-meter measurements at a gaging station are interpolated and extended to cover all gage heights at the station by means of curves drawn on cross-section paper. To construct these curves, the discharges of the channel in second-feet as computed from the current-meter discharge measurements, the corresponding mean velocities in feet per second, and the cross-sectional areas in square feet for each measurement are plotted as abscissas each to a convenient scale with the common gage heights as ordinates. The most probable curves are then drawn through these plottings by smoothing out irregularities disclosed by the related velocities, areas and discharges. From the discharge curve thus constructed, a rating table is prepared for the station from which the discharge at any gage height is taken. From the gage heights and this table, the total discharge over any length of period is computed.

SURFACE CURVES

22-21. Causes of Surface Curves.—At every change in the grade of a water channel there is likely to be a change in the water depth that will produce a surface curve above the point of the changed grade. If the depth of the water in the channel is increased, a backwater curve is produced; if it is decreased, a drop-down curve is produced. The construction of a dam across a channel creates a backwater curve; the construction of a canal, chute, tail race, or spillway may produce a drop-down curve. A backwater curve

will cause increased depths of water in the adjacent stretch of channel above the dam and may thereby cause overflows or increased discharge at intake gates situated within its influence. A drop-down curve will cause decreased water depths in the channel and may, therefore, cause a destructive velocity or a decreased discharge at intake gates situated within its influence. An accurate solution of the problem of surface curves is, therefore, essential in the proper design of dams, canals, chutes, tail races, or spillways.

22-22. Use of Bresse's Functions.—Leading works on hydraulics give formulas for the solution of this problem facilitated in use by Bresse's tables of backwater and drop-down functions, and applicable to stretches of a channel with a uniform slope of bed and a rectangular cross-section of such width that the hydraulic radius may be taken equal to the depth and the wetted perimeter equal to the width. There are many channels, however, that are more nearly trapezoidal in cross-section, and formulas applicable both to trapezoidal and rectangular channels are desirable. There is here given the development of such formulas.

22-23. Symbols Used.—For the purpose of this development, let the following symbols have the significance assigned them:

b' = mean width of channel in feet for normal flow for discharge Q.
c' = Kutter's coefficient of channel in normal flow for discharge Q.
p' = wetted perimeter in feet of channel for normal flow Q.
p = wetted perimeter in feet of channel for changed flow Q.
w = width in feet of channel bottom.
Q = discharge of channel in second-feet.
ϕ' = angle in degrees between channel bed and horizontal plane.
θ = angle in degrees of side slopes of channel cross-section.
a = water depth in feet in channel with discharge Q at any point in surface curve.
a_1 = water depth in feet in channel with discharge Q at upper end of any stretch L.
a_2 = water depth in feet in channel with discharge Q at lower end of the stretch L.
b = mean width of channel in feet in any length L for discharge

c = Kutter's coefficient in any length L of channel for discharge Q.
g = acceleration of gravity in feet per second per second.
w' = weight per cubic foot of water.
L' = total length in feet of channel under consideration.
M = depth of water in channel under the known unobstructed flow Q.
L = length in feet of any uniform stretch of channel above change of grade.
r = hydraulic radius in feet of channel cross-section where water depth is a.
v = mean velocity in feet per second in channel cross-section where water depth is a.

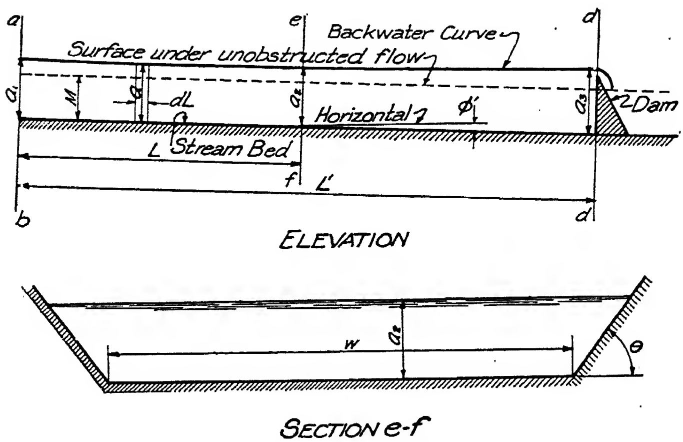


FIG. 84.—Diagram of surface curve.

22-24. Significance of Signs Used.—Let the surface curve formula be developed in terms of the length L, measured along the slope of the canal bed and of the depth a, measured perpendicularly to the plane thereof as shown in Fig. 84. L will be considered positive when measured in the direction of flow and a will be so considered when measured upward. da will, therefore, be inherently negative for the drop-down curve and inherently positive for the backwater curve. The velocity head, on the other hand, will have this condition of the signs reversed as the velocity increases with L in the drop-down curve and decreases therewith in the backwater curve.

22-25. Development of Equation of Surface Curve.—In an elementary length dL, $\sin \phi' \, dL - \cos \phi' \, da$ will represent the fall of the water surface, which must be equal to the sum of the friction head and the change of velocity head in the same length. Whence the following equation applies both to the backwater and to the drop-down curves:

$$\sin \phi' \, dL - \cos \phi' \, da = \frac{v^2 dL}{c^2 r} + \frac{v \, dv}{g}. \tag{316}$$

Since the velocity in the channel is equal to the discharge divided by the area, and the area is equal to $(w + a \cot \theta)a$, or approximately to

$$\left(w + \frac{a_1 + a_2}{2} \cot \theta\right)a$$

for short stretches of channel with obstructed flow, it follows that:

$$v = \frac{Q}{ba}, \tag{317}$$

in which $b = w + a \cot \theta$ or

$$w + \frac{a_1 + a_2}{2} \cot \theta, \text{ approximately.}$$

Also since the hydraulic radius equals the area divided by the wetted perimeter, it follows that,

$$r = \frac{ba}{p}, \tag{318}$$

in which b has the same significance as in Eq. (317), and

$$p = w + 2a \csc \theta$$

or

$$w + (a_1 + a_2) \csc \theta, \text{ approximately}$$

for short stretches of channel with obstructed flow. By differentiation of Eq. (317),

$$dv = -\frac{Q \, da}{ba^2}. \tag{319}$$

Substituting v, r, and dv in Eq. (316) and solving for L, there is obtained

$$L = \frac{\cos \phi'}{\sin \phi'} \int_{a_1}^{a^2} \left(\frac{1 - \dfrac{Q^2}{g \cos \phi' \, b^2 a^3}}{1 - \dfrac{Q^2 p}{c^2 \sin \phi' \, b^3 a^3}} \right) da. \qquad (320)$$

Now it may be shown by use of Chezy's formula that

$$M = \sqrt[3]{\frac{Q^2 p'}{c'^2 \sin \phi' \, b'^3}}. \qquad (321)$$

Also for convenience in computation, let it be assumed that

$$\frac{c'^2 \sin \phi' \, b'}{g p' \cos \phi'} = N. \qquad (322)$$

For the usual cases met in practice b', c', and p' in Eqs. (321) and (322) equal b, c, and p in Eq. (320), approximately, for short stretches of channels of widths great as compared to depths. Noting these equalities and substituting M and N for their equivalents in Eq. (320),

$$L = \frac{\cos \phi'}{\sin \phi'} \int_{a_1}^{a_2} \left[\frac{1 - N(M/a)^3}{1 - (M/a)^3} \right] da. \qquad (323)$$

By integrating the variable a between the limits a_2 and a_1, the following desired general equation is found:

$$L = \frac{\cos \phi'}{\sin \phi'} \left[a - M(1 - N) \left(\frac{1}{6} \log_e \frac{\left(\frac{a}{M}\right)^2 + \frac{a}{M} + 1}{\left(\frac{a}{M} - 1\right)^2} - \frac{1}{\sqrt{3}} \operatorname{arc\,cot} \frac{2\left(\frac{a}{M}\right) + 1}{\sqrt{3}} \right) \right]_{a_1}^{a_2}. \qquad (324)$$

Now assume,

$$\frac{1}{6} \log_e \frac{\left(\frac{a}{M}\right)^2 + \frac{a}{M} + 1}{\left(\frac{a}{M} - 1\right)^2} - \frac{1}{\sqrt{3}} \operatorname{arc\,cot} \frac{2\left(\frac{a}{M}\right) + 1}{\sqrt{3}} = \phi\left(\frac{a}{M}\right), \qquad (325)$$

and apply the limits a_2 and a_1, whence

$$L = \frac{\cos \phi'}{\sin \phi'}\left\{a_2 - a_1 + M(1 - N)\left[\phi\left(\frac{a_1}{M}\right) - \phi\left(\frac{a_2}{M}\right)\right]\right\}. \quad (326)$$

By computing the values of M from Eq. (321), N from Eq. (322), and $\phi(a/M)$ for the values a_1 and a_2, and substituting them in Eq. (326), the length L for the change of depth from a_1 to a_2 can be found. Bresse's "Tables of Backwater and Drop-down Functions" can be used to great advantage to obtain the values of $\phi(a/M)$. These functions may be found in Merriman's "Treatise on Hydraulics" and in other important works on the subject of hydraulics.

HYDRAULIC JUMP

22-26. General Statement.—The hydraulic jump is one of the very interesting and useful phenomena in the science of hydraulics. It consists of a sudden increase in the depth of a rapidly flowing stream, with a corresponding decrease in velocity beyond the jump. It is a familiar spectacle at the ocean beaches, where water from successive waves, receding at a shallow depth and high velocity, reaches the deeper body of water. An abrupt rise in the surface occurs, accompanied by great turbulence and entrainment of air. The sharpness of the rise is confused by the foam rolling over onto the shallower flow.

The hydraulic jump is now utilized extensively in engineering works because it provides a positive and simple method for decreasing erosive velocities below overflow dams, at the end of reservoir spillways, etc. Not only may part of the velocity head be converted into pressure head by means of the jump, but a considerable portion of the total energy of the flow may be dissipated owing to the accompanying turbulence.

The results of valuable studies and tests of the hydraulic jump have been published by the Miami Conservancy District,[1] and a very complete review of the subject has been prepared by Julian Hinds.[2]

[1] *Technical Reports*, Part III.

[2] "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," *Eng. News-Record*, Vol. 85, No. 22, p. 1034.

22-27. Symbols Used.—In the discussion of the hydraulic jump, the symbols used and their significance are as follows:

L = length in feet of the portion of a channel under consideration.
s = slope of the channel bottom = rise/length.
a, a_1, a_2 = depth of water, in feet at the point referred to.
v, v_1, v_2 = average velocity, in feet per second at the point referred to.
w = weight of water, in pounds per cubic foot.
g = acceleration, in feet per second, due to gravity.
M = mass of water = Qw/g.
F = loss of head, in feet, due to friction.
Q = rate of flow, in cubic feet per second.
A = area of channel, in square feet.
H = total energy head, in feet.
b = breadth of channel bottom.
P = total pressure of water on a cross-section, in pounds.
Z = total pressure of water on a cross-section plus rate of change of momentum, in pounds.

22-28. Fundamental Concepts Involved in Theory of Hydraulic Jump.—Two fundamental laws of physics involved in this subject are (1) the law of the conservation of energy and (2) the law of the conservation of momentum.

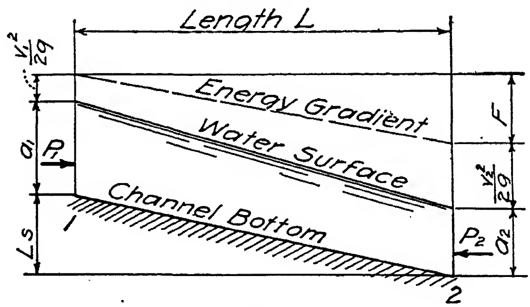


FIG. 85.

Bernoulli's theorem, which is based on the former, requires that in an open flowing stream the depth plus the velocity head at one cross-section is equal to the depth plus the velocity head minus the intervening fall in the channel bottom plus any intermediate friction or other losses at a cross-section further downstream. Thus referring to Fig. 85, it is evident that

$$Ls + a_1 + \frac{v_1^2}{2g} = a_2 + \frac{v_2^2}{2g} + F. \qquad (327)$$

The law of the conservation of momentum requires that the rate of change in the linear momentum of a mass along any line equals the algebraic sum of the external forces acting upon the mass along that line. Thus unless some *external* force, such as that of gravity or an obstruction, acts upon a flowing stream, the momentum in the direction of flow cannot be altered, regardless of the interplay of forces within the mass of water. It follows that even during the extreme turbulence of a mass of water as it passes through the hydraulic jump, the momentum remains constant except as it is affected by the increase of pressure due to greater depth after the jump. Thus, since momentum equals mass times velocity, and referring again to Fig. 85,

$$Mv_1 + \frac{wa_1^2}{2} = Mv_2 + \frac{wa_2^2}{2} \tag{328}$$

for a channel of unit width.

The method herein presented is applicable to a channel of any cross-section. The actual velocity is assumed to be equal to the average velocity; *i.e.*, $v = Q/A$, which results in an error that is, in general, of negligible magnitude. All cross-sections of the stream are taken on vertical planes, regardless of the channel slope, and the velocities referred to and the direction of momentum are thus horizontal.

22-29. Energy of Flow.—The total energy head of an open flowing stream at any section is made up of the depth of water a (potential energy) and the velocity head $v^2/2g$ (kinetic energy) or

$$H = a + \frac{v^2}{2g}; \tag{329}$$

and since for a *rectangular* section $v = Q/ab$,

$$H = a + \frac{Q^2}{2a^2b^2g}, \tag{330}$$

from which

$$a^3 - a^2H + \frac{Q^2}{2gb^2} = 0. \tag{331}$$

Equation (331) relating a and H has three roots, one of which is negative and hence unreal. The two positive roots indicate that, *for any value of the total energy head, there are two possible depths.* These are known as "alternate energy stages."

For that depth a_c at which the energy head is the *minimum*, the first derivative of $H = 0$, thus, from Eq. (330),

$$\frac{dH}{da} = 1 - \frac{Q^2}{a^3b^2g} = 0,$$

or

$$a^3 = \frac{Q^2}{b^2g} = \frac{a^2b^2v^2}{b^2g} = \frac{a^2v^2}{g};$$

i.e.,

$$a = \frac{v^2}{g}, \text{ and } \frac{a}{2} = \frac{v^2}{2g}. \tag{332}$$

Substituting this value in Eq. (329), solving for a, and noting that this minimum value of $a = a_c$, the critical depth,

$$a_c = \tfrac{2}{3}H, \text{ or } H = \tfrac{3}{2}a_c; \tag{333}$$

i.e., the critical depth in a rectangular channel equals two-thirds of the total energy head and equals twice the velocity head.

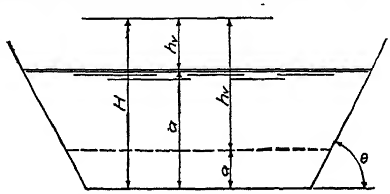


FIG. 86.—Section of trapezoidal channel.

By a similar process, it may be shown that in a trapezoidal channel, in which b represents the bottom width and θ the angle of the sides with the horizontal, as shown in Fig. 86,

$$a_c = \frac{4H\cot\theta - 3b + \sqrt{9b^2 + 16H^2\cot^2\theta + 16bH\cot\theta}}{10\cot\theta}. \tag{334}$$

Similarly, it may be shown that, where v_c is the critical velocity corresponding to a_c,

$$v_c = \sqrt{\frac{9(w + a_c \cot \theta)a_c}{w + 2a_c \cot \theta}}. \tag{335}$$

In Fig. 87 is shown a curve representing the energy head

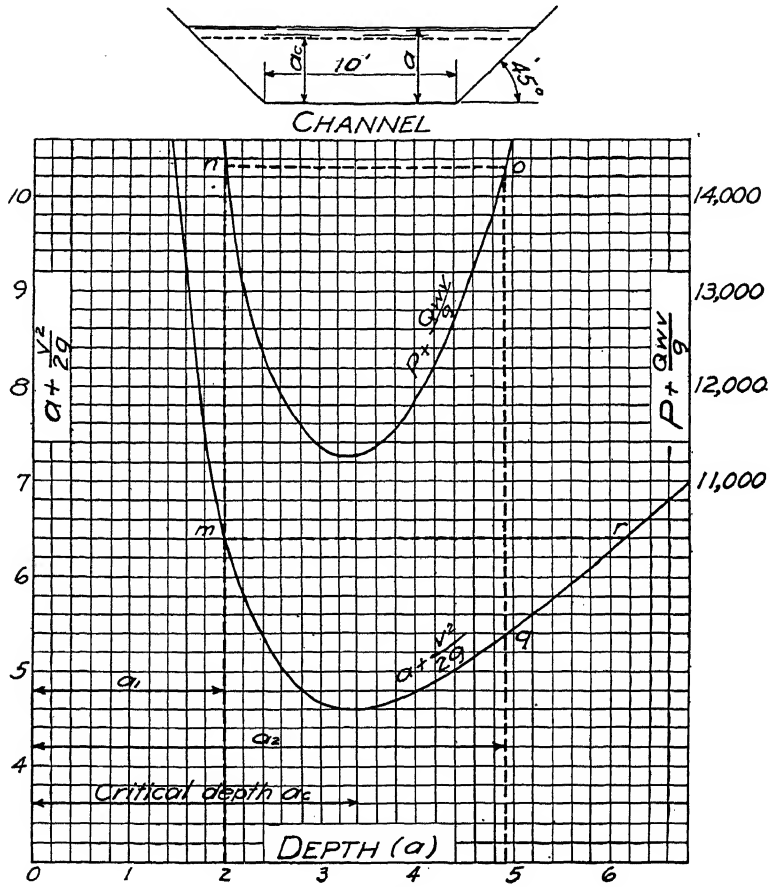


FIG. 87.

$\left(a + \frac{v^2}{2g}\right)$ plotted against the depth of water, for a trapezoidal channel wherein $b = 10$ ft., $\theta = 45$ deg., and $Q = 400$ cu. ft. per second. It will be noted that the curve is not symmetrical about the line representing critical depth.

22-30. Momentum of Flow.—What may be termed the total *momentum head* of an open flowing stream at any section is, like the energy head, made up of two items, the pressure P (potential) and the mass times velocity (active); *i.e.*,

$$Z = P + Mv. \tag{336}$$

In a rectangular channel, $P = wa^2b/2$, $M = Qw/g$, and $Mv = Qwv/g = Q^2w/abg$. Thus,

$$Z = \frac{wa^2b}{2} + \frac{Q^2w}{abg}. \tag{337}$$

For a minimum value of Z,

$$\frac{dZ}{da} = wab - \frac{Q^2w}{a^2bg} = 0. \tag{338}$$

Solving, $a^3 = Q^2/b^2g$; and, since $Q = vab$, $a = v^2/g$, which is the same value as for the critical depth given by Eq. (333). This proves that the minimum value of the pressure plus momentum occurs at the same depth as does the minimum value of the energy gradient.

Figure 87 shows a curve of the quantity $P + \dfrac{Qwv}{g}$ plotted against the depth of water for the trapezoidal channel described in the preceding paragraph. The curve reaches its minimum value at depth 3.4, which, as noted, is the critical depth a_c. At the same depth the minimum value of $a + \dfrac{v^2}{2g}$ is reached.

22-31. Significance of the Hydraulic Jump.—Referring again to Fig. 87, it will be noted that there are two alternate depths for any given value of the total energy head and of the momentum head. The action of the hydraulic jump is to change from the lesser to the greater depth, always crossing the critical depth. When the flow is at the depth of, for instance, 2 ft., it requires 6.35 units of total energy head as shown at m and 14,300 units of total momentum head as shown at n. It might at first glance be assumed that after passing through the jump the depth

would be 6.1 ft. as shown at r, this being the alternate energy depth. This is not possible, however, as that depth would involve an *increase* in the total of pressure plus momentum and this must remain constant at 14,300 (n or o on the curve) regardless of the turbulence or other head losses. The curve indicates at o that the depth should increase to only 4.92 ft. after the jump. The only conclusion possible from this apparent inconsistency is that the hydraulic jump has resulted in a loss of energy head of 6.50 − 4.92, or 1.58 ft. The "lost" energy has been transformed into heat, because of the friction and impact of the molecules upon one another and upon the channel walls. Both the loss of total energy head and the decrease of velocity resulting from the hydraulic jump are of great importance in its use for decreasing erosion below dams or spillways.

It is usually necessary to obtain the relation between the depths a_1 and a_2. In a simple jump such as that forming in a horizontal or nearly horizontal channel, these values may be related through the momentum equation; but for jumps of less regular type, experimental methods must be used. Referring to Eq. (328), noting that $M = Qw/g$, and transposing terms,

$$\frac{Qw}{g}(v_1 - v_2) = \frac{wa_2^2}{2} - \frac{wa_1^2}{2}. \tag{339}$$

Substituting v_1a_1 for Q and v_1a_1/a_2 for v_2 and dividing by w,

$$\frac{v_1a_1}{g}\left(v_1 - \frac{v_1a_1}{2}\right) = \frac{a_2^2}{2} - \frac{a_1^2}{2}, \tag{340}$$

from which

$$v_1^2 = \frac{ga_2}{2a_1}(a_2 + a_1). \tag{341}$$

Solving for a_1 and a_2,

$$a_1 = -\frac{a_2}{2} + \sqrt{\frac{2v_2^2a_2}{g} + \frac{a_2^2}{4}}, \tag{342}$$

$$a_2 = -\frac{a_1}{2} + \sqrt{\frac{2v_1^2a_1}{g} + \frac{a_1^2}{4}}. \tag{343}$$

22-32. Solution of Problems Involving the Hydraulic Jump.—In practical design, it is recommended that problems involving the hydraulic jump be solved with the aid of graphical methods. Figure 88 shows the toe of an overflow dam with water flowing at accelerated velocity over the curved bucket. At *o*, the end of the bucket, the pressure plus momentum is shown considerably above that of the main stream. It is therefore not possible for the jump to occur at this point. As the flow proceeds downstream from *o*, the slope of the apron is not sufficient to maintain

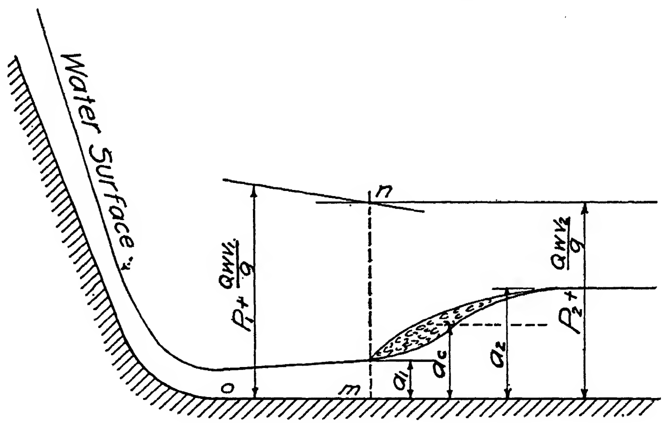


FIG. 88.

the high velocity owing to frictional resistance; hence velocity and momentum are lost, until at *m* the pressure plus momentum curves intersect and the jump occurs at that location. Since it is important owing to the erodibility of the channel bottom that the high-velocity flow be prevented from traveling beyond the end of a concrete apron, it is good practice to construct the apron so as to force the formation of the jump within a definite range. This may be done by depressing the entire bucket to a depth below the stream bed or by erecting a barrier wall at or shortly upstream from the end of the apron. In each case the effect is to increase sharply the slope of the $P_2 + \frac{Qwv_2}{g}$ curve so that its intersection with the $P_1 + \frac{Qwv_1}{g}$ curve will be certain to occur within the allowable limit.

Figure 89 shows the profile of a channel and stilling basin of the type frequently constructed below a spillway. The basin should be so proportioned that at maximum flow the jump will form on the slightly sloped portion BC. The slope of BC provides a factor of safety to allow for velocities higher than expected, which frequently result owing to the assumption of too great a value of Kutter's n for the approach channel. It is better practice to construct the basin with the sloping section as shown than to extend the steep slope to E, since the latter may result in a submerged and imperfect jump with waves and disturbed

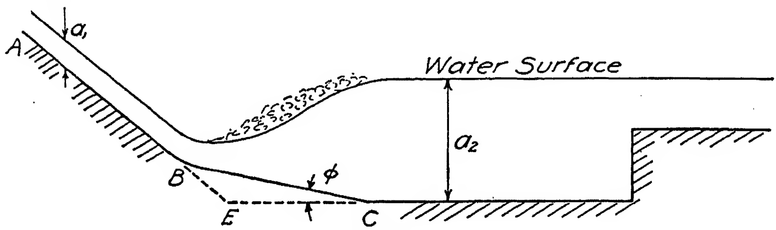


FIG. 89.

water extending downstream past the normal end of the jump. It will be noted from Fig. 87 that a slight increase in the depth greatly increases the value of $P + \frac{Qwv}{g}$. The length of basin should, of course, be made sufficient to include all of the disturbed water. It has been established by many tests that the length of the jump if completely formed is approximately five times the depth a_2. The basin should be somewhat longer than the value thus determined. If the jump forms on a sloping surface, there is obviously an additional active force which enters into the momentum equation, *viz.*, the forward component of the upward pressure of the bottom. This component is equal to the weight of the water times the sine of the angle of slope. Hence, the momentum equation for conditions before and after the jump will read

$$P_1 + Qw \sin \phi + \frac{Qwv_1}{g} = P_2 + \frac{Qwv_2}{g}. \qquad (344)$$

At rates of flow lower than the maximum, the jump will form on the steep slope AB and will be of the submerged type. The hydraulics of this type of jump have not been entirely solved; but it will generally be safe to assume that if the stilling basin is properly proportioned to take care of the maximum flow with a fully formed jump, it will also be safe for lesser flows wherein the jump forms on the slope.

A great deal of valuable material has been published recently on the subject of the hydraulic jump. The work of Bakhmeteff[1] is particularly noteworthy. The design of stilling basins wherein the jump is used has been the subject of extensive experiments by C. Maxwell Stanley.[2]

HYDRAULICS OF CHANNELS WITH UNIFORM SIDE SLOPES

22-33. General Statement.—If the channel below the spillway apron is of uniform cross-section, the carrying capacity at various depths of flow can be determined by the well-known Kutter-Chezy formula, for which constants may be found in paragraph 22-15. In the usual case of such channels, however, it is not possible or desirable to maintain a uniform section throughout, thus requiring special study to determine the effect on the flow caused by the changes in section. In the following mathematical treatment, a method is presented for computing the loss of head due to contractions or expansions of a waterway, either modified or unmodified by reducing sections.

22-34. Symbols Used.—The following symbols will be utilized in this discussion:

n = proportion of wetted perimeter on which contraction or expansion is suppressed.
c = Kutter's coefficient.
a = water depth in feet in reducing section.

[1] "Hydraulics of Open Channels," BAKHMETEFF, B. A., Engineering Societies Monographs, McGraw-Hill Book Company, Inc.

BAKHMETEFF, B. A., and A. E. MATZKE, "The Hydraulic Jump in Sloped Channels," *Trans. Am. Soc. Mech. Eng.*, February, 1938.

[2] "Study of Stilling-basin Design," *Trans. Am. Soc. Civ. Eng.*, Vol. 99, p. 490.

g = acceleration of gravity in feet per second per second.
h_c = loss of head in feet due to contraction of channel cross-section.
h_e = loss of head in feet due to expansion of channel cross-section.
h_f = friction head in feet in reducing section.
L = distance in feet of any cross-section of a reducing section from its upper end.
L' = length of the reducing section in feet.
h_v = difference in feet between velocity heads above and below change of cross-section.
$H = h_v - h_f$.
m = weir and orifice coefficient of contraction.
Q = channel capacity in second-feet.
r = hydraulic radius in feet of cross-section of reducing section at distance of L from its upper end.
v = mean velocity in feet per second at distance L from upper end of reducing section.
v_1 = mean velocity in channel in feet per second above change of cross-section.
v_2 = mean velocity in channel in feet per second below change of cross-section.
w = channel bottom width in feet above change in cross-section.
ϕ = one-half of angle between converging sides of reducing section.
θ = angle of side slopes of reducing section.

22-35. Changes in Channel Sections and Resulting Losses of Head.—When it is of interest to reduce head losses so as to preserve a low slope in the channel, changes in section are made gradually, the reducing or expanding zone consisting of wings or side walls making definite angles with the axis of the channel. There results an increase or decrease of velocity head together with losses of head due to partial contraction or expansion of the section and increased friction. The sum of these three heads is equal to the drop of the water surface in the zone.

22-36. Change in Velocity Head.—The velocities in the channel above and below the reducing or expanding zone may be computed by the Kutter-Chezy formulas. Then the increase or decrease in the velocity head is,

$$h_v = \frac{1}{2g}(v_2^2 - v_1^2). \tag{345}$$

22-37. Computation of Loss of Head Due to Change of Section.—In determining the value of h_c, the loss of head

due to contraction, the discharge may be considered to take place in two parts, that passing through the upper portion above the normal water level under weir action, and that passing through the lower portion under submerged-orifice action. The average coefficient for these two types of flow is about 0.62, which value must, however, be corrected for the suppression of bottom and side contractions. It may be safely assumed that the coefficient of discharge for full contraction is increased due to suppression in proportion to the extent of the wetted perimeter suppressed up to a limit of about 0.1 for complete suppression. Then the general equation for the coefficient of discharge will be,

$$m = 0.62 + 0.1n. \qquad (346)$$

In the usual case of an apron below a spillway, the side contractions are in the form of splay walls set at an acute angle with the channel center line as shown in Fig. 90. It will be sufficiently accurate to assume that the equivalent side suppression is equal to full side suppression multiplied by the cosine of this acute angle. By giving the foregoing conditions due weight in computing the value of m, the loss of head due to contraction of cross-section, may be computed by the equation,

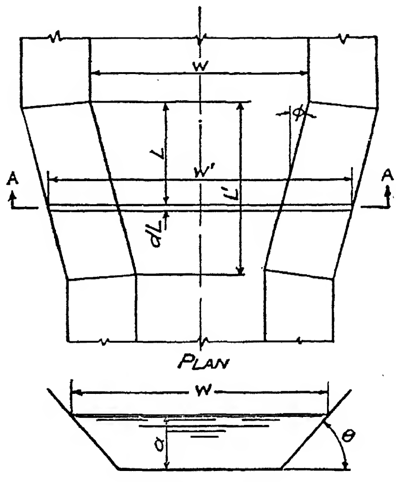


FIG. 90.—Diagram for channel of varying section.

$$h_c = \left(\frac{1}{m} - 1\right)^2 \frac{v^2}{2g}. \qquad (347)$$

The loss of head due to expansion of cross-section may be derived from the analogous formula for pipes, *i.e.*,

$$h_e = \frac{(v_1 - v_2)^2}{2g}.$$

Introducing the term n, as in the case of a contracting channel, the result becomes,

$$h_e = \frac{(1 - n)}{2g}(v_1 - v_2)^2. \tag{348}$$

22-38. Friction Loss in Reducing Section.—The friction loss in the reducing section may be determined by certain modifications of the Kutter-Chezy formulas to provide for variable velocity and cross-section. The summation of the head losses for the length L' of transition may be expressed as

$$h_f = \frac{1}{c^2}\int_0^{L'} \frac{v^2 dL}{r}. \tag{349}$$

From the fact that the velocity equals the discharge divided by the area, and from the characteristics of the figure,

$$v = \frac{Q}{a(w + a \cot \theta - 2L \tan \phi)}, \tag{350}$$

and since the hydraulic radius equals the area of the stream divided by the wetted perimeter,

$$r = \frac{(w + a \cot \theta - 2L \tan \phi)a}{w + 2a \csc \theta - 2L \tan \phi}. \tag{351}$$

By substituting these values of v and r in the equation for h_f and integrating between the limits L' and 0, there results,

$$h_f = \frac{Q^2}{4a^3c^2 \tan \phi}\left[\frac{2w + 2a \csc \theta + a \cot \theta - 4L' \tan \phi}{(w + a \cot \theta - 2L' \tan \phi)^2} - \frac{2w + 2a \csc \theta + a \cot \theta}{w + a \cot \theta)^2}\right]. \tag{352}$$

22-39. Total Required Head in Changing Section.—The sum of the heads as computed from Eqs. (345), (347) or (348), and (352) gives the total of the required head in the reducing or expanding section. If the change in section of the channel is from a smaller to a larger one, the value of h_v, as given in Eq. (345), will still be correct, the algebraic sign, of course, resulting as negative. Also the value of h_f, as given in Eq. (352), will still obtain if the angle ϕ therein be taken as negative. By making an algebraic

summary of the values of h_v, h_f, and h_c or h_e as the case may require, the gain or loss of head in the changing section may be determined.

CHANNELS WITH VARIABLE SIDE SLOPES

22-40. General Statement.—In case the channel reducing or expanding section has varying side slopes, the formulas for channels with uniform side slopes will not apply, as a correction must be made for the effect of the slope variation. In the usual method of constructing warped sections, the change in the mean width of the wetted cross-section is made proportional to the distance from the initial end of the reducing section. Hence, y, the unit rate of change of the mean width, may be expressed as

$$y = \frac{a(\cot\theta_1 - \cot\theta_2)}{L'},$$

in which θ_1 and θ_2 are the angles of the side slopes above and below the reducing section. If it is assumed that x, the rate of change in the wetted perimeter, is also proportional to the distance from the initial point, then

$$x = \frac{2a(\csc\theta_1 - \csc\theta_2)}{L'}.$$

This is not exactly true, as the unit rate of change is also a function of csc θ, but it is very close to correct for channels in which the depth is not great as compared with the width. In case the channel is deep as compared with the width, greater accuracy may be obtained by computing the friction loss for shorter lengths of channel and taking the sum for the total loss. Such accuracy, however, is seldom warranted. The case of a decreasing channel section will first be considered.

22-41. Loss of Head Due to Contraction.—There will be little, if any, loss of head due to contraction of the section, as this arrangement practically eliminates subsequent enlargement of the stream. This loss will, therefore, be neglected.

22-42. Change in Velocity Head.—There will be a marked change in the velocity head, which change may be expressed by the usual equation,

$$h_v = \frac{1}{2g}(v_2^2 - v_1^2). \tag{353}$$

22-43. Loss of Head Due to Friction.—The friction head is usually small in short reducing sections, and may be neglected or computed by the usual approximate methods. In long sections with great changes in velocity, a direct and accurate formula is more necessary. Such a formula may be developed by applying calculus to the Chezy formula. Assuming c to be constant, the value of h_f is as follows:

$$h_f = \frac{1}{c^2}\int_0^{L'} \frac{v^2 dL}{r}. \tag{354}$$

From the relation of velocity, area, and discharge, it follows that

$$v = \frac{Q}{a[w + a \cot \theta_1 - (y + 2 \tan \phi)L]},$$

and by definition of the hydraulic radius,

$$r = \frac{a[w + a \cot \theta_1 - (y + 2 \tan \phi)L]}{w + 2a \csc \theta_1 - (y + 2 \tan \phi)L}.$$

Substituting the above values of v and r, and integrating between the limits indicated, there results,

$$h_f = \frac{Q^2}{2a^3c^2(y + 2\tan\phi)^2} \left[\frac{(w + 2a\csc\theta_1)(y + 2\tan\phi) + [w + a\cot\theta_1 - 2(y + 2\tan\phi)L'](x + 2\tan\phi)}{[w + a\cot\theta_1 - (y + 2\tan\phi)L']^2} - \frac{(w + 2a\csc\theta_1)(y + 2\tan\phi) + (w + a\cot\theta_1)(x + 2\tan\phi)}{(w + a\cot\theta_1)^2} \right]. \tag{355}$$

By adding the results of Eqs. (353) and (355), the total change of head in the reducing section may be obtained.

22-44. Loss of Head in Increasing Channel Section.—In case of diverging sections, Eq. (353) is directly applicable, the value of h_v being negative, indicating a surplus head. In Eq. (355) the signs of ϕ, x, and y should be changed. The loss of head due to expansion of the section will be negligible due to the gradual change.

CHAPTER XXIII

FAILURES OF DAMS

INTRODUCTION

23-1. General Statement.—Valuable information can be obtained from a careful study of the design of dams that have failed and the causes and nature of the failure. There is danger, however, in drawing false conclusions as to the cause and nature of the failure of a dam on account of lack of complete information about all of the numerous conditions affecting the structure. It is impossible here to give complete details of dam failures, and the reader who desires elaborate information should refer to the fuller discussions given in general engineering literature to which references are made in the following pages.

23-2. Lists of Dam Failures.—In Tables XXXV to XL, inclusive, are given extensive lists of dams that have failed after having been put into service. In these lists the reader will find references to sources of more complete information and brief statements as to the general assumed causes and natures of the failures. The dams in the lists have been classified into the various types discussed in this work such as (*a*) solid-gravity dams, (*b*) deck dams, (*c*) single-arch dams, (*d*) multiple-arch dams, (*e*) rock-fill dams, and (*f*) earth dams. Although quite voluminous, the lists are not complete and only brief statements of the probable causes and natures of the failures are given, as a complete discussion of each failure would require an extensive volume in itself, and is, therefore, outside of the scope of this work. An attempt is made in each list not only to give the cause, but the nature of the failure as both are essential data to an intelligent understanding of the case.

FAILURES OF SOLID-GRAVITY DAMS

23-3. List of Failures.—Table XXXV contains a list of 17 solid-gravity dams that have failed in various parts of the United States and foreign countries. These examples may be taken to be fairly representative of failures of dams of this type.

23-4. Causes of Failures.—The failures of dams of the solid-gravity type are usually caused by the action of water on the foundations, the overtopping of the structure by unexpected floods, thrusts or bending moments produced by ice, or improper methods of construction. In the list of 17 solid-gravity dams contained in Table XXXV, it is thought that six failures can be ascribed to the action of water on the foundation, three of which are ascribed to piping, one to decreased friction on the foundation through water soaking, one to softening of the foundation material, and one to the eroding of the foundation material. The failure of five of the structures is assumed to have been caused by the overtopping of the structures by unexpected flood waters. Three are ascribed to defective construction material and three to action of ice. The lesson to be learned from these failures is that insufficient study is often given to the design of the foundation of dams, to the study of the water supply of the streams, and to the general action of ice. Ice effects have too often been ascribed to pressure alone, whereas considerable overturning moment may be produced on a dam through the lowering or raising of the water in the reservoir when the ice is strongly frozen to the dam. Three of the failures, it will be noted, have been caused by defective construction material. At the present time construction materials are more generally inspected and the work is more carefully done so that the chances for failures from this cause are not as great as they formerly were when the proper handling and strength of concrete were not fully understood.

23-5. Nature of Failures.—It is generally considered that the nature of the failure of a gravity dam consists in the crushing of the masonry of the dam, the sliding of

TABLE XXXV.—FAILURES OF SOLID-GRAVITY DAMS

Dam	Location	Height, feet	Date of failure	References	Probable cause of failure	Probable nature of failure
1. Habra	Algiers	110	1881	WEGMANN, "Design and Construction of Dams"	Faulty construction	Crushing
2. Housatonic	Connecticut	40	1891	*Jour. Electricity*, Vol. 44, Mar. 15, 1920	Undercutting of foundation	Overturning
3. Borizey	France	72	1895	*Inst. C. E.*, Vol. 125	Overtopping of dam	Overturning
4. Los Angeles	California	52	1895	*Jour. Electricity*, Vol. 44, Mar. 15, 1920	Piping of foundation	Washout of foundation and collapse of dam
5. St. Anthony Falls	Minneapolis	18	1899	*Eng. Record*, May 13, 1899	Ice pressure	Sliding
6. Colorado	Texas	60	1900	*Eng. News-Record*, Mar. 22, 1928	Overtopping of dam	Sliding
7. Anderson	North Carolina	44	1902	*Eng. News-Record*, Mar. 22, 1928	Overtopping of dam	Overturning
8. Columbus	Georgia	39	1902	*Eng. News-Record*, Mar. 22, 1928	Overtopping of dam	Overturning
9. Lower Tallassee	Alabama	30	1902	*Eng. News-Record*, Mar. 22, 1928	Overtopping of dam	Overturning
10. Austin	Pennsylvania	50	1911	*Eng. Record*, Oct. 7, 1911	Piping of foundation	Sliding
11. Port Angeles	Washington	110	1912	*Eng. News-Record*, Mar. 22, 1928	Piping of foundation	Washout of foundation. Dam remained standing
12. Rockport	New York	...	1912	*Eng. Record*, June 22, 1912	Ice pressure	Overturning
13. Lincoln Pond	New York	25	1912	*Eng. News*, June 6, 1912	Smooth foundation	Sliding
14. Bow River	Alberta	...	1912	*Eng. Record*, Oct. 5, 1912	Ice pressure	Washout of foundation and collapse of dam
15. Lake Gleno	Italy	143	1923	WEGMANN, "Design and Construction of Dams"	Defective construction. Combination gravity and multiple arch	Crushing
16. Dolgarrog	North Wales	20	1926	*Eng. News-Record*, Mar. 22, 1928	Defective construction and foundation	Washout of foundation and collapse of dam
17. St. Francis	California	205	1928	*Eng. News-Record*, Mar. 22, 1928 and May 10, 1928; Report of Governor's Commission, Mar. 24, 1928; *Western Construction News*, Apr. 10, 1928	Foundation softened under water pressure	Washout of foundation and collapse of dam

the dam on its foundation or on a horizontal section, or the overturning of the dam, or the crushing, piping, softening, slickening, or eroding of the foundation material. The data in Table XXXV indicate that out of the 17 failures listed for solid-gravity dams, seven probably failed by overturning, two by crushing of faulty material, four by sliding on the foundation, and five by eroding or washing out of the foundation.

FAILURES OF DECK DAMS

23-6. List of Failures.—Table XXXVI contains a list of five failures of deck dams. The list is necessarily short because the use of this type of dam is comparatively recent; and, therefore, the dams built have had the advantage of the recent improvements in concrete fabrication and design.

23-7. Causes of Failures.—It will be seen from Table XXXVI that the failures of all of these dams are ascribed to piping of the foundation. The reasons for this are obvious. This type of dam generally has a short path of percolation of the water under the dam, and is often built on a pervious foundation of considerable depth, making a tight deck cutoff difficult. The inference to be drawn from the failures is that foundation percolation should be more carefully provided against than is sometimes done.

23-8. Nature of Failures.—The causes of the failures listed indicate that the failures naturally take place in the foundation and this is, or is not, followed by the failures of the dams themselves. In the list given, it will be observed that three of the dams slid on their bases, one dam collapsed after the foundation washed out, and still another stood intact even after the foundation had washed out.

FAILURES OF SINGLE-ARCH DAMS

23-9. List of Failures.—There have been very few failures of single-arch dams, partly because of the comparatively recent construction, in large number, of this type of dam; partly because of their use on good foundations

*

TABLE XXXVI.—FAILURES OF DECK DAMS

Dam	Location	Height, feet	Date of failure	References	Probable cause of failure	Probable nature of failure
1. Ashley	Massachusetts	40	1909	*Eng. News*, Apr. 1, 1909	Piping of foundation	Sliding
2. Canaseraga Creek	New York	..	1909	*Eng. Record*, Jan. 1, 1910	Piping of foundation	Sliding
3. Janesville	Wisconsin	..	1912	*Eng. Record*, Jan. 13, 1912	Piping of foundation	Washout of foundation. Structure not damaged
4. Stony River	West Virginia	50	1914	*Eng. News*, Jan. 22, 1914	Piping of foundation	Sliding
5. Plattsburg	New York	94	1916	*Eng. News*, June 8, 1916	Piping of foundation	Washout of foundation and collapse of part of dam

only, and partly because of the relatively large factor of safety used in their design. Table XXXVII contains data covering two failures.

23-10. Causes of Failures.—The data given in Table XXXVII indicate that the failures of single-arch dams thus far have been due to piping or erosion of the foundation material. This is as might be expected from the general strength usually given to the arches themselves, both as to dimensions and quality of construction material.

23-11. Nature of Failures.—The nature of the two failures has been washouts of the foundations under the abutments; and in each case the main structure has remained intact in spite of the damage to the abutment foundation.

FAILURES OF MULTIPLE-ARCH DAMS

23-12. List of Failures.—Table XXXVIII shows but one record of failure of a multiple-arch dam. There are few such failures because these structures are of recent adoption, and are of a type requiring careful analysis and design. Another explanation is doubtless that there are few of these dams built. Their use is, however, increasing in popularity at the present time due to a more thorough understanding of the principles involved in design.

23-13. Cause and Nature of Failures.—The failure given in the list is ascribed to defective design and defective construction, which is unusual for a structure built at the present time, and is, therefore, not to be understood as a characteristic weakness of this type of dam. In general, it may be assumed that the failures of multiple-arch dams are likely to be of the same cause and nature as for deck dams, as already discussed.

FAILURES OF ROCK-FILL DAMS

23-14. List of Failures.—Table XXXIX presents a list of six rock-fill dams that have failed, together with a statement of the probable cause and nature of the failure of each dam. All of these dams were built prior to the year

TABLE XXXVII.—FAILURES OF SINGLE-ARCH DAMS

Dam	Location	Height, feet	Date of failure	References	Probable cause of failure	Probable nature of failure
1. Moyie River.......	Idaho	53	1925	*Jour. Electricity*, Vol. 44, April, 1920	Piping under abutment	Washout of foundation. Main structure remained intact
2. Lake Lanier........	North Carolina	62	1926	*Jour. Electricity*, Vol. 44, April, 1920	Failure of foundation under abutment	Washout of foundation. Main structure remained intact

TABLE XXXVIII.—FAILURES OF MULTIPLE-ARCH DAMS

Dam	Location	Height, feet	Date of failure	Reference	Probable cause of failure	Probable nature of failure
1. Lake Gleno....................	Italy	143	1923	WEGMANN, "Design and Construction of Dams"	Defective design and construction	Sliding and collapse

TABLE XXXIX.—FAILURES OF ROCK-FILL DAMS

Dam	Location	Height, feet	Date of failure	References	Probable cause of failure	Probable nature of failure
1. English	California	131	1883	SCHUYLER, J. D., "Reservoirs"	Overtopping of dam	Washout of dam
2. Walnut Grove	Arizona	110	1890	WEGMANN, "Design and Construction of Dams"	Overtopping of dam	Washout of dam
3. Lake Avalon	New Mexico	58	1893	MERRIMAN AND WIGGIN, "American Civil Engineers' Handbook"	Overtopping of dam	Washout of dam
4. Lake Avalon	New Mexico	63	1904	MERRIMAN AND WIGGIN, "American Civil Engineers' Handbook"	Burrowing animals	Washout of dam
5. Zuni	New Mexico	70	1909	*Eng. News*, July 15, 1909	Defective foundation	Washout and settlement of dam
6. Lower Otay	California	130	1916	*Eng. News*, Feb. 3, 1916, and Feb. 17, 1916	Overtopping of dam	Washout of dam

*

1900 except the Zuni Dam which was built in 1907. They, therefore, did not conform with the present accepted principles of design and construction.

23-15. Causes of Failures.—As would be expected, the majority of failures of this type of dam has been due to overtopping, resulting from a lack of knowledge of the maximum floods to be expected in the streams on which they were built. Even at the present time, the maximum runoff of streams is not accurately known, and at the time these dams were constructed, even less data on this subject were available. These failures point to the fact that a large factor of safety should always be used in estimating the spillway capacity of a reservoir created by a dam which cannot withstand overtopping, and that ample freeboard should always be provided.

23-16. Nature of Failures.—It will be observed that all of the failures of rock-fill dams have been in the nature of washouts. It may be accepted as an axiom that once the water has attained a erosive velocity, either over or through a rock-fill dam, it will result in a complete failure of the structure, as the material will generally wash away faster than the water level of the reservoir falls.

FAILURES OF EARTH DAMS

23-17. List of Failures.—Table XL presents a list of 33 failures of earth dams. The list is not complete, as a very large number of minor structures might be added.

23-18. Causes of Failures.—The most common cause of failure of earth dams is overtopping. As in the case of rock-fill dams, if the water rises above the top of an earth dam, failure of the structure is almost certain, the erosion of the embankment taking place faster than the fall of the water surface, except in very small reservoirs. Another common cause of failure is shown to be the placing of conduits through the embankment. It is accepted practice to require conduits to be placed in cut rather than in fill, and to require ample number and size of collars around the conduits to act as cutoffs. Thorough puddling and

TABLE XL.—FAILURES OF EARTH DAMS

Dam	Location	Height, feet	Date of failure	References	Probable cause of failure	Probable nature of failure
1. Mill River	Massachusetts	43	1874	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Piping of foundation	Erosion
2. Linde Brook	Massachusetts	27	1876	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Leakage around culvert	Erosion
3. Staffordville	Connecticut	26	1877	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Leakage around culvert	Erosion
4. Lebanon	Ohio	30	1882	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Overtopping of dam	Erosion
5. Mud Pond	Massachusetts	15	1886	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Poor construction	Erosion
6. Johnstown	Pennsylvania	75	1889	*Eng. Record,* June 8 and 15, 1889	Overtopping of dam	Erosion
7. Spring Lake	Rhode Island	18	1889	*Eng. Record,* Aug. 31, 1889	Leakage around culvert	Erosion
8. Gunnison	California	20	1890	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Leakage along drain pipe	Erosion
9. Ansonia	Connecticut	....	1894	*Eng. Record,* Nov. 10, 1894	Leakage along drain pipe	Erosion
10. Melzingah, Reservoir 1 and 2	New York	30 and 24	1897	*Eng. Record,* July 17, 1897	Overtopping of dam	Erosion
11. Grand Rapids	Michigan	25	1900	*Eng. Record,* July 14, 1900	Overtopping of dam	Erosion
12. Wilmington	Delaware	....	1900	*Eng. Record,* Oct. 20, 1900	Leakage along drain pipe	Erosion
13. Middlefield	Massachusetts	20	1901	*Eng. Record,* May 4, 1901	Overtopping of dam	Erosion
14. Victor	Colorado	25	1901	*Eng. Record,* June 8, 1901	Overtopping of dam	Erosion
15. Bonney Irrigation Reservoir	Colorado	34	1903	*Eng. Record,* Apr. 25, 1903	Break in dam	Erosion
16. Shelton	Connecticut	....	1903	*Eng. Record,* Feb. 22, 1903	Burrowing animals and piping	Erosion
17. Jeanette	Pennsylvania	20	1903	*Eng. Record,* July 11, 1903	Overtopping	Erosion
18. Scottdale	Pennsylvania	60	1904	*Eng. Record,* Aug. 4, 1904	Soaking of embankment	Erosion
19. Norwick	New York	34	1905	*Eng. Record,* Sept. 14, 1905	Overtopping of dam	Erosion
20. Bridgeport	Connecticut	....	1905	*Eng. Record,* Aug. 12, 1905	Overtopping of dam	Erosion
21. Leroux Creek	Colorado	25	1905	*Eng. Record,* July 22, 1905	Overtopping of dam	Erosion
22. Ketterling	England	46	1905	*Eng. Record,* Sept. 30, 1905	Settlement of puddle core	Erosion
23. Yuba River	California	....	1907	*Eng. Record,* Aug. 8, 1907	Overtopping of dam	Erosion
24. Blue Water	New Mexico	35	1909	*Eng. Record,* Sept. 30, 1909	Overtopping of dam	Erosion
25. Zuni Black Rock	New Mexico	70	1909	*Eng. Record,* Dec. 2, 1909	Undermining of dam	Erosion
26. Dells and Hatfield	Wisconsin	....	1911	*Eng. Record,* Oct. 10, 1911	Overtopping of dam	Erosion
27. Toronto	Canada	35	1912	*Eng. Record,* Apr. 27, 1912	Overtopping of dam	Erosion
28. Dalton	New York	....	1912	*Jour. Electricity,* Vol. 44, Mar. 15, 1920	Failure of sand bank at one end of dam	Erosion
29. Lebanon	Pennsylvania	....	1912	*Eng. Record,* Aug. 24, 1912	Softening of embankment caused slide	Erosion
30. Colorado Springs	Colorado	....	1912	*Eng. Record,* Aug. 24, 1912	Seepage through porous embankment and slide	Erosion
31. Horse Creek	Colorado	65	1914	*Eng. Record,* Feb. 14, 1914	Seepage under dam	Erosion
32. Hebron	New Mexico	56	1914	....	Leakage through gopher holes	Erosion
33. Turlock Irrigation District	California	39	1914	*Eng. Record,* July 9, 1914	Leakage around outlet structure	Erosion

compacting of the back-fill around such outlets are essential to prevent seepage. Neglect of these precautions is the probable cause of most of such failures of earth dams. The list indicates that poor construction, faulty foundation treatment, and miscellaneous causes such as burrowing animals are occasionally responsible for failure. Attention to modern principles in construction will generally obviate such difficulties.

23-19. Nature of Failures.—It will be observed that all failures listed have been in the nature of erosion. From the nature of this type of dam, this is almost invariably the case.

23-20. Accidents during Construction.—Earth dams, and especially those constructed by the hydraulic-fill process, are subject to accidents during construction. These accidents, when conservative face slopes are used, consist mainly of slides due to internal liquid pressure in the hydraulic-fill type, or to an overplastic condition of the material in the rolled-fill type. As a matter of fact, very few accidents occur during construction except in the hydraulic-fill type. The slides in this type may be prevented by the use of coarser, thicker, and more porous drainage flanks adjacent to the faces of the dam. It is evident that slides in the rolled-fill type may be prevented by limiting the clayey or fine materials in the central part of the dam, by using coarser materials on the flanking portions, and by using water sparingly. The amount of water required is that sufficient to so saturate the material as to secure thorough compacting in the rolling process.

*

APPENDIX I

RUNOFF AND RAINFALL TABLES

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed, Maximum and date	Expected peaks: Frequent	Expected peaks: Rare
1	Wailua. South Fork, Oahu, Hawaii	0.23	1,955, Jan., 1916		
2	Beacon Creek, near Fishkill, N. Y.	0.25	3,200, ... 1897		
3	Bull's Run, Long Level, Pa.	0.58	4,170, July, 1914		
4	Docker's Hollow, North Braddock, Pa.	0.60	4,000, June, 1917		
5	Mann's Run, Creswell Station, Pa.	0.67	2,540, July, 1914		
6	Manoa, West Branch, Oahu, Hawaii	1.00	3,250, Jan., 1916		
7	Manoa, East Branch, Oahu, Hawaii	1.10	2,810, Jan., 1921		
8	China Wash, near Hurricane, Utah	1.10	500, Aug., 1916	300	600
9	Budlong Creek, near Utica, N. Y.	1.13	120, ... 1904		
10	Sylvan Glen Creek, New Hartford, N. Y.	1.18	57, ... 1904		
11	Mad Creek, Leroy, N. Y.	1.5	2,300		
12	Nuuanu Reservoir No. 4, Hawaii	1.5	1,600, Feb., 1907		
13	Pequest Hunt's Pond, N. J.	1.7	25, ... 1904		
14	Green Branch, Bridgeville, Pa.	1.7	1,595, July, 1914		
15	South Arroyo, near Pueblo, Colo.	1.8	1,060, June, 1921		
16	Provo (headwaters), Wall Lake, Utah	2.0	350, June, 1911	200	400
17	Cherryvale Creek, Cherryvale, Kans.	2.0	930		
18	Indian Run, Letort, Pa.	2.1	1,930		
19	Canodochly Branch, East Prospect, Pa.	2.2	1,630, July, 1914		
20	Rio Grande, near Culebra, Panama	2.4	161		
21	Colvin Run, Grindstone, Pa.	2.7	480, July, 1912		
22	San Pedro Creek, San Antonio, Tex.	2.7	750, Sept., 1921		
23	Canodochly Creek, Branch, Pa.	3.2	1,120, July, 1914		
24	Starch Factory Creek, N. Y.	3.4	209, Sept., 1905	130	
25	Provo, Washington Lake, Utah	3.4	300, June, 1911	200	350
26	Estanguela, near Monterey, Mexico	3.5	825, Aug., 1909	300	700
27	Switzer Canyon, San Diego, Calif.	3.5	188, Jan., 1916		
28	Nuuanu, U. S. Geological Survey Gage, Hawaii	3.5	952, Jan., 1916		
29	Venison Branch, near Mulga, Ala.	3.9	53		
30	North Canyon, near Centerville, Utah	4.0	450, Aug., 1923	50	500
31	Reel's Creek, near Dearfield, N. Y.	4.4	670, ...	55	
32	Kaukonahua, Upper Dame, Hawaii	4.5	1,605, Jan., 1916		
33	Mill Brook, Sherbourne, N. Y.	5.0	262, ... 1905		
34	Hall's Gulch, Boise, Idaho	5.0	1,000, July, 1913		
35	Breakbeck Run, Bullskin Tp., Pa.	5.2	310, May, 1902		
36	Kaneohe, Pali, Oahu, Hawaii	5.3	2,070, Jan., 1916		
37	Brush Creek, Jeannette, Pa.	6.0	500, July, 1903		
38	Skinner Creek, Mannsville, N. Y.	6.4	124, July, 1891		
39	Cold Spring, Brook, Mass.	6.4	48, ... 1886		
40	Blue Ribbon Creek, Pueblo, Colo.	6.7	1,360, June, 1921		
41	Honey Creek, East Fork, New Carlisle, Pa.	6.7	2,210, July, 1918		
42	Farmington Creek, Farmington, Utah	7.0	350, Aug., 1923		
43	Templeton Gap, Colorado Springs, Colo.	7.1	862, May, 1922		

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Fre-quent	Rare
44	Cameron Arroyo, near Pueblo, Colo.	7.3	1,900, June, 1921		
45	Camp Branch, Ensley, Ala.	7.4	68.8, ... 1909		
46	Osteen Arroyo, near Pueblo, Colo.	7.8	1,160, June, 1921		
47	Croton, South Branch, N. Y.	7.8	74, ... 1869		
48	Burgoon's Run, Pa.	8.0	400, May, 1894		
49	Independence Creek, Calif.	8.5	34, ...	27	
50	Israel, Jefferson Highlands, N. H.	8.7	63, ...	41	
51	Arroyo, Indiole, N. Mex.	8.9	1,105, July, 1915		
52	Fred Rohr Gulch, Pueblo, Colo.	9.3	104, June, 1921		
53	Woodhall, Reservoir, Herkimer, N. Y.	9.4	78, ... 1869		
54	Mill Brook, near Sherbourne, N. Y.	9.4	241, Sept., 1905		
55	Dry Run, Ohio	9.8	460, ... 1912		
56	Baker Creek, Baker, Nev.	10.0	17, ... 1914		
57	Spring Creek, Harrisburg, Pa.	11.6	259, Feb., 1908		
58	Honey Creek, East Fork, New Carlisle, Pa.	11.8	1,285		
59	Boulder Creek, near Julian, Calif.	12.0	217, Jan., 1916		
60	Mill Creek, Erie, Pa.	12.9	1,000, Aug., 1915		
61	Goodyear Creek, Goodyear Bar, Calif.	12.2	97		
62	Stony Brook, Boston, Mass.	12.7	121		
63	Tomer, above Reservoir, Holyoke, Mass.	13.0	52, ...	33	
64	Manhan River, Holyoke, Mass.	13.0	182, Feb., 1900		
65	Connoquenessing Creek, Oakland, Pa.	13.6	315, Aug., 1903		
66	Chandler Creek, near Pueblo, Colo.	13.6	118, June, 1921		
67	Canodochly Creek near Mouth, Pa.	13.9	359, July, 1914		
68	Great River, Westfield, Mass.	14.0	71		
69	Panther Creek, Iowa	14.0	520, June, 1905		
70	Grand Central, below Forks, Alaska	14.6	100, ... 1906		
71	Rocky Creek, near Ellisville, Miss.	15.0	1,110, May, 1882		
72	North Arroyo, near Pueblo, Colo.	15.6	619, June, 1921		
73	Rio des Perca, Clayton Road, St. Louis, Mo.	15.6	410, Aug., 1915		
74	Broad Creek, Millgreen, Md.	16.4	31, ...	17	
75	Arroyo Seco, near Pasadena, Calif.	16.4	192, Jan., 1916		
76	Williamstown, Williamstown, N. Y.	16.5	34		
77	Alazon Creek, San Antonio, Tex.	17.1	1,515, Sept., 1921		
78	Waiawa Waste, Oahu, Hawaii	17.5	343, Jan., 1910		
79	Ford Creek, Augusta, Mont.	18.0	40, ...	22	
80	Little Devil's Creek, Iowa	19.0	560, June, 1905		
81	Yuba, Bowman Dam, Calif.	19.0	32		
82	City Creek, Salt Lake City, Utah	19.2	8, ...	6	50
83	Rush Creek, near Pueblo, Colo.	19.6	238, June, 1921		
84	Martinez Creek, San Antonio, Tex.	19.6	1,223		
85	Ridley Creek, near Philadelphia, Pa.	20.0	750, Aug., 1843		

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile			
			Observed		Expected peaks	
			Maximum and date		Frequent	Rare
86	Chase Creek of Gila River, Ariz	20.0	647	, Dec., 1906		
87	Willow Creek, near Heppner, Ore	20.0	1,800	, June, 1903		
88	Croton, West Branch, N. Y	20.5	54	, ... 1874		
89	Beaverdam Creek, Altmar, N. Y	20.7	111			
90	Israel, below South Branch, N. H	21.2	49	, ...	34	
91	Mill Creek, Salt Lake City, Utah	21.3	6	, ...	4	30
92	Brush Hollow Creek, Pueblo, Colo	21.9	243	, June, 1921		
93	Crum Creek, near Philadelphia, Pa	22.0	410	, Aug., 1843		
94	Cane Creek, Bakersville, N. C	22.0	1,341	, May, 1901		
95	Penitencia Creek, San Jose, Calif	22.0	....		40	59
96	Coal Creek, near Pueblo, Colo	22.3	167	, June, 1921		
97	Dry Run, Decorah, Iowa	22.3	720	, Mar., 1915		
98	Ritchie Gulch, near Pueblo, Colo	22.6	41	, June, 1921		
99	Trout Brook, Centerville, N. Y	23.0	51			
100	Wailua, near Lihue, Kanai, Hawaii	23.0	1,960	, Jan., 1916		
101	Rio des Perca, near St. Louis, Mo	23.8	256	, Aug., 1915		
102	Apache Creek, San Antonio, Tex	23.8	948	, Sept., 1921		
103	Mill, North Fork, Pinkbed, N. C	24.0	49	, ...	32	
104	Six Mile Creek, near Pueblo, Colo	24.6	77	, June, 1921		
105	Trout Brook, Brooksport, N. Y	25.0	158			
106	Pequonnock, Bridgeport, Conn	25.0	157	, July, 1905		
107	Middle Creek, Bozeman, Mont	26.0	31	, ...	18	50
108	Smith Creek, Augusta, Mont	26.0	31	, ...	23	60
109	Olmos Creek, San Antonio, Tex	26.0	948	, Sept., 1921		
110	Boggs Creek, near Pueblo, Colo	26.0	582	, June, 1921		
111	Spring Creek, above Dayton, Ohio	27.0	210	, Mar., 1913		
112	Bear Grass Creek, Louisville, Ky	27.5	100	, ... 1908		
113	Donnels Creek, above Dayton, Ohio	27.5	147	, Mar., 1913		
114	Little Cottonwood, Salt Lake City, Utah	27.7	25	, June 1912	13	40
115	Cottonwood Creek, near Fresno, Calif	28.0	....		20	26
116	Wantuppa Lake, Fall River, Mass	28.5	72	, ... 1875		
117	Emigration Creek, Salt Lake City, Utah	29.0	6	, ...	3	20
118	American Fork, above Power Plant, Utah	30.0	12	, May 1914	10	50
119	Pinal Creek, Globe, Ariz	30.0	440	, Aug., 1904		
120	Arroyo Seco, near Pasadena, Calif	30.5	374	, May, 1914		
121	Donner Creek, near Truckee, Calif	30.5	32	, ...	19	34
122	Moosa Creek, near Bonsall, Calif	31.0	269	, Jan., 1916		
123	Pequest, Huntsville, N. J	31.4	19			
124	Ink's Creek, near Red Bluff, Calif	34.0	....		120	153
125	San Antonio, San Antonio, Tex	34.3	691	, Sept., 1921		
126	Peck's Creek, near Pueblo, Colo	34.4	564	, June, 1921		
127	Sawkill, near Mouth, N. Y	35.0	229	, Apr., 1895		
128	Chalk Creek, Fillmore, Utah	38.0	13	, ... 1914		

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
129	Whippany River, Whippany, N. J.	38.0	84 , ... 1896		
130	Turtle Creek, above Dayton, Ohio	35.0	175 , Mar., 1913		
131	San Francisquito Creek, Calif.	38.0	....	51	62
132	Lake Roland, Md.	39.0	230 , ... 1868		
133	Cuyadutta, Johnstown, N. Y.	40.0	72 , ... 1896		
134	Mill, South Fork, Sitton, N. C.	40.5	51 , Jan., 1916	39	
135	Red Creek, near Pueblo, Colo.	40.6	22 , June, 1921		
136	San Pablo Creek, Berkeley, Calif.	41.0	....	38	50
137	Little Johns Creek, near Stockton, Calif.	41.0	....	70	107
138	Bassic Creek, Donedrold, N. Y.	41.0	81		
139	Westfield Little, Blandford, Mass.	43.0	50 , ...	34	
140	Little Gun Powder, Bel Air, Md.	43.0	37 , ...	20	
141	Sweetwater River, Descano, Calif.	43.7	326 , Jan., 1916		
142	Elkhorn Creek, Keystone, W. Va.	44.0	1,363 , June, 1901		
143	Cameron Creek, Hurley, N. Mex.	44.0	135 , Aug., 1913		
144	San Leandro Creek, Oakland, Calif.	44.0	....	38	50
145	Six Mile Creek, Ithaca, N. Y.	46.0	185 , June, 1905		
146	San Pedro Creek, below Apache Creek, Tex.	46.5	698 , Sept., 1921		
147	Wallowa, Joseph, Ore.	47.0	16 , ...	14	
148	Dog River, Northfield, Vt.	47.0	72 , ... 1913		
149	West Canada Creek, Mott's Dam, N. Y.	47.5	34		
150	Dry Creek, near Fresno, Calif.	48.0	....	42	51
151	Pine Creek, Paris, Tex.	48.0	410 , May, 1920		
152	Prosser Creek, Hobart Mills, Calif.	48.0	19 , ...	11	
153	Darby Creek, near Philadelphia, Pa.	48.0	580 , Aug., 1843		
154	Turkey Creek, near Pueblo, Colo.	48.0	188 , June, 1921		
155	Big Cottonwood, Salt Lake City, Utah	48.5	17 , ...	9	30
156	Conemaugh Branch (Johnstown flood), Pa.	48.6	206 , May, 1889		
157	Parley's Creek, Salt Lake City, Utah	50.1	6 , ...	3	20
158	Sangquoit Creek, New York Mills, N. Y.	51.5	53		
159	Kosh Creek, near Henderson, Calif.	51.9	44		
160	Cottonwood Creek, North Fork, Ono, Calif.	52.0	78		
161	Lost Creek, above Dayton, Ohio	52.0	571 , Mar., 1913		
162	Guadalupe River, Calif.	52.0	....	55	75
163	Rockaway, Dover, N. J.	52.5	53		
164	Santa Ysabel Creek, Mesa Grande, Calif.	53.4	395 , Jan., 1916		
165	Tawawa Creek, above Dayton, Ohio	54.0	239 , Mar., 1913		
166	Yakima, Martin, Wash.	56.0	11 , ...	5	
167	Santa Maria Creek, Ramona, Calif.	57.3	125 , Jan., 1916		
168	Oneida Creek, Kenwood, N. Y.	59.0	41 , ... 1890		
169	Rock Creek, near Pueblo, Colo.	59.0	913 , June, 1921		
170	Wildcat Creek Group, Stockton, Calif.	59.0	....	70	97
171	Lytle Creek, San Bernardino, Calif.	60.0	267 , Jan., 1916		

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed: Maximum and date		Expected peaks: Frequent	Expected peaks: Rare
172	Flat, R. I.	61.0	120	, ... 1843		
173	Camden Creek, Camden, N. J.	61.4	24	, ... 1889		
174	Pequannock, Macopin, N. J.	62.0	91			
175	Chester Creek, near Philadelphia, Pa.	62.0	1,000	, Aug., 1843		
176	Los Angeles, near Los Angeles, Calif.	62.0	117	, Jan., 1916		
177	Nine Mile Creek, Slittville, N. Y.	62.6	125	, ... 1898		
178	Kachess, near Easton, Wash.	63.0	36	, ...	24	
179	Naches, Niles, Wash.	63.6	341	, ...	137	
180	Wissahickon Creek, Philadelphia, Pa.	64.6	43	, ... 1898		
181	Eight Mile Creek, near Pueblo, Colo.	65.0	154	, June, 1921		
182	Ludlow Creek, above Dayton, Ohio	65.0	266	, Mar., 1913		
183	American Fork, American Fork, Utah	66.0	13	, ...	7	
184	San Jacinto, at Hemet Reservoir, Calif.	65.8	145	, Jan., 1916		
185	Owens Creek Group, near Merced, Calif.	66.0	....	....	33	44
186	Daulton Creek Group, Calif.	66.0	....	....	40	51
*187	Carson, West Fork, Woodfords, Calif.	67.0	23	, May, 1906		
188	Oak Creek, near Pueblo, Colo.	68.0	41	, June, 1921		
189	Sandy Creek, Allendale, N. Y.	68.4	89	, ... 1891		
190	Jamul Creek, near Otay, Calif.	69.8	259	, Jan., 1916		
191	Bear Creek, near Madera, Calif.	71.0	....	....	36	50
192	Dutchman Creek Group, Madera, Calif.	72.0	....	....	40	54
193	Laramie Reservoir Outlet, Laramie, Wyo.	72.0	97	, Mar., 1913		
194	Butte Creek, Butte Valley, Calif.	73.0	23	, ...	16	
195	Brushy Creek, Round Rock, Tex.	74.7	462	, Sept., 1921		
196	San Vincent Creek, Foster, Calif.	74.9	248	, ... 1916		
197	Mission Creek Group, San Jose, Calif.	77.0	....	....	43	59
198	Rock Creek, Washington, D. C.	77.5	126			
199	Sonoma Creek Group, Sonoma, Calif.	78.0	....	....	52	66
200	Sudbury, Farmington, Mass.	78.0	41	, ... 1897		
201	Pequannock, Pompton, N. J.	78.0	56	, ... 1902		
202	Hockanum River, Conn.	79.0	78			
203	Chittenango Creek, Chittenango, N. Y.	79.0	24	, ...	17	
204	Dry Creek, near Auburn, Calif.	79.0	....	....	100	124
205	Bear Creek, near Cape Mendocino, Calif.	82.0	....	....	130	162
206	Claremont Creek Group, Claremont, Calif.	83.0	....	....	40	50
207	Lagunitas Creek, near Sansalito, Calif.	84.0	....	....	32	42
208	San Mateo Creek Group, Calif.	84.0	....	....	44	58
209	Kra'z Gamepa Outlet, Salmon, Alaska	84.0	51	, ... 1902		
210	Nashua River, Mass.	84.5	71	, ... 1850		
211	San Antonio, below San Pedro Creek, Tex.	85.0	499	, Sept., 1921		
212	Roach Roach, Me.	85.0	23	, ...	19	
213	Dry Creek, near Pueblo, Colo.	86.0	283	, June, 1921		

* Maximum daily flow.

Table II.—Maximum Observed Flood Discharge Rates and Expected Runoff from Various Drainage Basins Arranged According to Area.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile			
			Observed		Expected peaks	
			Maximum and date		Frequent	Rare
214	Willow Creek, near Augusta, Mont.	90.0	42	, May, 1917		
215	Gallinas, Hot Springs, N. Mex.	90.0	129	, Sept., 1904		
216	Putah Creek, near Guenoc, Calif.	91.0	270	, Mar., 1904		
217	Independence Creek, Crandall, N. Y.	93.2	66	, ... 1869		
218	Boulder, Bruffeys, Mont.	94.0	17	, ...	16	
219	Asay Creek Hatch, Utah	96.0	17	, ... 1913		
220	North Fork Creek, Brookville, Pa.	97.0	124	, July, 1912		
221	Malabu Creek, Cabasas, Calif.	97.0	70	, ...	27	
222	Yokohl Creek Group, near Tulare, Calif.	98.0	....	....	30	38
223	Otay, Lower Otay Dam, Calif.	98.6	379	, Jan., 1916	40	300
224	Churn Creek Group, Calif.	100.0	100	, ...	100	133
225	Passaic, Chatham, N. J.	101.0	30	, ...	20	
226	Wanague, Pompton, N. J.	101.0	84	, ...	.....	24
227	Deer River, Deer River, N. Y.	101.0	78	, ... 1869		
228	Swift Current, near Babb, Mont.	101.0	45	, ...	22	
229	Tolrickon Creek, Mt. Pleasant, Pa.	102.0	138	, ... 1894	41	
230	Little Stony Creek, near Ladaga, Calif.	102.0	69			
*231	Mariposa Creek, near Merced, Calif.	103	....	....	43	56
232	Fish Creek, East Branch, Point Rocks, N. Y.	104	805	, ... 1897		
233	Valley, Tomotia, N. C.	106	97	, ...	47	
234	San Jacinto River, near San Jacinto, Calif.	108	278	, Jan., 1916		
235	Onondago Creek, Syracuse, N. Y.	108	30	, ... 1913		
236	Nashua, Mass.	109	104	, ... 1848		
*237	Red Bank Creek Group, near Tulare, Calif.	109	....	....	67	89
238	Whiterocks, Whiterocks, Utah	110	10	, ...	.....	6
*239	Deer Creek, near Red Bluff, Calif.	110	....	....	32	38
240	Sandy Creek, North Branch, Adams, N. Y.	110	67	, ... 1897		
241	Santa Ysabel Creek, near Ramona, Calif.	110	258	, Jan., 1916		
242	Lewiston Reservoir, Outlet, Ohio	111	58			
243	Sweetwater River, near Dehesa, Calif.	112	217	, Jan., 1916		
244	Rockaway, Boonton, N. J.	118	49			
245	Ohanapecosh, near Lewis, Wash.	116	65	, ... 1909		
246	Laurel Hill Creek, Confluence, Pa.	118	42	, ...	25	
247	Scantic, North Branch, Conn.	118	52			
248	Ramapo, Mahwah, N. J.	118	105	, ... 1903		
249	Clear Creek, Buffalo, Wyo.	118	7	, ...	6	
250	Antelope Valley Group, near Los Angeles, Calif.	119	....	....	54	87
251	Los Gatos Creek, near Hanford, Calif.	119	....	....	27	38
252	Cottonwood Creek, Moorena, Calif.	119	128	, Jan., 1916		
253	Hobble Creek, Springville, Utah	120	7	, ...	3	
254	Los Gatos Creek Group, near Redwood, Calif.	121	....	....	46	62
255	Martells Creek Group, near Stockton, Calif.	122	....	....	70	97

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed: Maximum and date	Expected peaks: Frequent	Expected peaks: Rare
256	Suisun Creek Group, near Suisun, Calif.	125	....	43	52
257	Rockaway, Boonton, N. J.	125	22 , ... 1902		
258	South Boulder, Marshall, Colo.	125	9 , ...	5	
259	Santa Ysabel Creek, near San Diego, Calif.	126	....	80	154
260	Laurel Hill Creek, Confluence, Pa.	126	40		
261	Patuxent, Laurel, Md.	127	40 , ... 1915		
262	Walnut Canyon, near Flagstaff, Ariz.	128	82 , Sept., 1923		
263	Fresno River, near Knowles, Calif.	134	34 , Feb., 1917		
264	Bear Creek Group, Calif.	137	....	100	131
265	Cedar, Ravensdale, Wash.	138	91 , ...	35	
266	San Antonio Creek, near Santa Barbara, Calif.	138	....	53	73
267	Petaluma Creek Group, near Petaluma, Calif.	139	....	46	57
268	Union River (West Branch), Maine	140	50 , Apr., 1923	15	
269	Neshaminy Creek, below Forks, Pa.	140	98 , ... 1894	33	
270	Dead, Forestville, Mich.	142	17 , ...	16	
271	Devil's Creek, near Viele, Iowa	143	600+ , June, 1905		
272	Oriskany Creek, Oriskany, N. Y.	144	51		
273	Turtle Creek, East Pittsburgh, Pa.	146	64		
274	Musconetcong, Bloomsburg, N. Y.	146	19 , ...	13	
275	Salado Creek, Salado, Tex.	148	966 , Sept., 1921		
276	Salmon Creek, Malott, Wash.	152	4 , ...	3	
277	Perkiomen Creek, Frederick, Pa.	152	116 , ... 1894	35	
278	Mohawk, Ridge Mills, N. Y.	153	46		
279	Bolinas Creek Group, Bolinas Bay, Calif.	158	....	20	26
280	Soquee, Demorest, Ga.	158	56 , ...	36	
281	Mora River, below Mora, N. Mex.	159	140 , Sept., 1904		
282	Gunpowder Falls, Glencoe, Ind.	160	35 , ...	34	
283	Ramapo, Pompton, N. J.	160	66 , ... 1882		
284	Kinzua Creek, Dew Drop, Pa.	162	20		
285	Weber, Oakley, Utah	163	25 , ...	17	
286	Mono Lake Group, Calif.	166	....	20	25
287	Little Truckee, Starr, Calif.	166	11 , ...	9	
288	Los Angeles River, Tributary, Calif.	167	....	70	130
289	Fish Creek, East Branch, Taberg, N. Y.	169	65 , ... 1913		
290	Cedar, Ravensdale, Wash.	170	64 , ...	27	
291	Burns Creek Group, Merced, Calif.	171	....	50	66
292	Sweetwater River, Jamacho, Calif.	172	250 , ... 1916		
293	Hardscrabble Creek, Pueblo, Colo.	173	19 , June, 1921		
294	St. Mary, near Babb, Mont.	177	45 , ...	19	
295	Backbone Creek Group, Red Bluff, Calif.	178	....	80	120
296	Bear Creek, Morrison, Colo.	180	48 , July, 1896		
297	Fish Creek, West Branch, McConnellsville, N. Y.	187	33 , ... 1885		
298	Smoke Creek Group, near Honey Lake, Calif.	188	....	10	11

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile			
			Observed		Expected peaks	
			Maximum and date		Frequent	Rare
299	Santa Ana River, Mentone, Calif	189	154	, Jan., 1916		
300	San Diego River, Lakeside, Calif	189	201	, Jan., 1916		
301	Seekonk, Providence, R. I	190	57	, ... 1867		
302	Tallulah, Tallulah Falls, Ga	191	41	, ...	......	26
303	Coyote River, Madrone, Calif	197	127	, Mar., 1911		
304	Fishkill Creek, at Glenham, N. Y	198	69			
305	Santa Ana River Tributaries, Mentone, Calif	199	146	, Jan., 1916		
306	Williams Fork, Sulphur Springs, Colo	200	7	, ...	5	
307	Canyon Padre, Diablo Jct., Ariz	200	55	, Sept., 1923		
308	Mt. Diablo Creek, Martinez, Calif	200	......	....	47	63
309	Cave Creek, Phoenix, Ariz	200	125	, Aug., 1921		
310	Limekiln Creek, Tulare, Calif	201	......	....	30	38
311	Unadilla, New Berlin, N. Y	204	40	, ... 1905		
312	Cle Elum Lake, Roslyn, Wash	205	86	, ...	36	
313	Santa Yuez, Santa Barbara, Calif	207	46	, ... 1907		
314	San Diego River, San Diego, Calif	207	72	, ...	......	10
315	Cantua Creek, Hanford, Calif	208	......	....	20	32
316	San Luis Rey River, Mesa Grande, Calif	209	280	, Jan., 1916		
317	St. Vrains Creek, Lyone, Colo	209	6	, ...	5	
318	Coon Creek, Sacramento, Calif	210	......	....	70	89
319	Catskill Creek, South Cairo, N. Y	210	100	, ... 1901		
320	Mojave River, Victorville, Calif	211	64	, Mar., 1903	65	109
321	French Creek, North Branch, Kimmeytown, Pa	212	43			
322	Beaver Creek, Pueblo, Colo	213	45	, June, 1921		
323	Arroyo Seco, Soledad, Calif	215	62	, ...	19	
324	Owens Lake Group, Calif	216	......	....	21	24
325	Sespe Creek, Sespe, Calif	216	86	, Jan., 1916		
326	Uinta, White Rocks, Utah	218	10	, ...	5	20
327	Logan, Logan, Utah	218	11	, ...	7	30
328	Frazler, Granby, Colo	220	9	, ...	8	
329	Sapella, Los Alamos, N. Mex	221	37	, ... 1904		
330	Salmon, Altmar, N. Y	221	28			
331	San Gabriel, Azusa, Calif	222	212	, Feb., 1884	50	250
332	Pescadero Group, San Mateo Co., Calif	222	......	....	35	48
333	Napa River Tributaries, Napa, Calif	226	......	....	45	61
334	Ventura River, Ventura, Calif	226	......	....	70	87
335	Smith River, Crescent City, Calif	227	187	, Nov., 1915		
336	Alcovy, near Covington, Ga	228	10	,		
337	Salmon Creek Group, Sonoma, Calif	230	......	....	25	32
338	Tocca, near Blueridge, Ga	231	53	, ... 1901	150	300
339	Chowchilla River, Merced, Calif	238	......	....	35	44
340	Salinas River Tributaries, Soledad, Calif	238	80	, Jan., 1916		
341	Esopus Creek, Olive Bridge, N. Y	239	64	, ...	......	130

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
342	Schoharie, Prattsville, N. Y.	240	55 ,		144
343	Cobbossecontee Stream, Gardiner, Me.	240	14 ,		8
344	Jalama Creek, Santa Barbara, Calif.	242			60
*345	West Walker River, Coleville, Calif.	245	17 , July, 1907		
346	South, Port Republic, Va	246	37		
347	Mattole River, New Petrolia, Calif.	249	223		
348	Patapsco, Woodstock, Md.	251	44 ,		28
349	Crow Creek, Cheyenne, Wyo.	251	37 , May, 1904		
350	Clear Creek, Redbluff, Calif.	251		70	105
351	Butte Creek Group, Oroville, Calif.	251		100	134
352	Susan, Susanville, Calif.	256	7 ,	5	
353	East Canada Creek, Dolgeville, N. Y.	256	54 , ... 1913	22	
354	Cahokia Creek, Poag, Ill.	259	14		
355	Salmon, Pulaski, N. Y.	260	42 ,	38	
356	Bear River, Van Trent, Calif.	262	336 , Feb., 1907	100	400
357	Mattole, Cape Mendocino, Calif.	264		130	164
358	Tule, Portersville, Calif.	264	25 , Jan., 1914	12	
359	Black, Forestport, N. Y.	268	39		
360	Whitewater, Whitewater, Calif.	269		40	67
361	Miller's Creek, Lovella, Ore.	270	25 , ... 1907		
362	Fresno, near Fresno, Calif.	270		36	46
363	Cottonwood Creek, Jamul, Calif.	270	22 ,	7	
364	Notteley, Ranger, N. C.	272	21 ,	15	
365	Choccolocco Creek, Jenifer, Ala.	272	43 ,	19	
366	Chewaucan, Paisley, Ore.	272	13 ,	6	
367	Navarro River, Ukiah, Calif.	273		54	67
368	Redwood Creek, Eureka, Calif.	275	57		
369	Goose Lake, Goose Lake, Calif.	275		12	13
370	St. Regis, St. Regis, Mont.	278	22 , ... 1913		
371	Crooked Creek, Hileman's Farm, Pa.	279	43		
372	San Gabriel, Tributary, Azusa.	280		65	95
373	Conejos, Mogote, Calif.	282	15 ,	10	
374	Sutter Creek Group, Lodi, Calif.	285		55	75
375	Piscataguis, Foxcroft, Me.	286	78 , ... 1909	43	
376	Black Fork, Hyrum, Utah.	286	7 ,	3	
377	Yamhill, Sheridan, Ore.	290	62 ,	53	
378	Brokenstraw Creek, Youngsville, Pa.	290	25		
379	Youghioheny, Friendsville, Md.	294	28		
380	Mora, Weber, N. Mex.	294	94 , ... 1904		
381	Panoche Creek, Mendota, Calif.	295		30	43
382	Antietam Creek, Sharpsburg, Ind.	295	23 , ... 1902	11	
*383	Carson, East Fork, Stateline, Nev.	298	12 , June, 1911		

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed, Maximum and date		Expected peaks: Frequent	Rare
384	San Dieguito, Bernardo, Calif.	299	241	, Jan., 1916		
385	Gunpowder River, Md.	302	83	, ... 1889		
386	Oil Creek, Rouseville, Pa.	302	28			
387	Big Thompson Creek, Loveland, Colo.	305	7	, ...	4	
388	Walker, W. Fork, Coleville, Calif.	306	14	, ...	7	
389	Honcut Creek, Oroville, Calif.	314	....		90	124
390	Rapid Creek, Rapid, S. Dak.	320	3	, ... 1904	2	
391	Carson, East Fork, Nev.	323	....		14	16
392	Soquel Creek Group, Santa Cruz, Calif.	324	....		50	73
393	Esopus, Kingston, N. Y.	324	63	, ...	42	
394	San Luis Rey, near Pala, Calif	325	231			
395	San Jacinto Tributary, Calif.	330	....		50	89
396	Zion Creek, near Springdale, Utah	330	11	, Aug., 1916	7	20
397	Silver Creek, near Lebanon, Ill.	335	16	, ... 1908		
398	Croton, Croton Dam, N. Y.	339	75	, ... 1867		
399	Canyon Diablo, Arch Bridge, Ariz.	340	104	, Sept., 1923		
400	Carrabassett, North Anson, Me.	340	40	, ...	27	
401	Bear Creek, at Forks, Colo.	345	7	, ...	4	
402	Moose, Moose River, N. Y.	346	20	, ...	17	
403	Great Westfield, Mass.	350	151	, ... 1878		
404	Umatilla, Gibbon, Ore.	353	28	, ...	10	
405	Westfield, Mass.	356	149	, ... 1878		
406	Ogden, Ogden, Utah	360	9	, ...	5	15
407	Battle Creek, Red Bluff, Calif.	366	....		90	121
408	West Canada Creek, Hinckley, N. Y.	372	105	, ... 1869	35	
409	Ocoee, McCays, Copperhill, Tenn.	374	48	, ...	19	
410	San Diego River, Santee, Calif.	375	187	, Jan., 1916		
411	Esopus Creek, Mt. Marion, N. Y.	378	65			
412	Surprise Valley Group, Cedarville, Calif.	379	....		9	11
413	Malibu River Group, Chatsworth, Calif.	379	....		60	77
414	Rondout Creek, Rosedale, N. Y.	380	51	, ...	35	
415	Pompton, Two Bridges, N. J.	380	62	, ... 1903		
416	Carson, East Fork, Gardnerville, Nev.	381	9	, ... 1904	6	
417	Saco, Center Conway, N. H.	385	36	, ...	27	
418	Roanoke, Roanoke, Va.	388	47	, ...	23	
419	Tule River, Tulare, Calif.	390	....		20	28
420	Fall, Fremont, Idaho	390	10	, ...	8	
421	Calaveras, Jenny Lind, Calif.	394	176	, Jan., 1911	170	229
422	Willow Creek Group, Colusa, Calif.	394	....		60	79
423	Middle Oconee, near Athens, Ga.	395	49	, ... 1902		
424	Pacolet, Spartanburg, S. C.	400	89	, ... 1903		
425	Mojave, below Victorville, Calif.	400	33	, ...	20	40
426	Blacklick Creek, Blacklick, Pa.	403	38	, ...	24	

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile			
			Observed		Expected peaks	
			Maximum and date		Frequent	Rare
427	Tygerts Valley, Belington, W. Va.	403	41	, ... 1902		
428	West Walker River, Calif.	405	....	....	20	27
429	Moose, Ayers Mill, N. Y.	407	31			
430	Watauga, Elizabethton, Tenn.	408	23	, ...	14	
431	Potomac, North Branch, Piedmont, W. Va.	410	33	, ...	20	
432	Hiwassee, Murphy, N. C.	410	54	, ... 1899	30	
433	East Walker River, Calif.	411	....	....	19	24
434	Mahoning Creek, Furnace Bridge, Pa.	412	30			
435	Carson, East Fork, Rodenbohs, Nev.	414	13	, ...	9	
436	Elder Creek Group, Hamilton, Calif.	414	....	....	47	63
437	Esopus, at Saugerties, N. Y.	417	132			
438	Buffalo Creek, New York	420	55	, ... 1902		
439	Oil Creek, near Pueblo, Colo.	423	6	, June, 1921		
440	Sangammon, South Fork, Taylorsville, Iowa	427	10			
441	Stony Creek, Johnstown, Pa.	428	70			
442	San Gabriel River, Georgeton, Tex.	431	371	, Sept., 1921		
443	Youghiogheny, Confluence, Pa.	435	55	, ...	27	
444	Laramie, Woods Landing, Wyo.	435	10	, ...	7	
445	Owens River, Round Valley, Calif.	439	3	, June, 1907		
446	Appalachie, Brickhead, Ga.	440	17	, ...	11	
447	Whetstone, Bigston, S. Dak.	441	3			
448	Battenkill, Greenwich, N. Y.	444	22			
449	Cow Creek, Red Bluff, Calif.	444	....	....	90	114
450	Missiquoi, below Richford, Vt.	445	23	, ... 1913		
451	Cherry Creek, Denver, Colo.	445	25	, July, 1912		
452	Bishops Creek Group, Owens Valley, Calif.	446	....	....	25	32
453	Whiteface, below Meadowlands, Minn.	446	13	, ... 1916		
*454	Truckee, Stateline, Calif.	447	33	, Mar., 1907		
455	Casselman, Confluence, Pa.	448	46	, ...	23	
456	Tionesta Creek, Nebraska, Pa.	451	20			
457	St. Mary, near Cardston, Canada	452	40	, ...	13	
458	Pine, below Pine River, Reservoir, Minn.	452	4	, ...	3	
459	Union River, near Junction, Me.	452	50	, Apr., 1923		
460	Adobe Meadows Group, Ownes Valley, Calif.	453	....	....	18	21
461	Chatanika, below Poker, Alaska	456	8	, ... 1911		
462	Mad River, Arcata, Calif.	457	42	, ... 1912	100	142
463	Santa Ana Tributaries, Calif.	460	....	....	50	128
464	Machias, Whitneyville, Me.	465	24	, ...	12	
465	Cattaraugus Creek, Versailles, N. Y.	467	53			
466	Lake Fork, Myton, Utah	475	6	, ...	4	
467	Caliente Creek, near Tehachapi, Calif.	471	....	....	30	48
468	St. Charles River, Pueblo, Colo.	482	149	, June, 1921		

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Fre-quent	Rare
469	Little Wolf, Royalton, Wis.	485	11 , ... 1914		
470	Ausable, Ausable Forks, N. Y.	487	45 , ... 1913		
471	Coquitlam, above Coquitlam Dam, B. C.	490	49 , ... 1921	25	50
472	Eagle Lake Group, Calif.	498	....	10	11
473	Lake Tahoe Basin, Calif.	499	....	20	34
474	Yadkin, North Wilkesboro, N. C.	500	36 , ...	25	
475	Cache Creek, Lower Lake, Calif.	500	8.7		
476	Grande, North Branch, Haley, N. Dak.	500	11.6 , ... 1913		
477	Yakima, Cle Elum, Wash.	500	51.2 , ... 1915	31	
478	Shoshone, South Fork, Marquette, Wyo.	500	10.6 , ...	7	
479	Deerfield, Shelburne Falls, Mass.	501	42.5 , ... 1909		
480	Feather, North Fork, Prattville, Calif.	506	19.5 , ...	12	
481	Kaweah, Three Rivers, Calif.	514	28.6 , Jan., 1916		
482	Olentangy, Columbus Ohio	514	98 , Mar., 1913	28	100
483	West Canada Creek, Middleville, N. Y.	518	24.9 , ... 1898		
484	Truckee, at Tahoe, Calif.	519	2.6 , ...	2	5
485	Dolores, Dolores, Colo.	524	5.6 , ...	4	
486	Owens River (Upper) Calif.	524	....	25	32
487	Price, Helper, Utah	530	8.5 , ... 1913		
488	Coosawattee, Carters, Ga.	531	31.9 , ... 1901	22	
489	Cache la Poudre, North Fork, Livermore, Colo.	532	38.0 , May, 1904		
*490	Cosumnes, Michigan Bar, Calif.	534	42.0 , Jan., 1911	40	55
491	Los Angeles, Tributaries, Los Angeles, Calif.	534	71.2 , Dec., 1889	50	100
492	Canyon Diablo, Leupp, Ariz.	544	82 , Sept., 1923	30	90
493	Santa Catarina, Monterey, Mexico	544	432 ± , Aug., 1909		
494	Occoquan Creek, Occoquan, Va.	546	38.3 , ... 1915		
495	Madeline Plains Group, N. E. Calif.	548	....	9	11
496	Conewanger, Frewsburg, N. Y.	550	20.9		
497	Uncompahgre, Montrose, Colo.	565	3.7 , ...	3	
498	San Luis Rey, Oceanside, Calif.	565	169.0 , Jan., 1916		
499	Sacramento, above Pitt River, Calif.	568	....	60	101
500	Poso Creek Group, Tulare, Calif.	576	....	20	26
501	Stony Creek, Fruto, Calif.	577	50.8 , Feb., 1909	28	
502	Hoosic, Buskirk, N. Y.	579	23.6 , ...	18	
503	Farmington, Conn.	584	41.7		
504	Tugaloo, near Madison, S. C.	593	36.9 , ...	26	
505	Sun, North Fork, Augusta, Mont.	600	54.0 , June, 1916		
506	Fort Pierce, Wash, Utah-Arizona Boundary	600	6.5 , Aug., 1909	4	10
507	Etowah, Canton, Ga.	604	31.5 , ... 1895	23	
508	Hoosick, Johnsonville, N. Y.	605	38.0 , ... 1913		
509	Pennigewasset, Plymouth, N. H.	615	49.8 , ...	27	
510	Gualala Group, Santa Rosa, Calif.	623	....	45	59

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
511	Smith River, N. W. Calif.	627	....	160	204
*512	Des Plaines, Riverside, Ill.	630	20.8, ... 1889		
513	Mokelumne, near Clements, Calif.	632	32.5, Jan., 1911	13	
514	Saranac, Plattsburg, N. Y.	634	7.5, ...	7	
515	White, Mecker, Colo	634	5.8, ...	5	
*516	Alameda Creek, Sunol Glenn, Calif.	639	23.0, Mar., 1911		
517	Provo, Provo, Utah	640	6.4, ...	4	
518	Sandy, Madison, Me.	650	21.2, ...	15	
519	Pigeon, Newport, Tenn.	655	30.9, ...	15	
520	Putah Creek, Winters, Calif.	655	91.6, Dec., 1913	28	
521	Monocacy, near Frederick, Md.	660	31.0, ... 1902	22	
522	Grande Ronde, Hilgard, Ore.	660	7.0, ...	5	
523	Milwaukee, Milwaukee, Wis.	661	8.0, ... 1915		
*524	Russian, Geyserville, Calif.	662	24.9		
525	Tuckasegee, Bryson, N. C.	662	58.2, ... 1889	34	
526	McCloud, Baird, Calif.	669	82.2, Feb., 1904	33	
527	Spanish Fork, Spanish Fork, Utah	670	2.9, ...	2	
528	Uinta, Fort Duchesne, Utah	672	6.7, ...	4	
529	Black, Neillsville, Wis.	675	34.2, ...	15	
530	Little Tennessee, Jackson, N. C.	675	85.3, ...1901	40	
531	Delaware, West Branch, Hancock, N. Y.	680	49.6, ...	28	
532	Santa Margarita, near San Diego, Calif.	690	....	40	67
533	Willamette, Coast Fork, Goshen, Ore.	690	45.4, ...	30	
534	Blue, Kremmling, Colo.	700	8.6, ...	8	
535	Moye, Snider, Idaho	717	11.1, ... 1913		
*536	San Jacinto, Elsmore, Calif.	718	19.5, Jan., 1916		
537	Trinity, Lewiston, Calif.	718	37.4, Jan., 1914		
538	Racquette, Piercefield, N. Y.	723	8.1		
539	Salmon, Somes Bay, Calif.	734	44.9, Feb., 1912		
540	Big Muddy, near Cambon, Ill.	735	15.0		
541	Indian Creek, Crescent Mills, Calif.	740	15.4, ...	9	
542	Purgatoire, Trinidad, Colo.	742	61.0, Sept., 1904		
543	West Fork, Enterprise, W. Va.	744	23.7, ... 1911		
544	Appomatox, Mottoax, Va.	745	15.7, ...	11	
545	Santa Ynez, near Lompoc, Calif.	750	55.3, Feb., 1915		
546	Chico Creek, near Pueblo, Colo.	750	38.0, June, 1920		
547	Huron, Geddes, Mich.	757	4.9, ...	3	
548	Catawba, Morganton, N. C.	758	42.4, ...	28	
549	Broad, near Carlton, Ga.	762	38.2, ... 1902	27	
550	Passaic, Little Falls, N. J.	773	24.2, ... 1882		
551	Chagres, Bohio, Panama	779	115.5		
552	Noyo River Group, N. W. Calif.	780	....	60	78

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed, Maximum and date	Expected peaks: Frequent	Expected peaks: Rare
553	Youghiogheny, Pa	782	58.9, ... 1888		
554	Little Muddy, Williston, N. Dak	800	5.4, ...	2	
555	Pryor Creek, Huntley, Mont	800	1.7, ...	1	
556	Escanaba, Escanaba, Mich	800	13.4, ...	9	
557	Eagle, Gypsum, Colo	800	7.5, ...	6	
558	Shasta River, Calif	803	....	12	13
559	Hudson, North Creek, N. Y	804	35.1		
560	North Point, Republic, Va	804	29.7, ... 1896		
*561	Raritan, Bound Brook, N. J	806	64.5, ... 1882	25	
562	Animas, Durango, Calif	812	9.6, ...	6	
563	Scott River, Scott Bar, Calif	813	....	30	39
564	Snake, South Fork, Moran, Wyo	820	25.5, ...	11	
565	Passaic, Dundee, N. J	823	43.4, ... 1903	15	
566	Kettle, near Sandstone, Minn	825	7.1, ... 1912		
567	Holston, South Fork, Bluff City, Tenn	828	39.8, ... 1902	24	
568	James, North Fork, Glasgow, Va	831	44.8, ... 1896	21	
569	West Gallatin, Salesville, Mont	860	12.5, ...	7	
570	Silvies, Burns, Ore	865	5.5, ...	3	
571	Dead, near The Forks, Me	870	20.7		
572	Youghiogheny, below Confluence, Pa	874	52.6		
573	Deschutes, Allen Ranch, Lava, Ore	880	2.4, ...	1	
574	San Pitch, Gunnison, Utah	886	10.6, Mar., 1915	0.6	
575	Fish, Wallagrass, Me	890	10.0, ...	8	
576	Potomac North Branch, Cumberland, Md	891	22.8, ... 1897		
577	Flint, Molena, Ga	892	7.4		
578	Black, Lyons Falls, N. Y	897	46.0, ... 1869		
579	Manistee, Sherman, Mich	900	3.2, ...	3	
580	Two Butte Creek, at Mouth, Colo	900	39.0, Oct., 1908		
581	Schoharie Creek, Fort Hunter, N. Y	900	55.1, ... 1901		
582	Tule Lake Group, near Mt. Shasta, Calif	901	....	14	16
583	Clarion, Clarion, Pa	910	43.2, ...	26	
584	Santa Clara Tributaries, Ventura, Calif	911	....	60	82
585	Delaware, East Branch, Hancock, N. Y	920	78.8, ... 1904	36	
586	Fountain Creek, Pueblo, Colo	932	54.0, June, 1921		
587	Cottonwood Creek, near Red Bluff, Calif	937	....	60	75
588	Clearwater, South Fork, Grangeville, Idaho	940	10.5, ... 1912		
589	Chelan, Chelan, Wash	950	10.3, ...	9	54
590	Truckee, near Stateline, Calif	955	16.0, ...	6	
591	Mahoning, Youngstown, Ohio	958	20.2, ...	0.15	
592	Teton, near St. Anthony, Idaho	960	7.9, ...	5	
593	Mill Creek Group, near Red Bluff, Calif	971	....	90	122
594	Cut Bank Creek, Cut Bank, Mont	971	9.1, ... 1908		

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed, Maximum and date	Expected peaks: Frequent	Expected peaks: Rare
595	Stanislaus, Knight's Ferry, Calif.	983	61.0, Jan., 1911	27	
596	Winooski, Richmond, Vt.	985	29.8, ... 1904	20	
597	French Broad, Asheville, N. C.	987	31.2, ...	16	
598	Flint, near Woodbury, Ga.	988	30.6, ... 1913	16	
599	Huron, Flat Rock, Mich.	1,000	2.8, ...	2	
600	Redwater, Belle Fourche, S. Dak.	1,006	8.0, ... 1904		
601	Virgin, Virgin City, Utah	1,010	11.9, ... 1912	6	13
602	San Luis Obispo Group, Monterey, Calif.	1,019	....	55	73
603	Hoosatonic, Gaylordsville, Conn.	1,020	31.0, ...	16	
604	Blackfoot, Preston, Idaho	1,020	2.3, ...	2	
605	Shenandoah, North Fork, Riverton, Va.	1,037	20.9, ... 1901		
606	Cahaba, Centerville, Ala.	1,040	16.4, ...	11	
607	Snake, North Fork, Ora, Idaho	1,040	5.2, ...	4	
608	Scioto, Columbus, Ohio	1,047	80.8, Mar., 1913	14	
*609	Merced, near Merced Falls, Calif.	1,054	36.0, Jan., 1911		
610	Saluda, Waterloo, S. C.	1,056	18.0, ... 1903		
611	Sacandaga, Hadley, N. Y.	1,060	27.4		
612	Cache la Poudre, Fort Collins, Colo.	1,060	5.3, ...	3	
613	Genesee, Mount Morris, N. Y.	1,070	39.2		
614	Truckee, Reno, Nev.	1,070	7.0, ... 1913		
615	French Creek, Carlton, Pa.	1,070	22.9		
616	Pajaro Tributaries, near Salinas, Calif.	1,070	....	44	62
617	Penobscot, East Branch, Grindstone, Me.	1,070	28.9, Apr., 1922	14	
618	Oconee, Greensboro, Ga.	1,100	62.0, ...	16	
619	Walker, Yerington, Nev.	1,100	1.5, ...	1	
620	Noclichucky, Greeneville, Tenn.	1,100	29.4, ...	14	
621	Naches, North Yakima, Wash.	1,120	19.6, ...	10	
622	Cowlitz, Mossy Rock, Wash.	1,170	43.5, ... 1906		
623	Roquette, Massena Springs, N. Y.	1,170	9.4, ...	7	
624	Chattahoochee, Norcross, Ga.	1,170	25.8, ...	15	
625	Neuse, Selma, N. C.	1,175	6.7		
626	Pea, Pera, Ala.	1,180	10.7, ...	8	
627	Hiwassee, Reliance, Tenn.	1,180	46.7, ...	24	
628	Cache Creek, Yolo, Calif.	1,195	17.4, Feb., 1909	12	
629	Umatilla, Yoakum, Ore.	1,200	20.0, ...	8	
*630	Yuba, near Smartsville, Calif.	1,200	92.5, Jan., 1909	47	
631	Wenatchee, Dryden, Wash.	1,200	20.1, ... 1913	14	
632	Deschutes West Ranch, Lava, Ore.	1,240	3.2, ...	2	
633	Heart, Richardton, N. Dak.	1,250	6.4, ...	3	
634	Thunder Bay, Alpena, Mich.	1,260	5.8, ...	4	
635	Dock, Columbia, Tenn.	1,260	20.3, ...	15	
636	Conecuts, Beck, Ala.	1,290	12.7, ...	8	

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
637	Animas, Axtec, N. Mex.	1,300	9.6,	5	
638	Mohawk, Little Falls, N. Y.	1,306	26.6, 1913	15	
639	Oneida, Euclid, N. Y.	1,310	19.5,	8	
640	Wapsipinicon, Stone City, Iowa	1,310	6.6,	5	
641	Chagres, near Gatun, Panama	1,320	93.9		
642	Youghiogheny, Connellsville, Pa.	1,320	41.2, 1907		
643	Tygart Valley, Felterman, W. Va.	* 1,327	26.4		
644	Orestimba Creek Group, Merced, Calif.	1,340		30	40
645	Tejon Creek Group, near Hanford, Calif.	1,341		21	35
646	Greenbriar, Alderson, W. Va.	1,344	46.5,	27	
647	Grande Ronde, Elgin, Ore.	1,350	6.2,	4	
648	Cheat, Morgantown, W. Va.	1,380	30.3,	20	
649	Hondo Reservoir, N. Mex.	1,387	4.6, 1904		
650	San Juan, Arboles, Colo.	1,390	28.8, 1911		
651	Ocmulgee, Jackson, Ga.	1,400	18.2,	13	
652	Rio Grande, Del Norte, Calif.	1,400	5.5,	3	
653	Geneseo, New York	1,410	12.5		
654	St. Croix, Woodland, Me.	1,420	14.3,	7	
655	Au Sable, Bamfield, Mich.	1,420	3.0,	2	
656	Potomac, Springfield, W. Va.	1,440	17.8		
657	Willamette, Middle Fork, Jasper, Ore.	1,450	64.2,	33	
658	Merrimac, Franklin Jct., N. H.	1,460	19.1,	13	
659	Kalamazoo, Allegan, Mich.	1,470	7.0,	3	
660	Shoshone, Cody, Wyo.	1,480	10.7,	8	
661	Canadian, French, N. Mex.	1,480	105.0, Oct., 1909		
662	Mattawamkeag, Mattawamkeag, Me.	1,500	16.3,	11	
663	Honey Lake Group, Calif.	1,507		11	14
664	Russian, N. W. Calif.	1,508		40	51
665	Truckee, Vista, Nev.	1,520	5.9,	3	
666	Oostanaula, Resaca, Ga.	1,527	14.5		
667	Chenango, Binghamton, N. Y.	1,530	23.5,	18	
668	Catawba, Catawba, N. C.	1,535	61.9, May, 1901	19	
669	Yakima, Umtaum, Wash.	1,540	26.7,	16	
670	Tuolumne, La Grange, Calif.	1,543	39.0, Jan., 1911		
671	Bitterroot, Grantsdale, Mont.	1,550	8.3,	6	
672	Chattahoochee, Oakdale, Ga.	1,560	31.3, 1901	22	
673	Kennebec, Forks, Me.	1,570	13.2, Apr., 1923	9	
674	Lower Scioto, Columbus, Ohio	1,570	70.5, 1913	20	
675	Shenandoah, South Fork, Front Royal, Va.	1,570	48.9, 1902	16	
676	Osgewatchie, Ogdensburg, N. Y.	1,580	10.0,	8	
677	Oostanaula, Branch, Mobile, Ga.	1,610	24.3,	15	
*678	San Joaquin, Friant, Calif.	1,631	36.6, 1880	11	

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
679	Santa Maria, Santa Maria, Calif.	1,634	....	40	61
680	Allegheny, Red House, N. Y.	1,640	25.0 , ...	15	
681	Weiser, Weiser, Idaho	1,670	10.7 , ...	6	
682	Kings, near Sanger, Calif.	1,694	35.3 , Jan., 1914	12	
683	Elk, Elkmont, Ala.	1,700	28.8 , ...	24	
684	Methow, Pateros, Wash.	1,710	7.0 , ...	7	
685	Yampa, Craig, Colo.	1,730	5.6 , ...	5	
686	Arkansas Tributaries, Pueblo, Colo.	1,740	57.0 , June, 1921		
687	Truckee, Clark, Nev.	1,740	3.0 , ... 1911		
688	King River, State Point, Calif.	1,742	25.2 , ... 1901		
689	Kiskiminatis, Avonmore, Pa.	1,750	44.3 , ...	26	
690	Umpqua, South Fork, Brockway, Ore.	1,800	39.2 , ...	28	
691	Etowah, Rome, Ga.	1,800	33.0 , ...	195	
692	Gallatin, Logan, Mont.	1,805	3.6 , ...	3	
693	Black, Carthage, N. Y.	1,812	21.2 , ... 1869	10	
694	Penobscot, West Branch, Millinocket, Me.	1,880	12.9 , ...	8	
695	Black Warrior, Cordova, Ala.	1,900	28.8 , ...	21	
696	American, near Folsom, Calif.	1,900	99.5 , Jan., 1862		
697	Schuylkill, Fairmont, Pa.	1,920	42.8 , ... 1898	16	
698	Feather, North Fork, Big Bend, Calif.	1,940	56.3		
699	Rogue River, Tolo, Ore.	2,020	23.9 , ... 1909	17	
700	Chemung, Elmira, N. Y.	2,055	67.1 , ... 1889		
701	Missouri, Red Bluffs, Mont.	2,085	5.0 , ...	3	
702	Flambeau, Ladysmith, Wis.	2,120	6.0 , ...	5	
703	Meadow Valley Wash, near Moapa, Nev.	2,150	3.8 , Jan., 1910		
704	Birch Creek, Fourteen Mile House, Alaska	2,150	6.9 , ... 1911		
705	Palouse, Hooper, Wash.	2,210	7.8 , ...	3	
706	San Francisco Bay, Drainage Basins, Calif.	2,219	....	40	51
707	Aroostook, Fort Fairfield, Me.	2,230	15.4 , ...	14	
708	Payette, Horse Shoe Bend, Idaho	2,240	8.1 , ...	6	
709	Tar, Tarboro, N. C.	2,290	6.4		
710	Hiwassee, Charleston, Tenn.	2,297	20.0 , ...	16	
711	Trinity, Fort Worth, Tex.	2,300	20.0 , Apr., 1922	13	
*712	Androscoggin, Rumford Falls, Me.	2,320	23.8 , ... 1869	11	
713	Klamath, N. W. Calif.	2,320	....	15	20
714	Merrimac, Garvins Falls, N. H.	2,340	18.3 , ...	14	
715	Susquehanna, Binghamton, N. Y.	2,350	26.6 , ... 1902	16	
716	Muskegon, Newdygo, Mich.	2,350	2.7 , ...	2	
*717	Genesee, Rochester, N. Y.	2,365	21.2 , ...	10	
718	Grand, Kremmling, Colo.	2,380	6.4 , ...	4	
719	Guadalupe, New Braunfels, Tex.	2,400	23.6 , Sept., 1921		
720	Kern, Bakersfield, Calif.	2,410	7.6 , Jan., 1914	2	

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed: Maximum and date	Runoff, second-feet per square mile: Expected peaks: Frequent	Runoff, second-feet per square mile: Expected peaks: Rare
721	Menominee, near Iron Mountain, Mich.	2,415	6.2 , ...	5	
722	Ocmulgee, Macon, Ga.	2,425	21.0 , ... 1902	14	
723	Miami, Dayton, Ohio	2,450	100. , Mar., 1913	19	
724	Little Tennessee, McGhee, Tenn.	2,470	28.3 , ...	15	
725	Elkhorn, near Norfolk, Nebr.	2,470	3.2 , ... 1903		
726	Tallapoosa, Sturdevant, Ala.	2,500	23.6 , ...	15	
727	Clark Fork, Fromberg, Mont.	2,500	4.3 , ...	3	
728	Sangamon, Riverton, Ill.	2,560	7.5 , ... 1911		
729	Sevier, Marysvale, Utah	2,560	1.2 , ...	1	
730	Rhine, Macon, Ga.	2,574	37.4		
731	Grand River, Ontario, Canada	2,600	14.3		
732	Marias, Shelby, Mont.	2,610	11.3 , ...	5	
733	Yellowstone, Corwin Springs, Mont.	2,630	8.7 , ... 1911		
734	Wisconsin, near Merrill, Wis.	2,630	8.0 , ...	6	
735	Pecos, Santa Rosa, N. Mex.	2,649	17.6 , ... 1904		
736	James, Buchanan, Va.	2,660	23.3 , ...	16	
737	Kennebec, near Waterville, Me.	2,700	48.6 , ... 1901		
738	Flint, Montezuma, Ga.	2,700	7.7 , ...	5	
739	Savannah, near Calhoun Falls, S. C.	2,712	27.8		
740	New, Radford, Va.	2,725	63.8 , ... 1900	30	
741	Salme, Beverly, Kans.	2,730	5.9 , ... 1896		
742	Duchesne, Myton, Utah	2,750	3.5 , ...	2	
743	Dan, Sonth Boston, Va.	2,750	16.1 , ...	8	
744	Hudson, Glens Falls, N. Y.	2,760	25.4 , ... 1913		
745	Canadian, Taylor, N. Mex.	2,832	32.1 , ... 1904		
746	Oconee, Milledgeville, Ga.	2,840	10.4 , ...	6	
747	Humboldt, Elko, Nev.	2,840	0.8 , ...	0.5	
748	Trinity, near Dunsmuir, Calif.	2,965	....	35	44
749	Catawba, near Rockhill, S. C.	2,987	50.5 , May, 1901	22	
750	Shenandoah, Millville, W. Va.	2,995	46.6 , ... 1896	15	
751	Salt Creek, at Mouth, N. M.	3,052	4.1 , ... 1904		
752	Holston, Rogersville, Tenn.	3,060	16.8 , ...	12	
753	Arkansas, Canon City, Colo.	3,060	2.2 , ...	2	
754	Verdigris, Liberty, Kans.	3,067	16.4 , July, 1904	13	
755	Eel, Scotia, Calif.	3,071	94.4		
756	Seneca, Baldwinsville, N. Y.	3,100	3.5 , ...	3	
757	Connecticut, Fairlee, Vt.	3,100	18.5 , ... 1913		
758	Link, Klamath Falls, Ore.	3,110	2.9		
759	Pearl, Jackson, Miss.	3,120	11.7 , ...	7	
760	Klamath, Keno, Ore.	3,150	2.7		
761	Wabash, Logansport, Ind.	3,163	18.0		
762	Delaware, Port Jervis, N. Y.	3,250	33.2 , ...	18	
763	Belle Fourche, Belle Fourche, S. Dak.	3,250	18.2 , ...	12	

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile — Observed — Maximum and date	Expected peaks — Frequent	Expected peaks — Rare
764	Bitterroot, Missoula, Mont.	3,260	11.5, ...	7	
765	Yakima, Union Gap, near Yakima, Wash.	3,300	19.3, ... 1904	9	
766	Chattahoochee, West Point, Ga.	3,300	26.9, ... 1901	14	
767	Connecticut, Orford, N. H.	3,305	15.0, ...	9	
768	Iowa, Iowa City, Iowa	3,320	5.9, ...	3	
769	Juniata, Newport, Pa.	3,380	53.8, ... 1889	18	
770	Mohawk, Rexford, N. Y.	3,384	23.1, ... 1892		
771	Saline, Saline, Kans.	3,311	2.4, ...	1	
772	Yadkin, Salisbury, N. C.	3,399	38.3, ... 1889	19	
773	Mohawk, Cohoes, N. Y.	3,472	28.5, ... 1913	14	
774	Staunton, Clarksville, Va.	3,546	10.3		
775	Eel, Calif.	3,547	....	90	121
776	Crow Wing, near Mouth, Minn.	3,580	2.8,		
777	Yellowstone, Livingstone, Mont.	3,580	7.5, ...	5	
778	Feather, Oroville, Calif.	3,627	51.6, Mar., 1907	26	
779	Cannon Ball, Stevenson, N. Dak.	3,650	1.6, ...	1	
780	Neosho, Iola, Kans.	3,670	20.3, July, 1904	14	
781	Dan, Clarksville, Va.	3,749	8.8		
782	South Platte, South Fork, Denver, Colo.	3,840	1.4, ...	0.5	
783	Tallapoosa, Milstead, Va.	3,840	18.2, ...1901		
784	St. Joseph, Buchanan, Mich.	3,940	4.7, ...	3	
785	Sevier, Gunnison, Utah	3,990	0.6, ...	0.3	
786	Spokane River, Spokane, Wash.	4,000	8.8, ... 1894	6	
787	Coosa, Rome, Ga.	4,006	16.0, ... 1901	14	
788	Pit, Bieber, Calif.	4,040	6.8, ...	3	
789	Salinas Tributaries, Salinas, Calif.	4,042	....	60	112
790	Merrimac, Lowell, Mass	4,085	19.8		
791	White, near Interior, S. Dak.	4,090	4.0, ... 1905		
792	Oconee, Dublin, Ga.	4,182	8.3, ...	6	
793	Malheur, Vale, Ore.	4,190	3.5, ...	2	
794	Kennebec, Waterville, Me.	4.270	35.4, ... 1901		
795	Tombigbee, Columbus, Miss.	4,440	11.4, ...	8	
796	Cape Fear, Fayetteville, N. C.	4,493	20.2, ...	12	
797	Wateree, Camdem, S. C.	4,500	8.1, ...	7	
798	Hudson, Mechanicsville, N. Y.	4,500	26.7, ... 1913	11	
799	Susquehanna, Williamsport, Pa.	4,500	11.6		
800	Mississippi, above Sandy River, Minn.	4,510	2.1, ...	1.4	
801	Colorado (Grand), Glenwood Springs, Colo.	4,520	6.1, ...	4.5	
802	Merrimac, Lawrence, Mass.	4,553	23.4, ...	12	
803	Arkansas, Pueblo, Colo., before 1914	4,600	2.4, ...	1.2	
804	Arkansas, Pueblo, Colo., after 1921	4,600	22.3, June, 1921	3	
805	Broad, Alston, S. C.	4,609	28.4, ... 1901	17	
806	Yadkin, Norwood, N. C.	4,614	13.7		

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
807	Potomac, Dam No. 5, Md.	4,640	22.2		
808	Clearwater, Kamiah, Idaho	4,850	15.8, ... 1913		
809	Williamette, Albany, Ore.	4,860	62.2, ... 1861	25	
810	Grand, Grand Rapids, Mich.	4,900	10, ... 1905		
811	Black Warrior, Tuscaloosa, Ala.	4,900	38.8, ... 1895	28	
812	Flint, Albany, Ga.	5,000	8.5, ...	6	
813	Guadalupe, near Cuero, Tex.	5,020	14.2, ... 1903		
814	John, Fort Kent, Me.	5,280	14.3, ...	12	
815	Little River, Leon Junction, Tex.	5,300	62.5, Sept., 1921		
816	Chippewa, Chippewa Falls, Wis.	5,300	12.1, ...	7	
817	Red Lake, Crookston, Minn.	5,320	2.7, ...	1.6	
818	Pit, Ydalpom, Calif.	5.346	8.8, Dec., 1913	5	
*819	Monongahela, Lock No. 4, Pa.	5,430	38.1, ... 1888		
820	Snake, South Fork, Lyon, Idaho	5,480	9.4, ...	6	
821	Yakima, Kiona, Wash.	5,520	11.5, ... 1906		
822	Sevier, Leamington, Utah	5,595	0.4, ...	0.3	
823	Susquehanna, Williamsport, Pa.	5,640	29.0, ...	19	
824	Salt River, Roosevelt, Ariz.	5,756	36.0, ... 1893		
825	Little Missouri, Medora, N. Dak.	5,780	3.3, ...	2	
826	St. Croix, St. Croix Falls, Minn.	5,950	6.0		
827	Missoula, Missoula, Mont.	5,960	6.0, ...	3.5	
828	Elkhorn, Arlington, Neb.	5,980	1.6, ...	1	
829	Bear, Collinston, Utah	6,000	1.9, ...	1	
830	Verde, McDowell, Ariz.	6,000	24.0, ... 1893		
831	Niobara, near Valentine, Neb.	6,070	1.1, ...	0.6	
832	Pecos, Fort Sumner, N. Mex.	6,191	7.3, ... 1904		
833	Fox, Rapid Croche Dam, Wis.	6,200	2.5, ... 1895		
834	New, Fayette, W. Va.	6,200	17.8, ... 1899	13	
835	James, Cartersville, Va.	6,230	13.6, ...	9	
836	Salt, McDowell, Ariz.	6,260	22.0, ...	7	
837	Rock, below Rockton, Ill.	6,290	4.3		
838	Cedar, Cedar Rapids, Iowa	6,320	8.3, ...	5	
839	Delaware, Riegelsville, N. J.	6,430	27.4, ...	15	
840	Penobscot, West Enfield, Me.	6,600	14.1, ...	9	
841	Chippewa, Eau Claire, Wis.	6,740	9.0, ... 1905	6	
842	Susquehanna, Northumberland, Pa.	6,800	17.5, ... 1889		
843	Solomon, Miles, Kans.	6,815	1.6, ...	0.8	
844	Coosa, Riverside, Ala.	6,850	10.5, ... 1898		
845	Delaware, Stockton, N. J.	6,855	37.2, ... 1841		
846	Little River, Cameron, Tex.	7,010	92.3, Sept., 1921		
847	Coosa (Branch Mobile), Riverside, Ala.	7,060	10.7, ...	8	
848	Mississippi	7,283	1.49,		

* Maximum daily flow.

Table II.—Maximum Observed Flood Discharge Rates and Expected Runoff from Various Drainage Basins Arranged According to Area.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile: Observed: Maximum and date	Expected peaks: Frequent	Expected peaks: Rare
849	Cheyenne, Edgemont, S. Dak.	7,350	1.5 , ...	1	
850	Green River, Green River, Wyo.	7,450	3.0 , ...	2	
851	Savannah, Augusta, Ga.	7,500	40.0 , ... 1884	15	
852	Penobscot, Bangor, Me.	7,700	14.94		
853	Connecticut, Sunderland, Mass.	7,700	13.4 , ...	10	
854	John Day, McDonald, Ore.	7,800	2.9 , ...	2	
855	Gunnison, Whitewater, Colo.	7,863	3.6 , ... 1905	7	
*856	Smoky Hill, Ellsworth, Kans.	7,980	2.6 , ... 1895		
857	Wisconsin, Kilburn, Wis.	8,000	10.0		
858	Bighorn, Thermopolis, Wyo.	8,184	2.1 , ...	1.6	
859	St. John, Van Buren, Me.	8,270	16.2 , Apr., 1923		
860	Roanoke, Old Gaston, N. C.	8,350	32.90, ... 1877		
861	Colorado, (Grand), Palisade, Colo.	8,546	4.9		
862	Connecticut, Holyoke, Mass.	8,660	21.10, ... 1854		
863	Allegheny, Kittanning, Pa.	8,690	26.7 , ...	14	
864	Tombigbee, Epes, Ala.	8,830	6.9 , ...	5	
865	Great Basin Drainage, Calif.	8,876	....	20	25
866	Kanawha, Charleston, W. Va.	8,900	13.49, ... 1875		
867	Jefferson, Sappington, Mont.	8,984	1.7 , ...	1.1	
868	Tennessee, Knoxville, Tenn.	8,990	17.5 , ...	10	
*869	Allegheny, Kittanning, Pa.	9,010	26.66		
870	Deschutes, Biggs, Ore.	9,180	3.3 , ...	2	
871	Sacramento, Red Bluff, Calif.	9,258	30.0 , ... 1909	15	
872	Blue, near Manhattan, Kans.	9,490	9.13, ... 1903		
873	Potomac, Point of Rocks, Md.	9,654	48.9 , ... 1889	12	
874	Susquehanna, Wilkes-Barre, Pa.	9,810	22.2 , ...	13	
875	Connecticut, Hartford, Conn.	10,234	20.3 , ... 1854	11	
876	Susquehanna, Danville, Pa.	11,100	27.4 , ... 1902	13	
877	Rio Grande, Rio Grande, N. Mex.	11,250	2.75, ... 1904		
878	Potomac, Great Falls, Md.	11,427	41.15, ... 1889		
879	Canadian River, Logan, N. Mex.	11,440	11.29, ... 1904		
880	North Platte, Pathfinder, Wyo.	12,000	1.0 , ...	0.8	
881	Yellowstone, Huntley, Mont.	12,000	4.03, ... 1907		
882	Salt River, Ariz.	12,000	24.7 , ... 1891		
883	Mississippi, Sank Rapids, Mich.	12,400	4.1 , ...	3	
884	Illinois, Peoria, Ill.	13,480	5.94, ... 1904	4	
885	Loupe, Columbus, Nebr.	13,540	5.17, ... 1896	3	
886	South Pacific Drainage, Calif.	13,589	....	50	86
887	Salmon, Whitebird, Idaho	13,600	5.97, ... 1913		
888	Humboldt, Oreana, Nev.	13,800	0.22, ... 1897		
889	Yazoo, Miss.	13,850	10.04		
890	Des Moines, Keosauqua, Iowa	14,300	7.0 , ...	4	

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Continued*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
891	Missouri, Townsend, Mont.	14,500	3.6 , ...	2	
892	Minnesota, Mankato, Minn.	14,600	3.0 , ...	1.6	
893	North Platte, Orin Junction, Wyo.	14,828	1.5 , ...	1.1	
894	Pecos, Roswell, N. Mex.	14,840	3.75, ... 1904		
895	Alabama, Selma, Ala.	15,400	9.5		
896	North Platte, Guernsey, Wyo.	16,200	1.9 , ...	6	
897	North Pacific Drainage, Calif.	16,543	....	50	72
898	Colorado (Grand), Fruita, Colo.	16,800	3.81, ... 1909		
899	Mississippi, Anoka, Minn.	17,100	2.87, ...	2	
900	Gila, Florence, Ariz.	17,750	7.5 , ... 1891		
901	San Joaquin Drainage, Calif.	18,178	....	30	39
902	Missouri, Cascade, Mont.	18,300	2.70, ... 1908	1.3	
903	White, Clarendon, Ark.	19,000	16.84, ... 1916	8	
904	Ohio, Pittsburgh, Pa.	19,100	22.98, ... 1907		
905	Big Horn, Hardin, Wyo.	20,700	1.97, ... 1908	1.4	
*906	Tennessee, Chattanooga, Tenn.	21,382	34.37, ... 1867	11	
907	Sacramento Drainage, Calif.	21,420	....	46	66
908	Republican, Bostwick, Nebr.	22,300	1.10		
909	Snake, South Fork, Minidoka, Idaho	22,600	2.4 , ...	1.9	
910	Clark Fork, Newport, Wash.	24,000	6.45, ...	4.3	
911	Susquehanna, Harrisburg, Pa.	24,030	30.60, ... 1889	12	
912	North Platte, Camp Clarke, Nebr.	24,800	0.95, ...	0.7	
913	Red, Grand Forks, N. Dak.	25,000	1.70, ... 1897	1.2	
*914	Republican, Junction, Kans.	25,837	1.80, ... 1903	0.7	
915	Susquehanna, McCall's Ferry, Pa.	26,766	25.1 , ... 1904		
916	Illinois, at Mouth, Ill.	27,914	4 48, ... 1904		
917	Rio Grande, San Marcia, N. Mex.	28,067	1.17		
918	Tennessee, Florence, Ala.	30,800	16.2 , ... 1897		
919	Brazos, Waco, Tex.	30,800	4.3 , ...	2	
*920	Colorado, Austin, Tex.	34,200	3.57, ... 1900	2	
*921	Ohio, Wheeling, W. Va.	34,800	18.10, ... 1913	11	
922	Mississippi, St. Paul, Minn.	36,085	3.32, ... 1881	1.7	
923	Green, Greenriver, Utah	38,200	1.6 , ...	1	
924	Mississippi, Prescott, Wis.	44,070	2.50		
925	Gila, Yuma, Ariz.	56,000	3.93, Jan., 1916	2	4
926	Platte, near Columbus, Nebr.	56,900	0.83, ... 1905	0.4	
927	Kansas Lawrence, Kans.	59,841	3.80, ... 1903	1	
928	Ohio, Cincinnati, Ohio	75,800	8.70, ... 1913		
929	Muskingum, Marietta, Ohio	78,500	32.00, Mar., 1913		
930	Mississippi, Clayton, Iowa	79,040	2.66, ... 1880		
931	Ohio, Louisville, Ky.	90,600	8.5 , ... 1913		
932	Red. Ark.	97,000	2.32		

* Maximum daily flow.

TABLE II.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES AND EXPECTED RUNOFF FROM VARIOUS DRAINAGE BASINS ARRANGED ACCORDING TO AREA.—(*Concluded*)

No.	River or tributary and location	Drainage area, square miles	Runoff, second-feet per square mile		
			Observed	Expected peaks	
			Maximum and date	Frequent	Rare
933	Yukon, Eagle, Alaska	122,000	2.08, ... 1911		
934	Colorado, above Gila Junction, Ariz	169,000	0.89, ... 1909	0.6	0.9
935	Arkansas, Little Rock, Ark	170,000	2.7 , ... 1916	1.35	
936	Mississippi, Grafton, Ill	171,570	2.10, ... 1883		
937	Arkansas and White, Ark	189,000	0.84		
938	Ohio, Paducah, Ky	205,750	7.00		
939	Colorado, below Gila Junction, Yuma, Ariz	225,000	1.05, Jan., 1916		
940	Ohio, Cairo, Ill	233,000	6.0 , ... 1913	3.6	
941	Columbia River, Dalles, Ore	237,000	5.87, ... 1894	3	
942	Missouri, Sioux City, Iowa	323,462	1.64, ... 1881		
943	Missouri, St. Charles, Mo	530,810	1.13		
944	Mississippi, St. Louis, Mo	702,380	1.28, ... 1883		
945	Mississippi, Cairo, Ill	902,900	2.23, ... 1912	1.3	
946	Mississippi, Columbus, Ky	930,540	1.59, ... 1858		
947	Mississippi, Helena, Ark	1,000,000	2.04, ... 1912	1.1	
948	Mississippi, above Arkansas Junction, Ark	1,050,000	2.31, ... 1912	1.3	
949	Mississippi River, Miss	1,244,000	1.19		
950	Mississippi, Carrollton, La	1,400,000	1.07, May, 1922	0.7	

TABLE V.—RAINFALL CHARACTERISTICS[1]
(Depths expressed in inches)

Station No.	Station	Elevation above sea, ft.	Years of record: For day or less	Years of record: For month and year	Years of record: Last year included	Maximum observed precipitation: Number of minutes: 5	15	60	Day of 24 hr.	Month	Annual precipitation: Minimum	Mean	Maximum	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	15	60	Mean for month of maximum record	Maximum: In 24 hr.	Maximum: In one calendar month	Annual: Minimum	Annual: Mean	Annual: Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
1	Eastport, Me.	53	..	58	1927	....	....	....	....	13.2	22.8	40.4	64.5	0.7	...	....	....	7.8	....	32.6	57	100	160
2	Bar Harbor, Me.	20	..	44	1920	....	....	....	....	14.4	32.1	48.3	63.9	0.8	...	....	....	10.3	....	29.8	66	100	132
3	Orono, Me.	129	..	56	1925	....	....	....	....	10.5	26.0	41.7	53.8	0.9	...	....	....	7.7	....	25.2	62	100	129
4	Upper Oxford Dam, Me.	1,470	..	40	1925	....	....	....	....	8.8	22.4	32.8	41.7	...	...	....	....	10.1	....	26.8	68	100	127
5	Lewiston, Me.	185	..	52	1927	....	....	....	....	12.3	32.7	44.2	58.6	...	...	....	....	9.8	....	27.8	74	100	133
6	Gardiner, Me.	163	..	90	1927	....	....	....	....	13.1	30.2	43.0	54.6	...	...	....	....	9.5	....	30.5	70	100	127
7	Portland, Me.	99	57	57	1927	0.42	1.09	1.58	5.3	10.9	32.0	41.7	59.7	0.8	1.0	2.6	3.8	7.9	12.7	26.2	77	100	143
8	Cornish, Me.	778	..	67	1924	....	....	....	....	12.3	33.4	46.1	63.3	...	...	....	....	8.1	....	26.6	72	100	138
9	Lakeport, N. H.	500	..	69	1925	....	....	....	....	12.6	31.5	42.2	55.6	...	...	....	....	9.1	....	29.9	75	100	132
10	Concord, N. H.	350	..	70	1927	0.55	1.55	2.48	4.1	11.6	26.2	38.4	54.3	0.8	1.4	4.0	6.5	8.7	10.7	30.2	68	100	141
11	Hanover, N. H.	603	..	81	1927	....	....	....	....	9.8	22.7	35.9	55.8	0.8	...	....	....	9.6	....	27.5	63	100	155
12	Plymouth, N. H.	500	..	40	1927	....	....	....	....	11.5	30.8	39.3	53.5	...	...	....	....	8.9	....	29.3	78	100	136
13	Mt. Washington, N. H.	6,293	..	21	1890	....	....	....	....	23.9	55.8	82.2	121.1	...	...	....	....	12.6	....	29.0	68	100	147
14	Bethlehem, N. H.	1,470	..	33	1925	....	....	....	....	9.8	28.0	36.7	46.1	0.7	...	....	....	10.4	....	26.6	76	100	125
15	Burlington, Vt.	404	22	93	1927	0.44	0.78	1.52	4.5	9.9	24.3	32.6	49.4	0.7	1.3	2.4	4.7	10.6	13.7	30.4	74	100	152
16	Northfield, Vt.	876	..	40	1927	0.85	1.64	1.91	7.6	11.4	27.4	33.2	45.9	0.6	2.6	5.0	5.8	8.5	22.9	34.4	83	100	138
17	Chelsea, Vt.	840	30	42	1927	....	....	....	4.5	10.9	27.1	35.8	48.1	...	...	....	....	8.7	12.5	30.5	76	100	134
18	Woodstock, Vt.	700	..	60	1927	....	....	....	....	12.9	29.7	38.2	51.3	...	...	....	....	8.0	....	33.8	78	100	134
19	Williamstown, Mass.	711	..	50	1926	....	....	....	....	14.8	30.5	38.1	54.5	0.9	...	....	....	11.2	....	38.8	80	100	143
20	Springfield, Mass.	200	..	79	1926	....	....	....	....	15.0	30.9	43.7	58.0	...	...	....	....	9.9	....	34.4	71	100	133
21	Amherst, Mass.	222	..	91	1926	....	....	....	....	14.5	31.0	43.9	58.0	0.8	...	....	....	9.9	....	33.0	71	100	132
22	Lowell, Mass.	80	..	101	1926	....	....	....	....	12.3	27.8	41.3	59.5	...	...	....	....	9.9	....	29.8	67	100	144

23	Boston, Mass.	124	57	110	1927	0.55	1.12	1.64	6.0	12.4	27.2	43.5	67.7	0.8	1.3	2.6	3.8	8.2	13.8	28.5	63	100	158
24	Cambridge, Mass.	40	..	165	1927	....	....	....	....	.....	24.5	42.3	62.5	...	...	....	....	....	....	.....	58	100	148
25	Cambridge, Mass (Harvard Obs)	75	..	81	1921	....	....	....	....	15.0	30.1	43.5	59.3	...	...	....	....	9.5	....	34.5	69	100	137
26	Cambridge, Mass (Fresh Pond)	40	..	40	1925	....	....	....	....	11.7	28.2	41.0	62.5	...	...	....	....	8.5	....	28.6	69	100	153
27	Waltham, Mass.	30	..	101	1925	....	....	....	....	14.0	28.4	42.3	62.1	...	...	....	....	9.7	....	33.2	67	100	147
28	Lake Cochituate, Mass.	140	..	74	1925	....	....	....	....	13.3	31.2	45.6	69.3	...	...	....	....	8.5	....	29.1	68	100	152
29	New Bedford, Mass.	100	..	113	1926	....	....	....	....	18.7	32.9	46.1	65.4	...	...	....	....	9.0	....	40.6	71	100	142
30	Nantucket, Mass.	15	33	52	1926	0.44	0.83	2.26	5.7	11.0	27.1	39.3	53.0	0.7	1.1	2.1	5.8	8.2	14.5	28.0	69	100	146
31	Block Island, R. I.	26	..	46	1926	0.21	0.52	1.53	....	9.8	27.2	41.2	63.1	0.8	0.5	1.3	3.7	8.4	....	23.8	66	100	153
32	Providence, R. I.	162	22	95	1926	0.45	1.11	1.98	4.8	11.3	30.5	45.1	63.5	0.8	1.0	2.5	4.4	8.5	10.6	25.0	68	100	141
33	Norwich, Conn.	150	..	55	1925	....	....	....	....	13.3	29.8	44.4	62.8	...	...	....	....	9.8	....	29.8	67	100	136
34	New London, Conn.	47	..	56	1926	....	....	....	....	16.4	30.0	44.1	60.6	...	...	....	....	9.6	....	37.2	68	100	137
35	Middletown, Conn.	125	..	67	1925	....	....	....	....	15.5	34.0	46.8	66.7	...	...	....	....	8.5	....	33.1	73	100	142
36	Hartford, Conn.	159	..	55	1926	0.49	1.30	2.64	....	15.3	33.5	44.7	61.0	0.8	1.1	2.9	5.9	9.7	....	34.2	75	100	136
37	Canton, Conn.	900	..	64	1922	....	....	....	....	18.0	34.8	48.8	75.2	...	...	....	....	8.6	....	36.8	71	100	154
38	New Haven, Conn.	127	..	76	1926	0.40	1.05	2.33	....	17.1	33.9	45.6	60.3	0.8	0.9	2.3	5.1	9.4	....	37.5	74	100	132
39	New York, N. Y.	314	26	56	1927	0.75	1.63	2.48	....	14.5	33.2	43.1	58.7	0.8	1.7	3.8	5.8	7.9	....	33.7	77	100	136
39-A	New York, N. Y.	314	56	101	1927	....	....	....	9.4	14.5	28.8	42.1	58.7	0.8	...	....	....	8.0	22.3	34.4	68	100	139
40	Yonkers, N. Y.	100	..	48	1925	....	....	....	....	22.7	25.2	48.4	85.2	...	...	....	....	10.4	....	47.0	52	100	176
41	West Point, N. Y.	167	..	76	1925	....	....	....	....	13.5	31.5	45.3	63.6	1.0	...	....	....	8.1	....	29.8	70	100	141
42	Boyd's Corners, N. Y.	600	..	58	1925	....	....	....	....	14.4	35.3	49.0	63.4	...	...	....	....	8.6	....	29.4	72	100	129
43	Albany, N. Y.	97	45	101	1926	0.57	1.17	1.83	4.7	13.5	26.4	37.4	56.8	0.7	1.5	3.1	4.9	8.7	12.5	36.0	71	100	152
44	Plattsburgh, N. Y.	183	..	73	1920	....	....	....	....	9.2	18.8	30.4	47.3	1.0	...	....	....	11.6	....	30.3	62	100	155
45	Canton, N. Y.	448	15	38	1920	....	....	....	3.5	8.9	28.3	36.0	46.2	...	...	....	....	9.5	9.7	24.7	78	100	128
46	Oswego, N. Y.	335	50	74	1920	....	....	....	3.8	9.8	23.4	36.5	56.1	0.6	...	....	....	8.9	10.4	26.8	64	100	154
47	Syracuse, N. Y.	597	19	24	1920	0.56	1.16	2.65	4.8	7.4	29.2	34.7	44.5	0.6	1.6	3.3	7.6	9.5	13.8	21.3	84	100	128
48	Rochester, N. Y.	500	50	92	1920	....	....	....	4.2	8.7	17.0	33.2	49.9	0.6	...	....	....	8.8	12.7	26.3	51	100	151
49	Fort Niagara, N. Y.	300	..	48	1899	....	....	....	....	7.3	16.4	27.5	50.5	...	...	....	....	10.2	....	26.6	60	100	184
50	Buffalo, N. Y.	824	..	71	1927	0.79	....	....	4.2	10.6	25.3	36.1	60.2	0.6	2.2	....	....	8.3	11.6	29.3	70	100	167
51	Jamestown, N. Y.	1,365	..	28	1918	....	....	....	....	11.2	34.4	43.3	52.7	0.6	...	....	....	11.0	....	25.9	79	100	122
52	Bolivar, N. Y.	1,800	..	30	1920	....	....	....	....	11.0	31.8	40.1	43.6	...	...	....	....	9.7	....	27.4	79	100	122
53	Ithaca, N. Y.	928	42	64	1915	....	....	....	4.2	8.4	20.2	32.9	46.4	0.6	...	....	....	9.7	12.8	25.6	62	100	141
54	Cooperstown, N. Y.	1,200	..	67	1920	....	....	....	....	10.0	29.9	40.9	58.1	0.7	...	....	....	10.6	....	24.4	73	100	142
55	Binghamton N. Y.	871	..	38	1927	....	1.00	2.03	4.5	8.1	22.4	34.0	42.6	0.7	...	2.9	5.9	10.3	13.2	23.7	66	100	125
56	Scranton, Pa.	805	..	33	1927	....	....	....	5.1	9.2	31.3	37.3	45.3	0.7	...	....	....	8.3	13.7	24.8	84	100	122
57	Philadelphia, Pa.	156	46	56	1927	....	....	2.35	5.9	12.1	29.3	41.2	55.3	0.7	...	....	5.7	11.2	14.5	29.4	71	100	134
58	Philadelphia, Pa.	156	46	101	1920	....	....	2.35	5.9	15.8	29.3	42.7	61.2	0.7	...	....	5.5	10.5	13.8	37.0	69	100	143

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No.	Station	Elevation above sea, ft.	Years of record: For day or less	Years of record: For month and year	Years of record: Last year included	Maximum observed precipitation: Number of minutes 5	15	60	Day of 24 hr.	Month	Annual precipitation: Minimum	Mean	Maximum	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods, Number of minutes: 5	15	60	Mean for month of maximum record	Maximum: In 24 hr.	In one calendar month	Annual: Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
59	West Chester, Pa.	455	59	72	1920	....	....	....	6.6	14.6	33.0	49.1	73.0	0.7	...	....	....	9.8	13.4	29.7	67	100	149
60	Reading, Pa.	325	30	60	1927	....	....	....	5.4	13.3	31.5	40.8	65.0	0.8	...	....	....	9.0	13.2	32.6	77	100	159
61	Lebanon, Pa.	469	26	82	1920	....	....	....	5.2	12.9	28.9	44.4	64.1	0.8	...	....	....	10.4	11.7	29.0	65	100	145
62	Harrisburg, Pa.	361	32	55	1920	....	....	....	6.2	13.6	26.0	39.4	57.1	0.8	...	....	....	9.7	15.8	34.7	66	100	146
63	Altoona, Pa.	1,615	25	37	1920	....	....	....	4.3	9.8	21.2	34.3	44.4	...	...	....	....	11.4	12.5	28.6	62	100	129
64	Johnstown, Pa.	1,184	28	40	1920	....	....	....	4.5	12.2	37.5	47.8	58.7	0.6	...	....	....	9.6	9.4	25.5	78	100	123
65	Freeport, Pa.	750	..	44	1920	....	....	....	....	8.9	30.8	42.0	57.9	...	...	....	....	7.9	....	21.2	73	100	138
66	Franklin, Pa.	955	25	55	1920	....	....	....	2.8	11.6	34.0	42.5	59.7	0.6	...	....	....	10.5	6.6	27.3	80	100	140
67	Erie, Pa.	714	46	48	1920	....	....	....	5.8	9.3	26.7	37.9	55.0	0.6	...	....	....	8.9	15.4	24.6	70	100	145
68	Pittsburgh, Pa.	842	57	57	1927	0.71	1.20	1.65	4.1	9.5	25.7	36.2	50.6	0.6	2.0	3.3	4.6	11.6	11.3	26.3	71	100	140
69	Pittsburgh, Pa.	842	57	91	1927	0.71	1.20	1.65	4.1	9.5	25.3	36.2	50.6	0.6	2.0	3.3	4.6	10.7	11.3	26.3	70	100	140
70	Somerset, Pa.	2,280	..	42	1920	....	....	....	4.4	14.3	32.3	51.2	66.3	0.7	...	....	....	9.8	8.6	28.0	63	100	130
71	Morgantown, W. Va.	1,250	..	51	1923	....	....	....	....	11.2	28.1	43.3	69.5	0.9	...	....	....	11.5	....	25.8	65	100	161
72	Parkersburg, W. Va.	638	25	43	1927	0.75	1.48	2.12	3.8	11.5	24.3	39.5	62.7	0.7	1.9	3.7	5.4	11.2	9.6	29.0	62	100	158
73	Weston, W. Va.	1,026	..	37	1924	....	....	....	....	15.1	31.2	47.3	68.2	0.8	...	....	....	10.6	....	31.9	66	100	144
74	Elkins, W. Va.	1,963	23	29	1927	0.56	1.05	2.37	4.3	11.1	38.8	47.1	65.4	0.6	1.2	2.2	5.1	11.7	9.2	23.5	82	100	139
75	Washington, D. C.	112	57	85	1927	0.77	1.51	2.79	5.8	14.4	18.8	40.5	61.3	0.8	1.9	3.7	6.9	10.0	14.4	35.5	46	100	152
76	Washington, D. C.	112	53	53	1923	0.77	1.51	2.79	5.8	14.4	30.8	42.2	61.3	0.8	1.8	3.6	6.6	9.8	13.8	34.2	73	100	145
77	Baltimore, Md.	115	57	111	1927	....	....	....	6.1	12.3	22.4	40.3	62.3	0.8	...	....	....	9.9	15.1	30.6	56	100	155
78	Baltimore, Md.	115	57	57	1927	....	....	....	6.1	12.3	31.6	42.3	62.3	0.8	...	....	....	10.4	14.4	29.1	75	100	147
79	Trenton, N. J.	60	28	43	1913	....	....	1.70	5.4	15.2	34.3	48.8	67.2	1.0	...	....	3.5	10.6	11.1	31.1	70	100	138
80	Newark, N. J.	159	..	78	1920	....	....	....	....	22.5	34.5	47.8	69.1	0.8	...	....	....	10.6	....	47.0	72	100	145
81	Atlantic City, N. J.	16	48	48	1920	....	....	4.73	9.2	14.9	28.1	40.5	61.1	0.8	...	....	11.6	10.7	22.7	36.8	69	100	151

82	Fortress Monroe, Va.	8	..	55	1890	....	....	....	....	18.0	19.3	42.6	74.1	1.0	...	....	....	10.9	....	42.2	45	100	174
83	Norfolk, Va.	91	57	57	1927	0.66	1.37	2.72	6.3	11.9	30.4	45.6	70.7	0.8	1.4	3.0	6.0	7.4	13.8	26.1	67	100	155
84	Richmond, Va.	144	22	57	1927	0.70	1.55	2.97	7.3	14.0	27.6	41.6	72.0	0.8	1.7	3.7	7.1	11.0	17.6	33.6	66	100	173
85	Charlottesville, Va.	854	..	45	1920	....	....	....	....	14.4	30.1	44.8	65.3	0.9	...	....	....	10.4	....	32.1	67	100	146
86	Lexington, Va.	1,060	..	58	1920	....	....	....	....	15.9	30.2	40.2	60.0	0.9	...	....	....	8.6	....	39.5	75	100	149
87	Lynchburg, Va.	681	17	50	1920	0.56	1.51	3.49	7.2	12.4	30.5	41.7	60.6	0.8	1.3	3.6	8.4	9.8	17.2	29.7	73	100	145
88	Danville, Va.	413	..	30	1920	....	....	....	....	11.9	32.0	42.1	54.9	1.0	...	....	....	11.1	....	28.2	76	100	130
89	Wytheville, Va.	2,293	18	48	1920	....	....	....	4.2	13.1	30.9	40.4	62.6	0.8	...	....	....	10.6	10.4	32.5	76	100	155
90	Asheville, N. C.	2,255	18	19	1920	0.83	1.71	2.70	7.92	11.7	30.1	39.9	51.1	0.8	2.1	4.3	6.8	11.8	19.9	29.3	75	100	128
91	Murphy, N. C.	1,614	..	49	1920	....	....	....	....	15.4	45.6	59.2	84.8	0.9	..	....	....	8.0	....	26.0	77	100	143
92	Rock House, N. C. next to maximums	3,100	..	29	1920	....	....	....	....	36.4	62.9	84.3	113.8 / 110.6	0.8	...	....	....	11.2	....	43.1	74	100	135
93	Charlotte, N. C.	779	18	44	1922	0.59	1.12	1.98	5.0	16.5	34.8	46.9	62.4	0.8	1.3	2.4	4.2	11.3	10.6	35.2	74	100	146
94	Raleigh, N. C.	390	41	41	1927	0.75	1.74	3.01	5.7	13.6	32.1	46.4	63.4	0.8	1.6	3.7	6.5	11.8	12.3	29.3	69	100	136
95	Hatteras, N. C.	11	..	51	1925	....	....	....	....	16.3	29.6	54.9	102.0	0.7	...	....	....	10.4	....	29.7	54	100	186
96	Wilmington, N. C.	52	50	50	1920	0.66	1.06	2.25	8.0	21.1	27.7	48.8	83.6	0.8	1.4	2.2	4.6	13.4	16.4	43.2	57	100	171
96-A	Effingham, S. C.	106	..	30	1920	....	....	....	13.2	24.9	34.3	48.9	70.1	...	...	....	....	11.8	27.0	50.9	70	100	144
97	Charleston, S. C.	48	23	136	1927	0.56	1.25	2.72	9.5	19.2	23.7	47.5	78.4	0.9	1.2	2.6	5.7	13.8	20.0	40.4	50	100	165
98	Columbia, S. C.	351	20	48	1920	0.74	1.28	2.28	4.9	10.1	30.9	44.0	63.2	0.9	1.7	2.9	5.2	7.6	11.2	23.0	70	100	144
99	Camden, S. C.	222	..	72	1920	....	....	....	....	14.9	30.1	44.5	57.3	1.0	...	....	....	11.8	....	33.5	68	100	129
100	Greenville, S. C.	1,039	11	35	1920	0.52	1.23	3.43	8.2	19.5	42.7	52.0	77.8	0.9	1.0	2.4	6.6	9.7	15.8	37.5	82	100	149
100-A	Anderson, S. C.	764	26	..	1920	....	....	....	11.6	18.4	38.4	47.4	64.3	...	...	....	....	11.3	24.5	38.8	81	100	136
101	Augusta, Ga.	180	18	66	1920	1.24	2.11	3.08	5.6	14.0	35.8	46.5	60.7	0.9	2.7	4.5	6.6	10.6	12.1	30.1	77	100	130
102	Atlanta, Ga.	1,218	48	48	1926	....	....	....	7.4	15.8	33.1	48.8	65.2	0.9	...	....	....	10.3	15.2	32.4	68	100	134
103	Macon, Ga.	370	28	53	1926	....	....	....	6.8	13.4	31.7	44.9	61.9	0.9	...	....	....	10.8	15.2	29.8	71	100	138
104	Savannah, Ga.	65	24	57	1927	0.72	1.21	3.56	10.1	22.9	32.6	48.5	73.9	0.9	1.5	2.5	7.4	11.2	20.8	47.2	67	100	152
104-A	St. George, Ga	76	..	15	1920	....	....	....	18.0	24.2	30.9	49.6	63.2	...	...	....	....	17.7	36.3	48.8	62	100	128
105	Jacksonville, Fla.	222	33	56	1926	0.78	1.65	3.13	9.9	21.8	32.6	50.7	82.0	0.8	1.5	3.3	6.2	14.6	19.6	43.1	64	100	162
106	St. Augustine, Fla.	10	..	62	1923	....	....	....	....	23.9	31.6	47.7	71.8	1.1	...	....	....	13.6	....	50.0	66	100	150
107	Jupiter, Fla.	21	24	25	1920	0.65	2.12	3.83	13.2	21.4	39.8	59.5	87.1	0.7	1.1	3.6	6.4	17.3	22.2	36.0	67	100	146
108	Miami, Fla.	83	16	45	1926	0.58	1.34	3.05	9.0	27.9	33.1	60.0	89.1	0.8	1.0	2.2	5.1	15.2	15.0	46.5	55	100	148
109	Key West, Fla. next to maximums	16	50	77	1920	0.65	1.45	3.35	11.9	19.8	20.5	37.3	69.7	0.9	1.7	3.9	9.0	13.7	31.9	53.0	55	100	187
110	Tampa, Fla.	103	31	68	1927	1.00	1.90	3.30	6.6	24.5	32.2	50.2	89.9	0.8	2.0	3.8	6.6	17.8	13.1	48.8	64	100	179
111	Apalachicola, Fla.	24	24	24	1926	0.78	1.65	2.72	10.1	27.7	30.7	57.4	77.8	0.8	1.4	2.9	4.7	14.8	17.6	48.2	53	100	135
112	Pensacola, Fla.	151	21	48	1927	0.78	2.27	4.27	10.7	18.6	35.6	57.4	86.0	0.9	1.4	3.9	7.4	13.6	18.6	32.4	62	100	150
113	Mobile, Ala.	84	..	57	1927	....	....	....	....	26.7	39.5	61.8	92.3	0.8	...	....	....	8.9	....	43.2	64	100	149
114	Montgomery, Ala.	240	56	56	1927	....	....	....	10.0	17.8	37.0	51.7	77.9	0.9	...	....	....	10.1	19.4	34.5	72	100	151
115	Birmingham, Ala.	701	..	40	1927	....	....	....	8.8	20.1	34.3	52.2	76.2	0.8	...	....	....	10.2	16.8	38.5	66	100	146
116	Florence, Ala.	574	..	37	1920	....	....	....	....	16.1	40.7	51.2	68.4	1.1	...	....	....	11.2	....	31.4	79	100	134

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No.	Station	Elevation above sea, ft.	Years of record: For day or less	Years of record: For month and year	Years of record: Last year included	Maximum observed precipitation: Number of minutes 5	15	60	Day of 24 hr.	Month	Annual precipitation: Minimum	Mean	Maximum	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods, Number of minutes 5	15	60	Mean for month of maximum record	Maximum: In 24 hr.	In one calendar month	Annual: Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
117	Meridian, Miss.	375	32	34	1920	0.62	1.53	3.66	9.5	20.1	34.9	52.7	73.4	0.9	1.2	2.9	7.0	8.8	18.0	38.2	68	100	140
118	Vicksburg, Miss.	247	41	69	1920	0.83	1.41	3.11	8.0	22.2	38.0	52.0	84.2	0.9	1.6	2.7	6.0	10.2	15.4	42.7	73	100	162
119	New Orleans, La.	51	57	81	1927	....	....	....	14.0	22.7	31.1	56.8	85.7	0.8	...	....	....	10.1	24.6	40.0	55	100	151
120	Shreveport, La.	249	56	56	1927	0.59	1.20	3.15	8.6	16.5	23.1	44.3	66.6	1.0	1.3	2.7	7.1	6.9	19.4	37.3	52	100	150
121	Dallas, Tex.	466	16	51	1927	0.81	1.62	2.33	6.1	13.7	18.0	37.3	59.5	1.1	2.2	4.3	6.2	8.3	16.4	36.8	48	100	160
122	Fort Worth, Tex.	670	30	34	1927	0.56	1.40	2.75	8.8	17.6	17.9	31.6	48.4	1.3	1.8	4.4	8.7	13.0	27.9	55.7	57	100	153
123	Palestine, Tex.	510	41	41	{1922 1920}	0.59	1.56	4.46	6.3	17.2	24.0	40.7	61.2	1.1	1.4	3.8	10.9	11.1	15.5	42.2	59	100	150
124	Taylor, Tex.	583	22	25	1922	0.90	2.53	4.25	23.1	25.9	14.6	31.9	51.5	1.1	1.8	7.9	13.3	10.8	72.4	81.2	46	100	161
125	Austin, Tex.	593	..	65	1920	....	....	....	....	19.8	15.6	33.2	64.7	1.6	...	....	....	19.6	....	59.6	47	100	194
126	Houston, Tex.	138	32	45	1926	....	....	....	7.5	19.2	30.6	47.4	72.3	...	...	....	....	10.4	15.8	40.5	65	100	154
127	Galveston, Tex.	69	56	56	1926	....	....	5.32	14.4	26.0	21.4	47.1	78.4	1.0	...	....	11.3	11.5	30.6	55.3	45	100	167
128	San Antonio, Tex.	701	36	50	1920	....	....	....	7.1	11.7	10.1	27.7	50.3	1.2	...	....	....	11.9	25.6	42.2	36	100	182
129	Corpus Christi, Tex.	20	41	41	1927	....	....	....	8.3	13.8	5.4	24.9	48.2	1.2	...	....	....	16.4	33.3	55.5	22	100	194
130	Brownsville, Tex.	38	..	50	1920	....	....	....	....	30.6	12.3	26.4	60.1	1.5	...	....	....	20.2	....	116.0	47	100	228
131	Eagle Pass, Tex.	800	..	45	1920	....	....	....	....	13.5	7.0	20.8	44.4	2.3	...	....	....	12.4	....	65.0	34	100	213
132	El Paso, Tex.	3,762	48	48	1926	....	....	....	6.5	9.2	2.2	9.2	19.2	2.0	...	....	....	21.6	70.7	100.0	24	100	209
133	Abilene, Tex.	1,738	41	41	1926	0.67	1.51	3.47	6.8	15.7	10.9	24.4	41.5	0.7	2.7	6.2	14.2	9.6	27.9	64.4	45	100	171
134	Amarillo, Tex.	3,676	36	46	1927	0.53	1.19	1.95	4.0	9.8	11.1	22.0	39.7	1.4	2.4	5.4	8.9	13.7	18.2	44.5	50	100	181
135	Oklahoma, Okla.	1,247	..	30	1920	....	....	....	....	12.3	15.7	31.0	52.0	1.2	...	....	....	11.3	....	39.6	51	100	168
136	Fort Gibson, Okla	556	..	50	1920	....	....	....	....	13.1	18.8	37.2	59.3	1.2	...	....	....	9.6	....	35.2	50	100	160
137	Bentonville, Ark.	1,303	17	17	1922	0.75	1.58	2.23	5.1	11.2	34.6	41.4	56.3	0.9	1.8	3.8	5.4	6.2	12.3	29.5	84	100	136
138	Fort Smith, Ark.	457	41	41	1922	0.57	1.14	2.06	5.1	15.0	19.8	38.3	64.6	1.1	1.5	3.0	5.4	8.8	13.3	39.2	52	100	169
139	Little Rock, Ark.	357	31	48	1927	0.60	1.12	2.42	9.6	16.9	31.6	48.2	75.5	0.9	1.2	2.3	5.0	8.0	19.9	35.0	66	100	156

140	Memphis, Tenn.	409	57	57	1927	0.78	1.08	1.97	9.7	18.2	33.4	48.5	73.5	0.9	1.6	2.2	4.1	7.9	20.0	37.5	69	100	152
141	Nashville, Tenn.	624	18	57	1927	0.75	1.26	2.09	6.0	14.5	32.7	47.2	67.2	0.8	1.6	2.7	4.4	9.9	12.7	30.7	69	100	142
142	Chattanooga, Tenn.	762	19	42	1920	0.60	1.31	2.55	7.6	15.3	32.7	51.3	70.6	0.8	1.2	2.6	5.0	9.3	14.8	29.8	64	100	138
143	Knoxville, Tenn.	996	50	53	1920	0.57	1.24	2.04	6.2	17.3	35.1	48.5	73.9	0.7	1.2	2.6	4.2	9.2	12.8	35.6	72	100	152
144	Lexington, Ky.	989	39	62	1920	0.34	0.91	2.37	5.4	11.1	28.7	44.5	63.3	0.7	0.8	2.0	5.3	8.7	12.1	25.0	64	100	142
145	Louisville, Ky.	525	50	57	1927	0.55	1.20	1.59	5.5	16.5	29.2	43.7	56.5	0.8	1.1	2.7	3.6	8.9	12.6	37.8	67	100	129
146	Evansville, Ind.	431	31	51	1927	0.53	1.19	2.79	6.9	14.6	29.0	43.0	70.6	0.9	1.2	2.8	6.5	7.4	16.1	34.0	67	100	164
147	Cario, Ill.	358	50	57	1927	0.34	0.88	3.15	5.7	15.0	26.5	41.3	64.1	0.9	0.8	2.1	7.6	9.4	13.8	36.4	64	100	156
148	St. Louis, Mo.	567	57	91	1927	0.56	1.07	2.95	7.0	17.1	23.4	39.7	68.8	0.9	1.4	2.7	7.4	11.4	17.6	43.0	59	100	173
149	Hannibal, Mo.	534	25	43	1922	0.56	1.39	2.09	5.4	11.6	18.3	33.4	48.6	0.9	1.7	4.2	6.3	11.3	16.2	34.8	55	100	146
150	Columbia, Mo.	784	20	37	1922	0.70	1.22	2.73	6.6	11.5	21.3	37.6	54.6	0.9	1.9	3.2	7.3	12.1	17.6	30.6	57	100	145
151	Springfield, Mo.	1,302	40	50	1927	....	....	....	6.5	15.2	26.2	42.3	65.3	...	...	....	....	11.6	15.4	36.0	62	100	155
152	Kansas City, Mo.	963	33	50	1922	0.57	1.15	4.74	7.0	16.2	19.0	36.8	49.5	1.0	1.5	3.1	12.9	11.5	19.0	44.0	52	100	125
153	Oregon, Mo.	1,048	30	68	1922	....	....	....	7.0	15.0	21.4	35.6	49.7	0.9	...	....	....	13.6	19.7	42.1	60	100	140
154	Leavenworth, Kans.	913	..	87	1922	....	....	....	....	15.8	14.6	34.4	59.6	...	...	....	....	14.5	....	46.0	42	100	173
155	Manhattan, Kans.	1,031	29	65	1922	....	....	....	5.9	13.0	15.1	31.4	44.8	...	...	....	....	14.4	18.8	41.4	48	100	143
156	Dodge City, Kans.	2,509	48	49	1922	0.55	1.25	3.47	6.0	12.8	10.1	20.1	33.5	1.3	2.7	6.2	17.3	14.4	29.8	63.7	50	100	166
157	Wallace, Kans.	3,303	30	53	1922	....	....	....	6.1	12.6	7.4	16.2	34.0	...	...	....	....	19.4	37.6	77.8	46	100	210
158	North Platte, Nebr.	2,821	..	52	1925	....	....	....	....	8.5	11.2	18.4	32.7	1.2	...	....	....	14.8	....	46.2	61	100	178
159	Lincoln, Nebr.	1,189	30	51	1926	....	....	....	8.4	14.2	15.1	28.1	41.2	1.0	...	....	....	12.9	29.9	50.6	54	100	147
160	Omaha, Nebr.	1,103	57	57	1927	0.67	1.27	2.53	7.0	12.7	15.5	28.5	48.9	1.0	2.3	4.5	8.0	16.6	24.5	44.6	54	100	172
161	Des Moines, Iowa	861	42	43	1920	0.66	1.36	2.65	5.1	15.8	18.2	32.1	56.8	0.9	2.1	4.2	8.3	14.6	15.9	40.2	57	100	177
161-A	Cedar Rapids, Iowa	733	39	39	1920	....	....	....	7.8	15.4	18.7	31.1	44.9	...	...	....	....	10.8	25.1	49.5	60	100	144
162	Dubuque, Iowa	698	18	68	1920	0.80	1.52	2.70	5.4	14.2	19.3	33.4	55.1	0.9	2.4	4.6	8.1	13.2	16.2	42.5	58	100	165
163	Davenport, Iowa	606	55	55	1926	0.54	1.38	2.71	5.2	12.7	17.3	32.4	46.8	0.9	1.7	4.3	8.4	10.9	26.1	39.2	53	100	145
164	Keokuk, Iowa	614	13	50	1920	0.56	1.18	2.38	4.8	12.7	21.8	33.6	51.6	1.0	1.7	3.5	7.1	11.4	14.3	37.8	65	100	154
165	Springfield, Ill.	644	48	49	1927	0.66	1.41	2.75	5.9	15.2	22.8	36.4	58.2	0.9	1.8	3.9	7.6	10.1	16.2	41.8	63	100	160
166	Peoria, Ill.	609	..	72	1927	0.48	1.04	2.24	5.5	12.3	23.2	34.7	53.3	0.9	1.4	3.0	6.4	11.1	15.8	35.5	67	100	154
167	Chicago, Ill.	824	50	84	1927	0.42	0.90	1.53	6.2	11.3	22.4	33.1	47.1	0.8	1.3	2.7	4.6	9.5	18.7	34.1	68	100	142
168	Fort Wayne, Ind.	856	24	52	1928	0.81	1.27	2.27	4.9	10.6	26.7	37.1	47.0	...	2.2	3.4	6.1	10.9	13.1	28.6	72	100	127
169	Indianapolis, Ind.	822	52	57	1927	0.83	1.30	2.66	6.8	13.1	30.3	40.4	57.6	0.8	2.0	3.2	6.6	9.7	16.8	32.5	75	100	143
170	Terre Haute, Ind.	576	22	34	1928	1.15	1.38	2.57	5.2	11.4	29.3	38.8	48.3	0.8	3.0	3.6	6.7	9.4	13.5	29.6	76	100	126
171	Cincinnati, Ohio	628	51	93	1927	0.78	1.78	2.60	5.2	11.7	18.0	40.7	65.2	0.8	1.9	4.4	6.4	8.9	12.8	28.7	44	100	160
172	Portsmouth, Ohio	527	..	92	1921	....	....	....	3.5	11.1	24.3	41.2	57.6	0.9	...	....	....	8.2	8.5	27.0	59	100	140
173	Marietta, Ohio	627	28	104	1921	....	....	....	4.3	12.4	26.0	41.2	61.8	0.8	...	....	....	11.2	10.4	30.1	63	100	150
174	Columbus, Ohio	918	44	50	1927	0.38	1.00	2.80	3.8	9.6	26.5	36.4	51.3	0.7	1.0	2.7	7.7	9.9	10.4	26.4	73	100	141
175	Cleveland, Ohio	762	51	56	1926	0.44	1.27	1.80	5.0	9.8	24.5	34.1	53.5	0.6	1.3	3.7	5.6	9.6	14.6	28.7	72	100	157
176	Sandusky, Ohio	629	44	61	1927	0.56	1.34	2.22	4.2	10.6	23.5	33.1	46.3	0.7	1.7	4.0	6.7	8.9	12.7	32.0	71	100	140
177	Toledo, Ohio	769	51	60	1920	0.60	1.77	3.58	6.0	10.2	21.3	31.7	45.9	0.8	1.9	5.6	11.3	8.0	18.9	32.1	67	100	145

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of Record: For day or less	Years of Record: For month and year	Years of Record: Last year included	Maximum observed precipitation: Number of minutes: 5	15	60	Day of 24 hr.	Month	Annual precipitation: Minimum	Mean	Maximum	Observed precipitation, in percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	15	60	Mean for month of maximum record	Maximum: In 24 hr.	In one calendar month	Annual: Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
178	Detroit, Mich.	730	57	57	1927	....	....	....	4.7	8.8	21.1	32.0	47.7	0.7	...	....	....	10.4	14.7	27.5	66	100	149
179	Lansing, Mich.	863	..	65	1927	....	....	....	....	11.3	19.4	30.8	48.4	0.9	...	....	....	11.7	....	36.7	63	100	157
180	Alpena, Mich.	609	56	56	1927	....	....	....	3.9	13.2	17.4	30.7	45.6	0.7	...	....	....	9.8	12.7	43.0	57	100	148
181	Marquette, Mich.	734	..	55	1925	....	....	....	....	8.6	19.7	31.0	41.2	0.6	...	....	....	11.1	....	27.7	64	100	133
182	Milwaukee, Wis.	681	57	57	1927	....	....	....	5.8	10.0	18.7	30.8	50.4	0.8	...	....	....	11.3	18.8	32.5	61	100	164
183	Madison, Wis.	974	61	61	1920	....	....	....	5.2	10.7	13.5	31.2	52.9	0.9	...	....	....	10.8	16.7	34.3	43	100	169
184	La Crosse, Wis.	714	..	48	1920	0.31	0.87	1.80	7.2	12.1	16.8	31.1	44.7	0.9	1.0	2.8	5.8	7.8	23.2	38.9	54	100	144
185	St. Paul, Minn.	940	56	90	1926	0.40	1.00	2.20	5.7	11.7	10.2	27.2	49.7	0.9	1.5	3.7	8.1	15.2	20.9	43.0	37	100	183
186	Duluth, Minn.	1,133	50	57	1927	0.41	1.06	2.26	5.3	13.3	18.1	28.5	45.3	0.8	1.4	3.7	7.9	12.2	18.6	46.7	64	100	159
187	Moorhead, Minn.	935	46	46	1926	0.53	1.12	2.21	5.2	9.6	9.9	23.1	34.0	0.9	2.3	4.4	9.6	12.4	22.5	41.6	43	100	147
188	Bismarck, N. Dak.	1,674	..	51	1925	....	....	....	....	9.9	11.0	17.0	30.9	1.1	...	....	....	20.3	....	58.2	65	100	182
189	Williston, N. Dak.	1,875	20	27	1920	....	....	....	4.2	8.8	8.6	14.4	22.0	1.1	...	....	....	22.3	29.1	61.0	60	100	153
190	Huron, S. Dak.	1,306	39	40	1920	....	....	....	3.9	11.6	10.2	20.9	30.1	1.0	...	....	....	18.4	18.6	55.4	49	100	144
191	Pierre, S. Dak.	1,572	29	30	1920	....	....	....	3.7	6.1	7.8	17.0	23.6	1.2	...	....	....	16.5	21.8	35.9	46	100	139
192	Rapid City, S. Dak.	3,251	33	33	1920	....	....	....	4.8	9.7	9.6	17.7	27.1	1.0	...	....	....	13.6	27.1	54.8	54	100	153
193	Sheridan, Wyo.	3,790	15	28	1920	0.49	0.85	1.32	2.1	6.6	8.0	14.4	22.5	0.9	3.4	5.9	9.2	18.6	14.6	45.8	56	100	156
194	Cheyenne, Wyo.	6,088	50	50	1920	0.30	0.88	1.18	4.7	7.7	5.0	14.0	22.7	1.1	2.1	6.3	8.4	12.9	33.6	55.0	36	100	162
195	Yellowstone Park, Wyo.	6,200	32	40	1927	0.23	0.33	0.63	2.3	8.9	7.9	18.0	28.6	0.8	1.3	1.8	3.5	8.2	12.8	49.4	44	100	159
196	Miles City, Mont.	2,371	31	43	1920	....	....	....	3.4	5.9	9.1	13.5	22.7	1.2	...	....	....	11.7	25.2	43.7	67	100	168
197	Havre, Mont.	2,505	19	41	1920	0.43	0.77	1.60	3.7	9.7	6.8	13.5	25.7	1.1	3.2	5.7	11.9	13.3	27.4	71.8	50	100	191
198	Kalispell, Mont.	2,973	24	26	1921	0.43	0.90	1.03	1.8	5.2	10.7	15.2	19.7	0.8	2.8	5.9	6.8	10.1	11.8	34.2	70	100	130
199	Helena, Mont.	4,110	42	47	1926	0.24	0.49	1.04	3.7	6.2	6.7	13.4	20.0	1.0	1.8	3.7	7.8	15.9	27.6	46.2	50	100	149

200	Pocatello, Idaho	4,477	..	..	1920	....	....	....	1.6	4.3	9.3	13.9	22.4	1.1	...	....	....	11.3	11.5	30.9	67	100	161
201	Boise, Idaho	2,739	46	64	1927	....	....	....	2.7	7.7	6.0	13.5	25.8	1.1	...	....	....	11.6	20.0	57.0	44	100	191
202	Lewiston, Idaho	757	..	41	1920	....	....	....	1.9	6.3	9.1	13.4	21.7	0.9	...	....	....	10.5	14.2	47.0	68	100	162
203	Spokane, Wash	1,943	19	46	1926	0.29	0.38	0.77	2.2	5.8	9.9	16.3	26.0	0.9	1.8	2.3	4.7	12.9	13.5	35.6	61	100	159
204	Seattle, Wash	248	34	37	1927	0.29	0.52	0.80	3.5	11.8	21.7	32.9	45.8	0.7	0.9	1.6	2.4	16.1	10.6	35.9	66	100	139
205	Forks (N. W.), Wash	480	..	15	1921	....	....	....	....	31.3	108.4	122.3	153.8	0.6	...	....	....	14.8	....	25.6	89	100	126
206	North Head, Wash	211	19	44	1926	0.27	0.37	0.88	4.3	17.3	39.9	55.5	78.6	0.5	0.5	0.7	1.6	16.5	7.7	31.2	72	100	142
207	Astoria, Ore	24	..	66	1922	....	....	....	....	24.7	49.4	77.0	101.4	0.5	...	....	....	15.7	....	32.1	64	100	132
208	Portland, Ore	57	57	57	1927	0.40	0.83	1.31	7.7	20.1	30.8	42.8	67.2	0.6	0.9	1.9	3.1	15.4	18.0	47.0	72	100	157
209	Cascade Locks, Ore	100	..	44	1922	....	....	....	....	35.1	50.1	78.4	100.6	0.6	...	....	....	16.7	....	44.8	64	100	128
210	The Dalles, Ore	96	..	65	1922	....	....	....	....	10.8	7.5	16.3	43.6	...	...	....	....	18.9	....	66.2	46	100	268
211	Baker, Ore	3,471	31	31	1920	....	....	....	1.8	4.0	7.7	12.3	18.2	1.0	...	....	....	10.7	14.6	32.5	63	100	148
212	Roseburg, Ore	510	45	45	1922	0.55	1.10	1.14	4.1	12.8	21.1	33.3	44.3	0.8	1.7	3.3	3.4	15.7	12.3	38.5	63	100	133
213	Monumental, N. W. Calif	2,700±	..	6	1910	....	....	....	....	43.8	69.3	109.1	153.5	...	...	....	....	19.4	....	40.0	63	100	140
214	Crescent City, Calif	125	..	34	1923	....	....	....	....	31.9	47.5	74.8	107.6	0.9	...	....	....	13.5	....	42.6	63	100	144
215	Fort Gaston, Calif	397	..	30	1892	....	....	....	....	34.5	34.8	54.7	71.0	...	...	....	....	13.6	....	63.0	64	100	130
216	Eureka, N. W., Calif	64	37	37	1923	....	....	....	5.1	19.5	22.4	41.8	64.5	0.8	...	....	....	16.3	12.2	46.6	54	100	154
217	Weaverville, Calif	2,162	..	34	1923	....	....	....	....	19.8	21.6	37.2	54.8	...	...	....	....	19.4	....	53.2	58	100	147
218	Delta, Calif	1,138	..	34	1916	....	....	....	....	53.3	28.1	65.0	114.8	...	...	....	....	19.6	....	82.2	43	100	177
219	Red Bluff, Calif	307	47	48	1925	0.26	0.57	1.72	6.1	20.7	12.9	24.6	49.0	1.4	1.1	2.3	7.0	19.2	24.7	84.0	52	100	199
220	Marysville, Calif	67	..	52	1923	....	....	....	....	14.4	10.8	19.6	36.7	...	...	....	....	20.2	....	73.5	55	100	187
221	Helen Mine, Calif	2,750	..	22	1922	....	....	....	....	71.5	50.8	86.3	137.6	...	...	....	....	23.9	....	83.0	59	100	160
222	Santa Rosa, Calif	181	..	35	1923	....	....	....	....	18.4	13.6	29.9	46.5	1.4	...	....	....	22.4	....	65.0	46	100	156
223	Fort Ross, Calif	100	..	49	1923	....	....	....	....	37.4	21.7	51.9	88.2	...	...	....	....	21.9	....	72.0	42	100	170
224	Point Reyes, Calif	490	32	43	1923	0.20	0.69	1.40	3.1	11.4	7.3	20.6	32.4	1.3	1.0	3.4	6.8	16.6	15.0	55.2	36	100	157
225	Mt. Tamalpais, Calif	2,375	22	26	1923	....	....	....	3.0	15.6	13.5	27.5	42.2	1.2	...	....	....	21.2	10.9	56.6	49	100	153
226	San Francisco, Calif	206	57	79	1927	....	....	....	4.7	24.4	9.0	22.5	38.8	1.4	...	....	....	21.3	20.9	108.0	40	100	172
227	San Jose, Calif	95	17	49	1922	....	....	....	4.6	12.4	6.6	15.1	25.5	1.6	...	....	....	20.2	30.4	82.0	44	100	169
228	Boulder Creek, Calif	470	..	29	1916	....	....	....	....	39.4	21.2	55.6	95.8	...	...	....	....	25.4	....	71.0	38	100	172
229	Monterey, Calif	15	..	51	1915	....	....	....	....	11.7	7.2	16.7	29.0	...	...	....	....	20.8	....	70.0	43	100	174
230	San Luis Obispo, Calif	201	28	56	1925	....	....	....	6.0	18.2	6.9	21.1	50.0	1.9	1.2	1.9	5.8	23.8	28.4	86.0	33	100	237
231	Santa Barbara, Calif	130	..	56	1922	....	....	....	....	17.2	7.2	18.7	43.2	2.3	...	....	....	23.8	....	92.0	39	100	231
232	Los Angeles, Calif	361	47	51	1927	....	....	....	5.1	15.8	4.8	15.2	40.3	2.6	...	....	....	17.4	33.5	104.0	32	100	265
233	Lowe Observatory, Calif	3,420	..	24	1919	....	....	....	....	19.9	8.3	26.7	58.5	...	...	....	....	22.7	....	74.5	31	100	219
234	Mount Wilson, Calif	5,850	..	16	1919	....	....	....	....	24.5	8.2	33.8	52.9	...	...	....	....	25.3	....	72.4	24	100	156
235	Riverside, Calif	851	..	40	1919	....	....	....	....	9.7	3.9	10.7	25.3	2.6	...	....	....	21.7	....	90.6	36	100	236
236	San Diego, Calif	87	56	79	1927	....	....	....	3.2	9.3	4.3	9.8	27.6	2.3	...	....	....	19.2	32.6	95.0	44	.00	283
237	Campo, Calif	2,543	..	31	1919	....	....	8.6	....	20.4	11.3	19.8	35.3	...	...	....	43.5	18.7	....	103.0	57	100	178
238	Cuyamaca, Calif	4,677	..	33	1919	....	....	....	....	36.5	19.2	38.8	66.1	1.7	...	....	....	19.8	....	96.9	49	100	170
239	Sterling, Calif	255	..	42	1919	....	....	....	....	5.4	Tr.	2.2	7.4	...	...	....	....	19.0	....	245.0	0	100	336
240	Indio, Calif	−20	..	43	1919	....	....	....	....	6.0	0.1	2.8	7.1	8.3	...	....	....	29.3	....	214.4	4	100	254

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of Record: For day or less	Years of Record: For month and year	Years of Record: Last year included	Maximum observed precipitation: Number of minutes: 5	Maximum observed precipitation: Number of minutes: 15	Maximum observed precipitation: Number of minutes: 60	Maximum observed precipitation: Day of 24 hr.	Maximum observed precipitation: Month	Annual precipitation: Minimum	Annual precipitation: Mean	Annual precipitation: Maximum	Observed precipitation, in percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	Maximum during short periods: Number of minutes: 15	Maximum during short periods: Number of minutes: 60	Mean for month of maximum record	Maximum: In 24 hr.	Maximum: In one calendar month	Annual: Minimum	Annual: Mean	Annual: Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
241	San Bernardino, Calif.	1,054	..	49	1919	....	....	....	....	15.5	5.5	16.1	37.1	2.2	...	...	...	22.2	....	96.4	34	100	231
242	Squirrel Inn, Calif.	5,280	..	17	1919	....	....	....	16.7	50.3	19.6	39.0	82.2	...	...	...	...	24.9	42.7	128.6	50	100	210
243	Barstow, Calif.	2,105	..	24	1919	....	....	....	....	3.9	0.8	4.1	11.0	6.7	...	...	...	13.0	....	95.0	20	100	269
244	Mohave, Calif.	2,751	..	38	1914	....	....	....	....	7.3	0.2	4.9	14.8	...	...	...	...	20.2	....	149.0	4	100	302
245	Tehachapi, Calif.	3,964	..	39	1914	....	....	....	....	6.3	4.0	10.3	21.4	...	...	...	...	24.0	....	61.0	39	100	207
246	Bakersfield, Calif.	404	..	33	1922	....	....	....	....	3.2	3.0	5.7	9.3	...	...	...	...	19.8	....	56.1	53	100	164
247	Death Valley, Calif.	−178	..	16	1927	....	....	....	....	1.9	0.4	1.8	4.5	...	...	...	...	17.2	....	105.8	22	100	250
248	Independence, Calif.	3,957	..	36	1919	....	....	....	....	12.2	1.1	5.3	21.0+	3.7	...	...	...	21.9	....	230.0	21	100	396+
249	Lemon Cove, Calif.	600	..	24	1922	....	....	....	....	15.2	6.2	15.4	30.6	2.4	...	...	...	19.8	....	97.3	40	100	199
250	Fresno, Calif.	293	40	47	1927	....	....	....	2.9	9.5	3.9	9.7	21.5	2.2	2.7	5.1	10.0	10.3	29.9	96.2	40	100	222
251	Yosemite, Calif.	3,960	..	18	1922	....	....	....	....	24.6	17.3	34.8	69.1	...	...	...	...	21.3	....	70.7	50	100	199
252	Merced, Calif.	173	..	51	1922	....	....	....	....	8.0	4.3	11.2	23.8	2.4	...	...	...	21.5	....	71.4	38	100	213
253	Lick Observatory, Calif.	4,209	..	42	1922	....	....	....	....	33.8	15.8	30.4	90.1	1.3	...	...	...	19.5	....	111.2	52	100	297
254	Stockton, Calif.	23	..	56	1922	....	....	....	....	11.5	6.4	14.4	26.4	...	...	...	...	19.0	....	79.9	44	100	183
255	Mokelumne Hill, Calif.	1,550	..	41	1922	....	....	....	....	23.0	15.2	31.4	54.9	1.6	...	...	...	18.4	....	73.4	48	100	175
256	Tamarack, Calif.	8,000	..	22	1922	....	....	....	....	39.8	27.8	50.6	94.3	...	...	...	...	22.8	....	78.7	55	100	187
257	Placerville, Calif.	1,925	..	49	1922	....	....	....	....	28.8	19.4	13.2	74.1	...	...	...	...	19.6	....	66.8	45	100	172
258	Sacramento, Calif.	71	25	79	1927	0.29	0.64	0.94	7.2	15.0	7.8	18.3	34.9	1.7	1.6	3.5	5.2	19.6	38.4	82.0	43	100	190
259	Folsom, Calif.	252	..	53	1923	....	....	....	....	21.1	8.5	24.3	46.1	...	...	...	...	20.8	....	86.8	35	100	190
260	Auburn, Calif.	1,360	..	54	1923	....	....	....	....	23.1	16.6	33.7	60.6	...	...	...	...	19.7	....	68.5	49	100	180
261	Nevada City, Calif.	2,580	..	60	1923	....	....	....	....	41.9	25.7	53.0	102.8	...	...	...	...	18.6	....	79.0	49	100	194
262	Lake Spaulding, Calif.	4,600	..	30	1923	....	....	....	....	49.0	36.9	69.8	119.4	...	...	...	...	20.0	....	70.4	53	100	172
263	Tahoe, Calif.	6,230	..	15	1923	....	....	....	....	21.2	21.5	31.3	40.4	...	...	...	...	24.0	....	67.5	69	100	129
264	Truckee, Calif.	5,819	..	50	1920	....	....	....	....	20.9	6.7	26.2	43.8	...	...	...	...	22.2	....	79.6	26	100	167

265	Lewer's Ranch, Nev.	6,282	..	27	1915	....	....	....	....	21.5	10.7	25.2	48.5	...	...	...	...	21.0	....	85.8	43	100	193
266	Reno, Nev.	4,532	39	39	1926	....	....	....	2.7	6.8	4.7	8.2	13.7	2.0	...	...	...	20.0	33.0	83.0	57	100	167
267	Winnemucca, Nev.	4,344	36	51	1920	....	....	....	1.6	5.2	4.9	8.4	18.4	1.4	...	...	...	11.4	19.0	62.0	58	100	219
268	Beowawe, Nev.	4,695	..	51	1920	....	....	....	....	3.7	2.2	6.3	14.9	3.8	...	...	...	9.4	....	58.8	35	100	237
269	Clover Valley, Nev.	6,000	..	27	1920	....	....	....	....	8.7	7.5	13.8	24.4	...	...	...	...	15.1	....	63.0	54	100	177
270	Wells, N. E., Nev.	5,628	..	46	1915	....	....	....	....	4.7	3.4	9.1	21.6	2.7	...	...	...	14.6	....	51.6	37	100	238
271	Kelton, N. W., Utah	4,230	..	43	1920	....	....	....	....	3.7	1.5	6.4	14.5	3.6	...	...	...	10.0	....	57.8	23	100	226
272	Corinne, Utah	4,240	..	51	1920	....	....	....	....	5.4	5.4	12.6	22.3	2.4	...	...	...	7.8	....	42.9	43	100	177
273	Salt Lake City, Utah	4,408	54	54	1927	....	....	....	2.7	5.8	10.3	16.4	23.7	1.1	...	...	...	12.4	16.4	35.3	63	100	144
274	Strawberry Tunnel, Utah	7,606	..	13	1927	....	....	....	....	6.0	12.6	21.3	28.3	1.1	...	...	...	11.5	....	28.1	59	100	132
275	Modena, S. W., Utah	5,479	27	27	1927	....	....	....	2.3	4.7	5.1	11.3	19.1	1.5	...	...	...	12.4	20.4	41.6	45	100	169
276	St. George, S. W., Utah	2,800	..	38	1927	....	....	....	....	10.2	3.5	8.7	18.7	2.8	...	...	...	13.3	....	117.4	40	100	215
277	Alton, S., Utah	7,000	..	19	1920	....	....	....	....	12.0	12.2	22.7	34.8	1.9	...	...	...	17.4	....	52.8	54	100	153
278	Moab, S. E., Utah	4,000	..	32	1920	....	....	....	....	6.6	4.3	9.2	16.0	1.9	...	...	...	9.2	....	71.8	47	100	174
279	Fort Duchesne, Utah	4,941	..	33	1920	....	....	....	....	5.9	4.4	7.1	12.1	2.5	...	...	...	15.9	....	83.1	62	100	170
280	Steamboat Springs, Colo.	6,683	..	32	1922	....	....	....	....	6.0	13.9	22.9	32.1	1.1	...	...	...	7.8	....	21.8	61	100	140
281	Grand Junction, Colo.	4,602	19	37	1922	0.39	0.59	0.74	2.5	3.8	5.4	8.4	11.6	1.4	4.6	7.0	8.8	10.2	29.8	45.2	64	100	138
282	Durango, S. W., Colo.	6,550	..	37	1922	....	....	....	....	7.4	8.9	20.0	34.3	1.2	...	...	...	9.0	....	37.0	44	100	171
283	Wagon Wheel Gap, Colo.	9,610	..	25	1922	....	....	....	....	6.5	7.8	16.1	25.0	1.2	...	...	...	16.0	....	40.4	48	100	155
284	Gunnison, Colo.	7,670	..	31	1922	....	....	....	....	4.2	6.2	9.8	13.4	1.5	...	...	...	14.7	....	42.9	63	100	137
285	Marshall Pass, Colo.	10,844	..	18	1916	....	....	....	....	6.6	12.5	17.2	23.6	...	...	...	...	11.0	....	38.3	73	100	137
286	Leadville, Colo.	10,248	..	35	1922	....	....	....	....	7.7	11.7	16.7	25.8	0.9	...	...	...	13.2	....	46.1	70	100	154
287	Long's Peak, Colo.	8,956	..	28	1922	....	....	....	....	7.8	13.9	21.4	29.8	1.2	...	...	...	14.3	....	36.4	65	100	139
288	Greeley, Colo.	4,649	..	35	1922	....	....	....	....	6.3	5.4	12.6	20.9	1.8	...	...	...	13.6	....	50.0	43	100	166
289	Denver, Colo.	5,292	{25 56}	56	1927	0.91	1.54	2.20	6.5	8.6	7.7	14.3	23.0	1.2	6.4	10.8	15.4	23.3	45.4	60.0	54	100	161
290	Pikes Peak, Colo.	14,111	..	18	1894	....	....	....	....	20.5	9.3	30.0	44.6	0.7	...	...	...	8.6	....	68.3	31	100	149
291	Lake Moraine, Colo.	10,265	..	28	1922	....	....	....	....	16.5	18.5	24.7	33.5	1.1	...	...	...	13.1	....	86.9	75	100	136
292	Colorado Springs, Colo.	6,098	..	50	1922	....	....	....	....	7.3	8.6	14.4	24.5	1.4	...	...	...	15.6	....	50.8	60	100	170
293	Pueblo, Colo.	4,685	35	54	1927	0.45	0.99	2.07	2.9	11.9	6.1	11.5	20.3	1.5	3.9	8.6	18.0	11.4	25.2	103.4	53	100	177
294	Las Animas, Colo.	3,899	..	58	1922	....	....	....	....	7.5	3.8	12.3	19.3	2.3	...	...	...	12.5	....	61.2	31	100	157
295	Raton, N. Mex.	6,660	..	26	1921	....	....	....	....	11.3	8.0	16.0	23.8	1.9	...	...	...	19.3	....	70.7	50	100	149
296	Anchor Mine, N. Mex.	10,600	..	10	1920	....	....	....	....	11.0	28.9	35.8	43.1	0.9	...	...	...	16.1	....	30.7	81	100	120
297	Taos Canyon, N. Mex.	8,959	..	13	1921	....	....	....	....	7.7	12.2	21.5	26.3	1.0	...	...	...	15.8	....	35.8	57	100	122
298	Fort Union, N. Mex.	6,835	..	62	1918	....	....	....	....	12.6	6.3	18.4	29.2	...	...	...	...	20.8	....	68.5	34	100	159
299	Santa Fe, N. Mex.	7,013	57	76	1927	....	....	....	2.5	7.9	5.0	14.1	24.8	1.1	...	...	...	15.4	17.7	56.1	35	100	176
300	Albuquerque, N. Mex.	5,100	..	63	1921	....	....	....	....	8.1	3.3	7.8	16.3	4.3	...	...	...	9.7	....	103.9	42	100	209
301	San Marcial, N. Mex.	4,447	..	59	1920	....	....	....	....	11.9	1.1	9.2	24.6	3.8	...	...	...	20.3	....	129.3	12	100	267
302	Roswell, N. Mex.	3,578	27	27	1920	0.19	0.53	1.40	5.6	9.6	4.9	14.1	22.7	1.8	1.3	3.8	9.9	13.8	39.7	68.2	35	100	162
303	Las Cruces, N. Mex.	3,865	..	66	1920	....	....	....	....	5.8	3.5	8.5	17.1	2.4	...	...	...	20.1	....	68.4	41	100	201
304	Fort Bayard, N. Mex.	6,152	..	51	1920	....	....	....	....	9.6	5.8	15.5	31.2	1.6	...	...	...	20.6	....	62.0	37	100	201
305	Fort Apache, Ariz.	5,200	..	45	1921	....	....	....	....	9.3	8.7	18.3	33.1	1.4	...	...	...	18.6	....	50.8	48	100	181
306	Flagstaff, Ariz.	6,907	..	32	1920	....	....	....	....	12.6	15.7	22.8	34.5	1.2	...	...	...	12.4	12.7	55.3	69	100	151

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of record: For day or less	Years of record: For month and year	Years of record: Last year included	Maximum observed precipitation: Number of minutes: 5	15	60	Day of 24 hr.	Month	Annual precipitation: Minimum	Mean	Maximum	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	15	60	Mean for month of maximum record	Maximum: In 24 hr.	In one calendar month	Annual: Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
307	Grand Canyon, Ariz.	6,866	..	24	1926	...	...	...	....	8.5	10.7	18.2	36.7	1.7	...	...	...	8.5	....	46.6	59	100	202
308	Fort Mohave, Ariz.	604	..	46	1913	...	...	...	....	11.2	1.6	5.1	21.4	6.7	...	...	...	17.3	....	219.5	31	100	420
309	Prescott, Ariz.	5,320	..	53	1920	...	...	...	....	8.8	10.0	17.8	39.5	1.6	...	...	...	16.6	....	49.5	56	100	222
310	Phoenix, Ariz.	1,108	32	51	1926	...	...	...	5.0	6.5	3.0	7.5	19.7	2.7	...	...	...	14.0	66.7	86.5	40	100	263
311	Pinal Ranch, Ariz.	4,520	..	29	1921	...	...	...	....	14.9	11.8	25.3	58.4	1.8	...	...	...	14.5	....	58.6	47	100	231
312	Tucson, Ariz.	2,423	..	55	1921	...	...	...	....	6.3	5.3	11.7	24.2	2.2	...	...	...	22.2	....	53.7	45	100	207
313	Yuma, Ariz.	141	57	57	1926	...	...	...	4.0	6.2	0.6	3.4	11.4	6.8	...	...	...	17.0	117.5	182.3	18	100	335
314	Chihuahua, Mexico	4,670	..	13	1918	...	...	...	....	7.6	9.4	15.0	20.4	...	...	...	...	17.4	....	51.0	63	100	136
315	Mazarlan, Mexico	13	..	38	1920	...	...	...	....	28.0	10.7	28.4	57.3	...	...	...	...	29.4	....	98.0	38	100	202
316	Monterrey, Mexico	1,732	..	26	1924	...	...	...	....	17.2	3.4	19.5	37.9	..	...	...	...	22.6	....	88.0	17	100	194
317	Leon, Mexico	5,930	..	40	1920	...	...	...	....	16.1	12.4	25.2	36.7	...	...	...	...	18.0	....	64.0	49	100	146
318	Mexico City, Mexico	7,400	..	47	1924	...	...	...	....	13.6	13.1	22.8	34.4	...	...	...	...	20.3	....	59.5	57	100	150
319	Puebla, Mexico	7,050	..	41	1920	...	...	...	....	14.2	22.2	33.3	50.1	...	...	...	...	20.3	....	42.6	67	100	150
320	Oaxaca, Mexico	5,130	..	25	1915	...	...	...	....	16.4	16.7	26.9	44.0	...	...	...	...	23.5	....	61.0	62	100	164
321	Merida, S. E., Mexico	72	..	26	1924	...	...	...	....	17.7	16.0	35.1	64.2	...	...	...	...	15.5	....	50.1	46	100	183
322	Corozal, Belize	......	..	16	1899	...	...	...	....	17.5	33.7	52.5	66.5	...	...	...	...	12.7	....	33.3	64	100	127
323	Belize, Belize	......	..	24	1915	...	...	...	....	30.2	63.8	84.5	130.9	...	...	...	...	12.5	....	35.8	75	100	155
324	Setal, Guatemala	2,430	..	17	1913	...	...	...	....	49.9	139.0	193.1	263.6	...	...	...	...	14.7	....	25.9	72	100	136
325	Chimax, Guatemala	4,260	..	20	1913	...	...	...	....	27.1	68.9	96.0	116.9	...	...	...	...	14.1	....	28.3	72	100	122
326	Magdalena, Guatemala	7,220	..	11	....	...	...	...	....	11.3	.....	36.6	51.6	...	...	...	...	15.4	....	30.9	..	100	141
327	Guatemala, Guatemala	4,855	..	29	1913	...	...	...	....	22.6	25.0	51.8	71.7	0.7	...	...	...	15.5	....	43.8	48	100	139
328	San Francisco, Guatemala	2,430	..	25	....	...	...	...	....	49.0	.....	168.8	238.8	...	...	...	...	16.7	....	29.0	..	100	141

329	San Salvador, Salvador	2,155	..	23	1920	....	....	....	....	18.0	58.1	68.3	83.4	...	....	....	....	16.6	....	26.3	85	100	122
		Ma	xima	9	1920																		
330	Rivas, Nicaragua	150	..	21	1900	....	....	....	....	33.8	31.8	69.8	123.4	0.8	....	....	....	24.9	....	48.5	46	100	177
331	Greytown, S. E., Nicaragua	50±	..	6	1900	....	....	....	....	55.4	201.6	254.9	296.9	...	....	....	....	14.3	....	21.8	79	100	117
332	Sarapiqui, Costa Rica	2,750	..	6	1903	....	....	....	....	52.2	180.2	209.4	268.5	...	....	....	....	14.3	....	25.0	86	100	126
333	San Jose, Costa Rica	3,720	..	34	1904	....	....	....	....	28.7	19.5	70.8	154.3	0.6	....	....	....	16.6	....	40.5	28	100	218
334	Balboa, Canal Zone	118	16	24	1921	....	....	....	7.2	20.3	45.6	69.4	91.4	...	....	....	....	14.8	10.4	29.2	66	100	132
		Ma	xima	16	1921																		
335	Gamboa, Canal Zone	......	16	39	1921	....	....	....	6.6	28.4	62.0	90.8	136.2	...	....	....	....	13.0	7.3	31.3	68	100	150
336	Colon, Canal Zone	36	16	57	1925	....	....	....	8.5	42.5	86.5	127.3	183.4	...	....	....	....	16.3	6.7	33.4	68	100	132
337	Porto Bello, Panama	50±	10	10	1921	....	....	....	10.9	58.2	136.6	160.0	237.3	...	....	....	....	10.5	6.8	36.5	86	100	148
338	Cartagena, Colombia	50±	..	12	....	....	....	....	....	....	....	35.6	.....	...	....	....	....	....	....	....	..	100	
339	Maracaibo, Venezuela	26	9	11	....	....	....	....	4.4	....	....	20.9	.....	...	....	....	....	15.0	21.1	....	..	100	
340	Caracas, Venezuela	3,420	16	35	1925	....	....	....	3.3	8.4	22.6	31.7	47.4	...	....	....	....	13.5	10.4	26.5	71	100	150
341	Ciudad Bolivar, Venezuela	125	7	9	....	....	....	....	3.6	....	....	35.1	.....	...	....	....	....	8.6	10.2	....	..	100	
342	Calabozo, Venezuela	328	5	7	....	....	....	....	4.1	....	....	47.1	.....	...	....	....	....	2.5	8.7	....	..	100	
343	Merida, Venezuela	5,384	9	11	....	....	....	....	3.9	....	....	71.8	.....	...	....	....	....	14.1	5.4	....	..	100	
344	Bogota, Colombia	8,700	..	50	1923	....	....	....	....	15.5	25.4	41.6	56.8	...	....	....	....	9.8	....	37.2	61	100	136
345	Medellin, Colombia	4,950	..	15	....	....	....	....	....	....	....	58.8	.....	...	....	....	....	....	....	....	..	100	
346	Buenaventura, Colombia	50±	..	7	....	....	....	....	....	....	....	280.6	.....	...	....	....	....	....	....	....	..	100	
347	Quito, Ecuador	9,350	..	14	1915	....	....	....	....	10.3	38.0	43.1	54.0	...	....	....	....	16.8	....	23.9	88	100	125
348	Guayaquil, Ecuador	50±	..	4	1899	....	....	....	....	16.1	29.0	38.6	46.0	...	....	....	....	25.7	....	41.7	75	100	119
349	Lima, Peru	520	9	18	....	....	....	....	0.2	....	....	1.9	.....	...	....	....	....	21.6	10.5	....	..	100	
350	Cuzco, Peru	11,300	..	12	1921	....	....	....	....	12.7	13.1	31.7	45.8	...	....	....	....	17.3	....	40.0	41	100	144
351	Cailloma, Peru	13,000	..	17	....	....	....	....	....	....	....	26.1	.....	...	....	....	....	....	....	....	..	100	
352	Arequipa, Peru	8,041	..	37	1926	....	....	....	....	8.1	0.02	4.3	12.0	...	....	....	....	40.5	....	188.2	5	100	279
353	La Joya, Peru	4,140	..	9	....	....	....	....	....	....	....	0.2	.....	...	....	....	....	....	....	....	..	100	
354	Mollendo, Peru	80	..	10	....	....	....	....	....	....	....	0.8	.....	...	....	....	....	....	....	....	..	100	
355	La Paz, Bolivia	11,910	..	5	....	....	....	....	....	....	....	21.2	.....	...	....	....	....	....	....	....	..	100	
356	Oruro, Bolivia	12,122	..	4	....	....	....	....	....	....	....	2.1	.....	...	....	....	....	....	....	....	..	100	
357	Sucre, Bolivia	9,344	..	32	....	....	....	....	....	....	....	26.2	.....	...	....	....	....	....	....	....	..	100	
358	La Serena, Chile	115	..	52	1920	....	....	....	....	9.3	0.3	5.4	16.2	...	....	....	....	16.1	....	172.0	6	100	300
359	Santiago, Chile	1,702	..	55	1921	....	....	....	....	13.9	5.0	14.2	32.3	...	....	....	....	22.3	....	98.0	35	100	228
360	Punta Galera, Chile	131	..	23	1921	....	....	....	....	27.2	53.3	87.5	137.5	...	....	....	....	14.2	....	31.1	61	100	157
361	Valdivia, Chile	49	..	52	1921	....	....	....	....	34.0	71.7	106.3	140.7	...	....	....	....	15.4	....	32.0	67	100	132

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No.	Station	Elevation above sea, ft.	Years of record			Maximum observed precipitation				Annual precipitation				Observed precipitation, percentages of mean annual rainfall									
			For day or less	For month and year	Last year included	Number of minutes			Day of 24 hr.	Month	Minimum	Mean	Maximum	Mean daily intensity on days of rainfall	Maximum during short periods			Mean for month of maximum record	Maximum		Annual		
						5	15	60							Number of minutes				In 24 hr.	In one calendar month	Minimum	Mean	Maximum
															5	15	60						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
362	Evangelists Is., Chile	186	..	23	1921	....	....	....	....	19.7	85.5	121.0	137.0	...	...	....	....	9.2	....	16.3	71	100	113
363	Punta Arenas, Chile	92	..	34	1921	....	....	....	....	4.4	8.7	15.3	22.2	...	...	....	....	8.2	....	28.8	57	100	145
364	South Orkney Is.	23	..	18	1923	....	....	....	....	3.4	11.0	15.9	20.7	...	...	....	....	10.8	....	21.4	69	100	130
365	South Georgia Is.	13	..	21	1926	....	....	....	....	13.9	40.4	51.3	66.5	...	...	....	....	9.8	....	27.1	79	100	130
366	Stanley, Falkland Is.	6	..	17	1920	....	....	....	....	6.5	20.7	27.0	33.8	...	...	....	....	10.2	....	24.1	77	100	136
367	Bahia Blanca, Argentine	82	..	65	1924	....	....	....	....	12.4	8.1	21.0	50.6	...	...	....	....	10.7	....	59.0	39	100	241
368	Ajo-General Lavalle, Argentine	49	..	67	1924	....	....	....	....	14.9	20.0	34.7	71.1	...	...	....	....	9.3	....	42.9	58	100	205
369	Buenos Aires, Argentine	82	..	64	1924	....	....	....	....	21.5	19.8	37.2	79.6	...	...	....	....	11.7	....	57.8	53	100	214
370	Cordoba, Argentine	1,389	..	52	1924	....	....	....	....	12.4	17.7	27.7	40.0	...	...	....	....	16.8	....	44.7	64	100	144
371	San Juan, W., Argentine	2,180	..	42	1924	....	....	....	....	3.6	0.2	3.7	6.7	...	...	....	....	20.0	....	97.3	5	100	181
372	Tucuman, N., Argentina	1,470	..	45	1924	....	....	....	....	15.4	16.5	41.9	57.0	...	...	....	....	13.9	....	36.7	39	100	136
373	Asuncion, Paraguay	305	..	29	1923	....	....	....	....	18.6	22.6	51.8	78.5	...	...	....	....	11.9	....	35.9	44	100	151
374	Montevideo, Uruguay	96	..	20	1924	....	....	....	....	15.6	22.6	38.8	94.5	...	...	....	....	9.3	....	40.1	58	100	244
375	Curitiba, Brazil	2,979	..	38	1924	....	....	....	....	16.2	35.5	55.0	75.1	...	...	....	....	10.2	....	29.5	65	100	136
376	Alto de Serra, Brazil	2,624	..	55	1924	....	....	....	....	50.1	89.3	141.6	219.0	...	...	....	....	8.6	....	35.5	63	100	155
377	Rio de Janeiro, Brazil	201	..	75	1925	....	....	....	....	18.4	28.8	43.2	65.0	...	...	....	....	12.3	....	42.5	67	100	150
378	Pernambuco, Brazil	97	..	48	1922	....	....	....	....	56.7	24.9	65.0	178.5	...	...	....	....	16.1	....	87.0	38	100	275
379	Ceara, Brazil	66	..	72	1920	....	....	....	....	27.2	18.4	56.3	109.4	...	...	....	....	23.9	....	48.4	33	100	194
380	Para, Brazil	33	..	27	1924	....	....	....	....	30.9	64.1	87.0	116.8	...	...	....	....	16.1	....	35.4	71	100	134
381	Cayenne, Guiana	20	..	41	1907±	....	....	....	....	....	....	118.5	.....	...	...	....	....	....	....	.....	..	100	
382	Georgetown, Guiana	6	..	30	1923	....	....	....	....	26.6	52.7	88.7	121.9	...	...	....	....	12.6	....	30.0	59	100	138

383	Port of Spain, Trinidad........	69	18	59	1920	....	....	....	3.2	17.5	39.5	63.7	92.6	...	...	....	....	13.9	5.0	27.5	62	100	145
384	Sangre Grande, Trinidad........	.....	..	27	....	....	....	....	....	.....	.....	111.9	.....	...	...	....	....	....	....	.....	..	100	
385	St. George, Granada............	512	12	30	1920	....	....	....	4.3	20.9	56.7	76.1	106.0	..	...	....	....	11.0	5.7	27.5	74	100	139
386	Barbados Is..................	182	7	65	1920	....	....	....	4.6	28.4	27.0	54.8	89.3	...	...	....	....	13.9	8.4	51.8	49	100	163
387	Fort de France, Martinique.....	13	..	58	1923	....	....	....	....	.....	.....	78.8	.....	...	...	....	....	....	....	.....	..	100	
388	Morne des Cadets, Martinique	1,676	..	9	1923	....	....	....	....	.....	.....	120.2	.....	...	...	....	....	....	....	.....	..	100	
389	Christiansted, Virgin Is........	23	..	47	1925	....	....	....	....	18.7	25.0	46.5	74.0	...	...	....	....	9.1	....	40.3	54	100	159
390	St. Thomas, Virgin Is..........	27	7	17	1924	....	....	....	5.3	.....	.....	40.9	.....	...	...	....	....	15.0	13.0	.....	..	100	
391	San Juan, Porto Rico..........	82	25	27	1925	....	....	....	10.5	15.4	45.9	60.5	79.0	...	...	....	....	9.5	17.3	25.5	76	100	130
392	Toro Negro Dam, Porto Rico	2,275	13	13	1924	....	....	....	8.0	.....	.....	92.6	.....	...	...	....	....	16.4	8.6	.....	..	100	
393	Santo Domingo, Haiti..........	60	10	14	1924	....	....	....	7.4	.....	.....	55.5	.....	...	...	....	....	13.1	13.3	.....	..	100	
394	Port Au Prince, Haiti..........	123	41	49	1925	....	....	....	6.1	19.3	32.2	54.0	76.3	0.8	...	....	....	17.3	11.3	35.7	60	100	142
395	Morant Point, Jamaica.........	8	29	50	1919	....	....	....	12.0	.....	.....	70.2	.....	0.6	...	....	....	11.6	17.1	.....	..	100	
396	Kingston, Jamaica............	24	30	53	1924	....	....	....	8.9	.....	.....	32.5	.....	1.3	...	....	....	8.6	27.5	.....	..	100	
397	Hill Gardens, Jamaica..........	4,900	29	49	1919	....	....	....	28.7	.....	.....	105.5	.....	...	...	....	....	9.9	27.2	.....	..	100	
398	Mooretown, Jamaica..........	.....	15	24	1919	....	....	....	21.0	69.4	.....	222.0	320.8	0.6	...	....	....	15.6	9.4	31.4	..	100	144
399	Santiago de Cuba.............	118	16	21	....	....	....	....	14.3	.....	.....	43.8	.....	1.1	...	....	....	15.2	32.7	.....	..	100	
400	Isle of Pines, near Cuba........	200	10	13	....	....	....	....	6.8	.....	.....	70.7	.....	...	...	....	....	15.6	9.7	.....	..	100	
401	Havana, Cuba.	161	21	67	1925	....	....	....	10.3	22.6	27.9	48.2	65.2	1.0	...	....	....	4.6	21.4	47.0	58	100	136
402	Nassau, Bahama Is............	99	52	56	1920	....	....	....	8.0	23.4	25.5	51.0	79.2	0.8	...	....	....	12.5	15.8	46.0	50	100	156
403	Prospect, Bermuda............	151	47	55	1924	....	....	....	10.7	21.3	39.6	57.8	87.6	0.6	...	....	....	9.4	18.5	37.0	69	100	152
404	Sable Island, Canada..........	25	..	30	1920	....	....	....	....	17.9	43.4	51.8	67.6	...	...	....	....	7.1	....	34.6	84	100	131
405	Charlottetown, Canada.........	45	..	50	1925	....	....	....	....	10.4	18.5	39.0	56.4	...	...	....	....	10.9	....	26.6	47	100	144
406	St. Johns, Newfoundland........	125	..	52	1925	....	....	....	....	14.0	42.9	54.4	69.0	...	...	....	....	9.8	....	25.8	79	100	127
407	Father Point, Canada..........	20	..	49	1925	....	....	....	....	8.1	19.1	33.6	47.3	...	...	....	....	13.8	....	24.2	57	100	141
408	Montreal, Canada.............	187	..	52	1925	....	....	....	....	8.6	29.2	40.4	53.0	...	...	....	....	8.6	....	21.3	72	100	131
409	Toronto, Canada..............	379	..	52	1925	....	....	....	....	9.1	24.3	31.2	48.5	...	...	....	....	8.8	....	29.2	78	100	156
410	Winnipeg, Canada.............	760	..	40	1925	....	....	....	....	10.1	13.8	20.0	27.2	...	...	....	....	14.6	....	50.5	69	100	136
411	Prince Albert, Canada.........	1,432	..	41	1925	....	....	....	....	8.0	7.4	15.8	29.9	...	...	....	....	14.8	....	50.8	47	100	189
412	Edmonton, Canada............	2,158	..	43	1925	....	....	....	....	11.1	8.2	17.3	27.8	...	...	....	....	19.6	....	64.0	47	100	160
413	Calgary, Canada..............	3,389	..	41	1925	....	....	....	....	8.9	7.9	16.1	34.6	...	...	....	....	13.8	....	55.3	49	100	215
414	Barkerville, Canada...........	4,180	..	38	1925	....	....	....	....	9.3	20.6	37.6	49.5	...	...	....	....	8.9	....	24.8	55	100	132
415	Victoria, Vancouver, Canada....	228	..	35	1925	....	....	....	....	13.0	20.7	30.4	51.0	...	...	....	....	19.0	....	43.0	68	100	168
416	Sitka, Alaska.................	65	45	64	1921	....	....	....	5.0	25.5	58.2	84.8	140.3	0.5	...	....	....	12.0	5.9	30.1	68	100	169

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of Record: For day or less	Years of Record: For month and year	Years of Record: Last year included	Maximum observed precipitation: Number of minutes: 5	Maximum observed precipitation: Number of minutes: 15	Maximum observed precipitation: Number of minutes: 60	Maximum observed precipitation: Day of 24 hr.	Maximum observed precipitation: Month	Annual precipitation: Minimum	Annual precipitation: Mean	Annual precipitation: Maximum	Observed precipitation, in percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	Maximum during short periods: Number of minutes: 15	Maximum during short periods: Number of minutes: 60	Mean for month of maximum record	Maximum: In 24 hr.	Maximum: In one calendar month	Annual: Minimum	Annual: Mean	Annual: Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
417	Juneau, Alaska	84	18	36	1925	0.11	0.18	0.53	5.5	20.5	44.6	81.6	106.5	0.5	1.3	2.2	6.5	8.4	6.7	25.2	55	100	130
418	Dawson, N. W., Canada	1,052	..	24	1925	....	....	....	....	4.2	9.2	14.4	17.7	...	...	....	....	8.2	....	29.2	64	100	123
419	Eagle, Alaska	838	20	25	1922	....	....	....	1.9	4.7	6.3	10.8	13.5	1.1	...	....	....	18.2	17.6	43.5	58	100	126
420	Fairbanks, Alaska	500	17	17	1922	....	....	....	1.4	4.6	8.0	11.7	18.7	1.2	...	....	....	6.4	12.0	39.3	68	100	160
421	Tanana, Alaska	223	18	25	1925	....	....	....	2.1	5.8	7.8	12.6	17.7	0.9	...	....	....	18.8	16.7	46.1	62	100	141
422	Nome, Alaska	28	15	16	1922	....	....	....	2.3	8.4	9.5	17.4	29.5	0.9	...	....	....	16.5	13.2	48.4	54	100	170
423	Rodiak, Alaska	18	14	33	1922	....	....	....	4.5	17.8	43.6	60.7	83.6	...	...	....	....	12.2	7.4	29.5	72	100	138
424	St. Paul Island, Alaska	20	..	19	1921	....	....	....	....	9.9	21.0	31.3	47.1	0.5	...	....	....	11.4	....	31.7	67	100	150
425	Dutch Harbor, Alaska	16	15	18	1922	....	....	....	4.1	20.2	40.9	62.7	72.1	0.6	...	....	....	13.4	6.6	32.3	65	100	116
426	Midway Island	......	9	9	....	....	....	....	6.3	.....	.....	46.3	.....	...	...	....	....	11.2	13.6	.....	..	100	
427	Kealia, Kauai, Hawaiian Is.	14	16	19	1918	....	....	....	8.7	24.3	16.7	40.8	62.5	...	...	....	....	16.7	21.3	59.5	41	100	153
428	Honolulu, Oahu, Hawaiian Is.	38	23	54	1927	....	....	....	13.5	24.9	11.3	30.9	55.9	0.6	...	....	....	12.6	43.6	81.0	37	100	180
429	Luakaha, Oahu, Hawaiian Is.	1,111	16	14	1918	....	....	....	10.7	36.9	96.8	160.3	205.0	0.4	...	....	....	8.3	6.7	23.0	60	100	128
430	Kaanapali, Maui, Hawaiian Is.	12	..	20	1918	....	....	....	....	17.5	5.5	20.2	33.5	...	...	....	....	20.9	....	91.4	27	100	166
431	Puu Kukui Maui, Hawaiian Is	5,000	..	7	1918	....	....	....	....	96.0	232.5	370.1	562.0	...	...	....	....	9.5	....	25.9	63	100	152
432	Waialuku Mill, Maui, Hawaiian Is.	175	16	27	1916	....	....	....	6.2	23.9	8.7	27.8	54.5	0.7	...	....	....	14.8	22.2	85.8	31	100	196
433	Keanea Valley, Maui, Hawaiian Is.	1,000	16	14	1918	....	....	....	21.9	71.0	129.0	253.6	397.3	0.3	...	....	....	11.4	8.6	27.9	51	100	156
434	Kawainui, Hawaii, Hawaiian Is.	4,080	..	12	1918	....	....	....	....	84.2	99.5	276.6	503.7	...	...	....	....	10.0	....	30.5	36	100	182
435	Waimea, Hawaii, Hawaiian Is.	2,700	..	28	1918	....	....	....	....	27.3	24.4	43.5	80.1	...	...	....	....	11.5	....	62.8	51	100	184
436	Humuula, Hawaii, Hawaiian Is.	6,685	..	8	1913	....	....	....	....	16.3	19.1	32.3	53.4	...	...	....	....	13.1	....	50.6	59	100	166
437	Hilo, Hawaii, Hawaiian Is.	40	16	38	1918	....	....	....	14.5	58.6	91.2	139.4	194.2	0.4	...	....	....	10.9	10.4	42.0	65	100	139

438	Volcano House, Hawaii, Hawaiian Is.	4,000	..	17	1916	....	....	....	....	41.3	47.6	83.6	132.3	0.7	...	...	...	13.6	....	49.3	57	100	158
439	Kapapala Ranch, Hawaii, Hawaiian Is.	2,150	..	33	1918	....	....	....	....	34.4	23.9	58.1	114.6	...	...	...	...	12.3	....	59.1	41	100	197
440	Hilea, Hawaii, Hawaiian Is.	320	..	29	1918	....	....	....	....	25.9	12.5	37.9	74.2	1.3	...	...	...	13.8	....	68.5	33	100	196
441	Fanning Is.	......	..	13	1917	....	....	....	....	.....	47.4	118.2	208.8	...	...	...	...	....	....	.....	40	100	177
442	Malden Is.	23	..	31	1919	....	....	....	....	25.7	3.9	28.6	93.6	1.4	...	...	...	16.6	....	90.0	14	100	327
443	Papeete, Society Is.	20	..	35	1909	....	....	....	....	42.4	12.3	55.5	130.3	...	...	...	...	14.8	....	76.5	22	100	235
444	Rarotanga, Cook Is.	20	..	20	....	....	....	....	....	38.3	46.0	82.7	116.6	...	...	...	...	14.2	....	46.3	55	100	141
445	Apia, Samoan Is.	9	29	31	1920	....	....	....	10.2	40.0	68.8	107.2	172.7	...	...	...	...	15.5	9.5	37.3	64	100	161
446	Pago Pago, Samoan Is.	......	21	22	....	....	....	....	20.0	.....	.....	196.2	.....	...	...	...	...	7.8	10.2	.....	..	100	
447	Delanasua, Fiji Is.	70	10	40	....	....	....	....	14.9	.....	.....	92.9	.....	0.7	...	...	...	17.5	16.1	.....	..	100	
448	Wainunu, Fiji Is.	......	..	39	....	....	....	....	....	.....	.....	149.5	.....	...	...	...	...	....	....	.....	..	100	
449	Suva, Fiji Is.	44	16	34	....	....	....	....	15.9	37.3	73.0	112.4	169.6	0.4	...	...	...	8.5	14.1	33.2	65	100	151
450	Auckland, New Zealand	125	..	71	1923	....	....	....	....	14.1	26.3	43.9	74.1	...	...	...	...	6.9	....	32.1	60	100	169
451	Wellington, New Zealand	10	..	62	1923	....	....	....	....	12.9	28.5	48.0	67.7	...	...	...	...	8.5	....	26.9	59	100	141
452	Christchurch, New Zealand	25	..	47	1923	....	....	....	....	8.0	13.5	25.2	35.3	...	...	...	...	10.4	....	31.8	54	100	140
453	Hokitika, New Zealand	9	..	45	1923	....	....	....	....	29.1	83.2	116.0	154.4	...	...	...	...	8.3	....	25.1	76	100	133
454	Noumea, New Caledonia	30	..	55	1914	....	....	....	....	19.1	19.6	43.5	75.8	0.8	...	...	...	13.4	....	44.0	45	100	174
455	Col D'Amieu, New Caledonia	1,150	..	5	1914	....	....	....	....	.....	.....	78.3	.....	0.6	...	...	...	....	....	.....	..	100	
456	Tulagi, Solomon Is.	7	..	21	1918	....	....	....	....	41.9	62.1	115.4	162.1	...	...	...	...	14.2	....	37.3	54	100	141
457	Ocean Island	85	12	12	1916	....	....	....	6.7	.....	.....	80.8	.....	...	...	...	...	11.2	8.3	.....	..	100	
458	Nauru, Gilbert Is.	26	16	16	1913	....	....	....	7.7	29.6	21.4	84.2	163.2	...	...	...	...	13.1	9.1	35.2	25	100	194
459	Jaluit, Marshall Is.	15	17	17	1913	....	....	....	11.1	.....	.....	158.6	.....	...	...	...	...	9.7	7.0	.....	..	100	
460	Ujelang Is.	30	17	17	1913	....	....	....	11.7	.....	.....	77.4	.....	...	...	...	...	12.4	15.2	.....	..	100	
461	Guam Is.	67	17	17	1922	....	....	....	10.5	.....	.....	91.1	.....	...	...	...	...	15.6	11.5	.....	..	100	
462	Yap, Caroline Is.	104	20	20	....	....	....	....	12.6	45.8	72.9	119.1	152.0	...	...	...	...	13.2	10.6	38.5	61	100	128
463	Namatanai, New Ireland	......	7	7	1913	....	....	....	8.8	.....	.....	110.6	.....	...	...	...	...	14.1	8.0	.....	..	100	
464	Paniola, New Britain	......	19	19	1913	....	....	....	7.5	25.3	42.2	84.0	133.3	0.6	...	...	..	6.3	8.9	30.0	50	100	158

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No. (1)	Station (2)	Elevation above sea, ft. (3)	Years of record: For day or less (4)	Years of record: For month and year (5)	Years of record: Last year included (6)	Maximum observed precipitation: Number of minutes 5 (7)	Maximum observed precipitation: Number of minutes 15 (8)	Maximum observed precipitation: Number of minutes 60 (9)	Maximum observed precipitation: Day of 24 hr. (10)	Maximum observed precipitation: Month (11)	Annual precipitation: Minimum (12)	Annual precipitation: Mean (13)	Annual precipitation: Maximum (14)	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall (15)	Maximum during short periods, Number of minutes 5 (16)	Maximum during short periods, Number of minutes 15 (17)	Maximum during short periods, Number of minutes 60 (18)	Mean for month of maximum record (19)	Maximum: In 24 hr. (20)	Maximum: In one calendar month (21)	Annual: Minimum (22)	Annual: Mean (23)	Annual: Maximum (24)
465	Peterhafen, French Islands	.....	10	10	1913	....	....	....	11.0	.....	.....	130.2	.....	...	...	...	...	13.5	8.4	.....	..	100	
466	Port Moresby, New Guinea	127	..	29	1920	....	....	....	....	18.5	21.6	40.8	72.0	...	...	...	...	18.4	....	45.4	53	100	176
466-A	Crohamhurst, Australia	1,400	20	20	1912	....	....	....	35.7	107.6	24.7	80.1	201.7	...	...	...	...	19.7	44.6	134.3	31	100	252
467	Brisbane, Australia	137	74	74	1925	....	....	....	18.3	40.4	16.2	45.3	88.3	...	...	...	...	13.8	40.5	89.0	36	100	195
463	Sydney, Australia	138	67	85	1924	0.50	1.35	2.90	8.9	25.4	21.5	47.9	82.8	...	1.5	2.8	6.1	11.6	18.6	53.0	45	100	173
469	Melbourne, Australia	115	67	70	1925	....	....	....	3.5	7.9	15.6	25.6	44.2	...	...	...	...	9.6	13.7	30.8	61	100	172
470	Hobart, Tasmania	177	59	83	1925	....	....	....	5.0	10.2	13.4	23.8	43.4	...	...	...	...	7.7	21.0	42.7	56	100	182
471	Adelaide, Australia	110	87	87	1925	....	....	....	5.6	8.6	11.4	21.2	30.9	...	...	...	...	14.9	26.5	40.6	54	100	146
471-A	Peechywarrina, Australia	.....	..	21	1903	....	....	....	....	4.3	1.2	4.5	8.8	...	...	...	...	12.0	....	95.5	27	100	196
472	Alice Springs, Australia	1,926	..	51	1924	....	....	....	....	11.3	4.2	11.1	28.6	...	...	...	...	14.2	....	102.0	38	100	258
472-A	Nullarbor, Australia	.....	..	24	1911	....	....	....	....	5.9	3.6	9.5	18.9	...	...	...	...	8.0	....	62.1	33	100	199
473	Perth, S. W. Australia	197	50	50	1925	....	....	....	3.9	12.8	20.2	34.0	46.7	...	...	...	...	20.4	11.5	37.7	59	100	138
474	Carnarvon, W. Australia	.....	..	11	....	....	....	....	....	.....	.....	7.8	.....	...	...	...	...	....	....	.....	..	100	
475	Darwin, N. W. Australia	97	23	55	1924	....	....	....	11.7	27.9	40.6	61.8	87.2	...	...	...	...	25.8	19.0	45.1	66	100	141
476	Kupang, East Indies	49	..	45	1923	....	....	....	....	42.3	30.4	58.2	87.6	...	...	...	...	27.1	....	72.9	52	100	150
477	Kajoenas, East Indies	3,050	..	36	1923	....	....	....	....	40.3	70.4	99.4	154.0	...	...	...	...	17.9	....	40.6	71	100	155
478	Pasuruan, Java	16	..	45	1923	....	....	....	....	23.4	28.9	51.5	90.2	...	...	...	...	19.3	....	45.4	56	100	175
479	Batavia, Java	23	..	60	1923	....	....	....	....	32.4	45.9	72.1	96.8	...	...	...	...	18.1	....	44.8	64	100	134
480	Medan, East Indies	66	..	45	1923	....	....	....	....	17.2	62.2	81.4	103.7	...	...	...	...	11.1	....	21.1	76	100	127
481	Padang, Sumatra	3	..	45	1923	....	....	....	....	36.3	136.4	174.9	205.5	...	...	...	...	11.4	....	20.7	78	100	118
482	Kuta-Raja, Sumatra	23	..	45	1923	....	....	....	....	22.7	47.0	65.0	86.5	...	...	...	...	10.4	....	35.0	72	100	133

483	Singapore, Malay	.....	..	..	....	....	....	....	....	.....	32.7	92.0	158.7	...	...	....	....	....	....	.....	36	100	173
484	Pontianak, Borneo	10	..	45	1923	....	....	....	....	33.9	99.3	126.1	167.1	...	...	....	....	12.0	....	26.8	79	100	132
485	Sandakan, Borneo	105	..	33	1920	....	....	....	....	61.1	57.8	119.7	177.6	...	...	....	....	15.4	....	51.0	48	100	148
486	Menado, East Indies	13	..	45	1923	....	....	....	....	47.4	62.6	103.7	158.9	...	...	....	....	17.6	....	45.6	60	100	153
487	Amboina, East Indies	3	..	45	1923	....	....	....	....	62.2	59.9	134.8	242.7	...	...	....	....	18.3	....	46.1	44	100	180
488	Zamboanga, Philippine Is	10	17	19	1919	....	....	....	6.3	13.1	18.0	39.1	60.0	...	...	....	....	13.0	16.1	33.5	46	100	154
489	Iloilo, Philippine Islands	25	17	25	1922	....	....	....	7.9	55.3	45.3	86.3	127.5	...	...	....	....	19.8	9.2	64.0	53	100	148
490	Legaspi, Philippine Is	31	17	20	1922	....	....	....	11.7	44.5	48.6	127.0	202.3	...	...	....	....	15.7	9.2	35.0	38	100	159
491	Manila, Philippine Is	56	37	62	1926	....	....	2.56	13.2	78.1	35.7	80.0	154.4	...	...	....	3.2	20.4	16.5	97.6	45	100	193
492	Baguio, Philippine Is	4,959	17	18	1919	....	....	3.55	{46.0 34.6}	133.0	125.7	182.0	355.8	...	...	....	1.9	22.1	25.3	73.0	69	100	196
493	Aparri, N. Philippine Is	20	17	27	1922	....	....	....	11.3	39.2	46.0	87.3	118.1	...	...	....	....	8.7	12.9	44.9	53	100	135
494	Kashoryo, Formosa	1,378	..	12	1918	....	....	....	....	68.7	184.8	282.5	331.0	...	...	....	....	9.8	....	24.3	65	100	117
495	Taihoku, Formosa	30	22	24	1920	....	....	....	11.3	37.0	63.0	83.2	110.0	...	...	....	....	14.0	13.6	44.4	76	100	132
496	Nagasaki, Japan	435	..	42	1920	....	....	....	....	38.8	40.5	76.9	106.3	...	...	....	....	17.6	....	50.5	53	100	139
497	Kioto, Japan	160	..	40	1920	....	....	....	....	24.7	44.5	61.9	81.1	...	...	....	....	12.2	....	40.0	72	100	132
498	Tokyo, Japan	70	..	45	1920	....	....	....	....	18.1	45.7	59.9	86.4	...	...	....	....	14.6	....	30.4	76	100	145
499	Miyako, Japan	99	..	37	1920	....	....	....	....	22.7	34.9	55.6	80.7	...	...	....	....	12.6	....	41.0	63	100	145
500	Nemuro, Japan	89	..	37	1920	....	....	....	....	11.9	25.2	37.8	57.5	...	...	....	....	11.5	....	31.5	67	100	152
501	Vladivostok, Siberia	98	..	14	1890	....	....	....	....	13.8	9.4	19.5	33.6	1.4	...	....	....	17.8	....	70.8	48	100	172
502	Chemulpo, Korea	222	..	16	1920	....	....	....	....	17.9	25.0	37.4	54.7	...	...	....	....	22.9	....	48.0	67	100	146
503	Peking, China	132	31	33	1882	....	....	....	9.9	19.8	9.5	24.6	41.9	1.3	...	....	....	34.2	40.3	80.7	39	100	170
504	Zi-Ka-Wei, China	23	..	48	1920	....	....	....	....	19.4	27.9	45.1	62.5	...	...	....	....	16.1	....	43.0	62	100	139
505	Hong Kong, China	108	..	40	1895±	....	....	....	....	.....	.....	90.0	.....	...	...	....	....	....	....	.....	..	....	....
				37	1920	....	....	....	....	48.8	45.8	84.3	119.7	...	...	....	....	13.8	....	58.0	54	100	142
506	Saigon, Indo-China	36	..	15	1920	....	....	....	....	23.7	61.9	77.2	107.0	...	...	....	....	14.4	....	30.8	80	100	139
507	Mergui, Burma	66	..	56	1920	....	....	....	....	62.7	132.0	162.3	213.0	...	...	....	....	18.6	....	38.6	81	100	131
508	Port Blair, Andaman Is	59	..	53	1920	....	....	....	....	41.5	83.9	115.6	150.3	...	...	....	....	16.4	....	36.0	73	100	130
509	Moulmein, Burma	77	..	71	1920	....	....	....	....	72.2	124.2	190.3	279.0	...	...	....	....	22.9	....	38.0	65	100	146
510	Ragoon, Burma	18	..	51	1920	....	....	....	....	35.7	69.0	99.6	143.4	...	...	....	....	21.6	....	35.8	69	100	144
511	Akyab, Burma	20	..	63	1920	....	....	....	....	97.8	142.0	203.8	323.5	...	...	....	....	26.3	....	47.8	70	100	158
512	Mandalay, Burma	250	..	35	1920	....	....	....	....	14.2	16.8	35.1	49.7	...	...	....	....	12.4	....	40.4	48	100	142

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No.	Station	Elevation above sea, ft.	Years of record: For day or less	Years of record: For month and year	Years of record: Last year included	Maximum observed precipitation: Number of minutes: 5	Maximum observed precipitation: Number of minutes: 15	Maximum observed precipitation: Number of minutes: 60	Maximum observed precipitation: Day of 24 hr.	Maximum observed precipitation: Month	Annual precipitation: Minimum	Annual precipitation: Mean	Annual precipitation: Maximum	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	Maximum during short periods: Number of minutes: 15	Maximum during short periods: Number of minutes: 60	Mean for month of maximum record	Maximum: In 24 hr.	Maximum: In one calendar month	Annual: Minimum	Annual: Mean	Annual: Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
513	Gauhati, N. E. India	196	..	73	1920	....	....	....	....	34.2	46.6	66.5	99.1	...	...	....	....	19.0	....	51.5	70	100	149
514	Shillong, N. E. India	4,920	..	55	1920	....	....	....	....	39.9	51.8	83.1	121.2	...	...	....	....	16.0	....	48.0	62	100	146
515	Cherrapunji, N. E. India	4,309	..	62	1920	....	....	....	40.0	366.1	282.6	456.9	905+	...	...	....	....	23.5	8.8	80.1	62	100	198
516	Calcutta, N. E. India	21	..	92	1920	....	....	4.0	....	45.4	39.4	64.3	93.3	...	...	....	....	15.5	6.3	70.8	61	100	146
517	Patna, E India	183	..	73	1920	....	....	....	....	30.3	21.8	45.9	77.1	...	...	....	....	26.0	....	66.0	48	100	168
518	Nagpur, Central India	1,017	..	73	1920	....	....	3.6	....	24.8	14.4	46.4	65.3	...	...	....	7.8	28.7	....	53.5	31	100	141
519	Waltair, India	38	..	55	1920	....	....	....	....	30.2	17.1	41.0	79.5	...	...	....	....	22.9	....	73.8	42	100	194
520	Masulipatam, India	15	..	58	1920	....	....	....	....	26.6	21.3	39.8	69.5	...	...	....	....	20.7	....	67.0	53	100	175
521	Madras, India	22	..	108	1920	....	....	0.94	25.0	42.8	18.4	49.6	88.4	...	...	1.9	....	27.5	50.0	86.3	37	100	178
522	Trincomalee, Ceylon	99	..	52	1920	....	....	....	....	35.2	34.9	62.4	95.1	...	...	....	....	22.6	....	56.4	56	100	152
523	Nuwara Eliya, Ceylon	6,188	..	52	1920	....	....	....	....	35.0	68.4	92.8	131.5	...	...	....	....	12.8	....	37.6	74	100	142
524	Colombo, Ceylon	40	..	51	1920	....	....	....	....	35.3	45.7	80.1	139.7	...	...	....	....	16.6	....	44.0	57	100	174
525	Kodlakanal, S. India	7,688	..	21	1920	....	....	....	....	16.8	46.6	62.2	79.7	...	...	....	....	15.5	....	27.0	75	100	128
526	Cochin, S. India	9	..	65	1920	....	....	....	....	49.5	80.6	114.8	165.0	...	...	....	....	24.2	....	43.2	70	100	144
527	Mangalore, S. India	72	..	66	1920	....	....	....	....	70.4	89.4	128.9	182.3	...	...	....	....	29.0	....	54.7	69	100	142
528	Bangalore, S. India	3,021	..	85	1920	....	....	....	....	19.5	16.0	35.3	56.6	...	...	....	....	17.0	....	55.1	45	100	160
529	Belgaum, S. India	2,562	..	69	1920	....	....	....	....	42.2	28.7	49.9	73.4	...	...	....	....	31.5	....	84.5	58	100	147
530	Bombay, S. India	37	..	104	1920	....	....	3.0	16.0	59.0	33.4	72.4	114.9	...	...	....	4.1	33.5	22.1	81.6	46	100	159
531	Ahmadabad, S. India	163	..	55	1920	....	....	....	....	37.5	4.8	28.9	51.2	...	...	....	....	39.2	....	130.0	17	100	177
532	Allahabad, S. India	309	..	73	1920	....	....	....	....	30.5	15.7	39.0	76.2	...	...	....	....	28.2	....	78.1	40	100	195
533	Jaipur, India	1,431	..	52	1920	....	....	....	....	21.8	4.7	23.9	55.3	...	...	....	....	30.5	....	91.0	20	100	232
534	Simla, India	7,232	..	59	1920	....	....	....	....	42.5	40.4	80.0	109.7	...	...	....	....	26.0	....	53.2	50	100	137
535	Leh, India	11,503	..	45	1920	....	....	....	....	4.0	0.4	3.2	9.1	...	...	....	....	5.0	....	125.0	12	100	284

536	Lahore, India	702	..	63	1920	0.90	2.00	4.50	9.0	20.4	6.2	20.1	37.8	...	...	....	....	25.0	....	101.5	31	100	188
537	Peshawar, N. W. India	1,113	..	62	1920	....	....	....	....	17.7	4.1	13.5	27.9	...	...	....	....	16.1	....	131.0	30	100	206
538	Quetta, Baluchistan	5,502	..	43	1920	....	....	....	....	7.5	3 9	9.3	21.6	...	...	....	....	19.5	....	80.8	42	100	233
539	Khelat, Baluchistan	6,630	..	41	1920	....	....	....	....	7.2	4.1	7.4	13.6	...	...	....	....	18.9	....	97.1	55	100	184
540	Hyderabad, N. W. India	96	..	55	1920	....	....	....	6.9	15.8	1.0	7.6	21.1	...	...	....	....	37.0	91.0	207.9	13	100	277
541	Karachi, N. W. India	13	..	65	1920	....	....	....	....	18.6	0.5	7.4	28.0	...	...	....	....	38.3	....	251.3	7	100	378
542	Muscat, E. Arabia	20	..	27	1920	....	....	....	....	4.5	0.9	4.2	10.4	5.5	...	....	....	27.6	....	107.1	21	100	247
543	Jask, Persia	13	..	28	1920	....	....	....	....	4.8	0.2	4.5	7.7	...	...	....	....	17.8	....	106.7	4	100	152
544	Bushire, Persia	14	..	44	1920	....	....	....	....	12.9	0.0	10.4	26.6	...	...	....	....	25.7	....	124.0	0	100	256
545	Meshed, Persia	3,104	..	28	1920	....	....	....	....	5.2	3.1	9.2	16.8	...	...	....	....	24.4	....	56.5	34	100	183
546	Teheran, Persia	4,002	..	27	1920	....	....	....	....	7.5	3.6	9.5	15.3	...	...	....	....	17.4	....	78.9	38	100	161
547	Baghdad, Mesopotamia	125	..	31	1920	....	....	....	2.5	7.9	1.3	7.1	22.3	3.7	...	....	....	19.6	37.2	111.1	18	100	314
548	Aden, S. Arabia	94	..	40	1920	....	....	....	....	6.6	0.0	1.8	8.6	...	...	....	....	26.6	....	366.7	0	100	478
549	Khartoun, Sudan	1,280	22	24	1922	....	....	....	3.5	7.4	1.3	5.7	14.4	...	...	....	....	45.6	61.5	130.0	23	100	253
550	Deim Zubeir, Sudan	2,168	14	15	1920	....	....	....	5.4	23.9	.....	45.7	78.5	...	...	....	....	17.0	11.8	52.3	..	100	172
551	Gore, Abyssinia	6,990	13	13	1920	....	....	....	3.5	17.0	.....	75.7	92.6	...	...	....	....	15.0	4.6	22.5	..	100	122
552	Adis-Ababa, Abyssinia	8,006	13	23	1920	....	....	....	3.0	22.5	.....	49.6	74.9	...	...	....	....	15.2	6.0	45.4	..	100	151
553	Entebbe, Uganda	3,847	..	25	1920	....	....	....	....	15.8	47.2	58.0	75.7	...	...	....	....	16.8	....	27.3	81	100	131
554	Dar-es-Salaam, E. Africa	249	..	20	1912	....	....	....	....	23.8	19.4	42.3	56.9	...	...	....	....	28.0	....	56.2	46	100	134
555	Zanzibar, E. Africa	56	..	30	1920	....	....	....	....	3.3	27.5	60.2	92.9	...	...	....	....	23.4	....	55.3	46	100	154
556	Seychelles Is., E. Africa	......	..	30	1920	....	....	....	....	31.4	66.5	94.5	132.8	...	...	....	....	17.9	....	33.3	70	100	140
557	Tananarivo, Madegascar	4,600	..	34	1923	....	....	....	....	26.4	27.2	53.7	69.6	...	...	....	....	21.4	....	49.1	51	100	130
558	Salisbury, Rhodesia	4,860	..	28	1923	....	....	....	....	18.8	21.7	31.9	42.6	...	...	....	....	23.2	....	59.0	68	100	134
559	Bulawayo, Rhodesia	4,443	..	27	1923	....	....	....	....	13.4	12.1	23.6	36.8	...	...	....	....	21.9	....	56.8	51	100	156
560	Belvedere, S. Africa	5,000	..	17	1925	....	....	....	....	.....	.....	81.4	.....	...	...	....	....	....	....	.....	..	100	
561	Johannesburg, S. Africa	5,925	..	37	1924	....	....	....	....	20.0	19.8	33.2	54.9	1.1	...	....	....	18.6	....	60.2	60	100	165
562	Kroonstad, S. Africa	4,300	..	45	1925	....	....	....	....	.....	.....	24.2	.....	...	...	....	....	....	....	.....	..	100	
563	Aliwal, S. Africa	4,352	..	42	1924	....	....	....	....	12.2	8.9	20.4	35.8	...	...	....	....	14.3	....	59.8	44	100	176
564	Kokstad, S. Africa	4,280	..	40	1925	....	....	....	....	.....	.....	27.4	.....	...	...	....	....	....	....	.....	..	100	
565	Durban, S. Africa	265	..	39	1920	....	....	....	....	26.7	27.2	42.7	85.5	0.8	...	....	....	12.0	....	62.6	64	100	200
566	Port Elizabeth, S. Africa	181	..	59	1924	....	....	....	....	9.3	10.4	22.5	34.1	1.3	...	....	....	9.3	....	41.2	46	100	152
567	Cape Town, S. Africa	40	..	87	1924	....	....	....	....	13.5	17.5	25.3	41.0	...	...	....	....	15.1	....	53.3	69	100	162
568	Carnarvon, S. Africa	4,060	..	46	1925	....	....	....	....	.....	.....	8.1	.....	...	...	....	....	....	....	.....	..	100	
569	Springbok, S. Africa	3,200	..	48	1925	....	....	....	....	.....	.....	9.6	.....	...	...	....	....	....	....	.....	..	100	
570	O'Okiep, S. Africa	3,035	..	42	1924	....	....	....	....	4.3	2.1	6.7	11.7	...	...	....	....	16.8	....	64.0	31	100	174
571	Port Nolloth, S. Africa	25	..	40	1925	....	....	....	....	.....	.....	2.5	.....	...	...	....	....	....	....	.....	..	100	

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of record			Maximum observed precipitation					Annual precipitation			Observed precipitation, percentages of mean annual rainfall									
			For day or less	For month and year	Last year included	Number of minutes			Day of 24 hr.	Month	Minimum	Mean	Maximum	Mean daily intensity on days of rainfall	Maximum during short periods, Number of minutes			Mean for month of maximum record	Maximum		Annual		
						5	15	60							5	15	60		In 24 hr.	In one calendar month	Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
572	Bethany, S. W. Africa	......	..	14	1913	....	....	....	....	4.1	1.2	4.0	9.1	...	...	...	...	23.7	....	102.5	30	100	227
573	Hasuur, S. W. Africa	......	..	14	1913	....	....	....	....	7.1	0.3	7.6	25.6	...	...	...	...	29.6	....	93.6	4	100	338
574	Steinkopf, S. W. Africa	3,000	..	36	1925	....	....	....	....	.....	.....	5.6	.....	...	...	...	...	....	....	.....	..	100	
575	Pella, S. W. Africa	1,500	..	47	1925	....	....	....	....	.....	.....	3.3	.....	...	...	...	...	....	....	.....	..	100	
576	Rietfontein, S. W. Africa	2,720	..	17	1925	....	....	....	....	.....	.....	7.2	.....	4.3	...	...	...	....	....	.....	..	100	
577	Kenhardt, S. Africa	2,704	..	45	1925	....	....	....	....	.....	.....	5.8	.....	...	...	...	...	....	....	.....	..	100	
578	Prieska, S. Africa	3,100	..	48	1925	....	....	....	....	.....	.....	9.6	.....	...	...	...	...	....	....	.....	..	100	
579	Hopetown, S. Africa	3,604	..	46	1925	....	....	....	....	.....	.....	12.6	.....	...	...	...	...	....	....	.....	..	100	
580	Kimberley, S. Africa	3,947	..	50	1923	....	....	....	....	9.0	8.5	16.9	31.3	1.6	...	...	...	17.4	....	53.2	50	100	185
581	Vryburg, S. Africa	3,890	..	33	1925	....	....	....	....	.....	.....	19.2	.....	...	...	...	...	....	....	.....	..	100	
582	Mafeking, S. Africa	4,194	..	23	1925	....	....	....	....	.....	.....	22.2	.....	1.8	...	...	...	....	....	.....	..	100	
583	Palapye Road, S. Africa	......	..	8	....	....	....	....	....	.....	.....	16.5	.....	...	...	...	...	....	....	.....	..	100	
584	Tsau, S. W. Africa	......	..	4	....	....	....	....	....	.....	.....	12.6	.....	...	...	...	...	....	....	.....	..	100	
585	Windhoek, S. W. Africa	5,461	..	22	1913	....	....	....	....	12.2	7.3	15.2	29.8	...	...	...	...	25.4	....	80.1	48	100	196
586	Swakopmund, S. W. Africa	26	..	15	1913	....	....	....	....	0.8	0.1	0.6	1.2	...	...	...	...	16.7	....	133.3	17	100	200
587	Waterberg, S. W. Africa	5,248	..	21	1913	....	....	....	....	20.3	7.3	21.1	41.6	...	...	...	...	26.3	....	96.0	35	100	197
588	Mossamedes, Angola	......	..	10	....	....	....	....	....	.....	.....	14.7	.....	...	...	...	...	....	....	.....	..	100	
589	Loanda, Angola	......	..	42	....	....	....	....	....	.....	.....	12.4	.....	...	...	...	...	....	....	.....	..	100	
590	Cazengo, Angola	......	..	10	....	....	....	....	....	.....	.....	42.2	.....	...	...	...	...	....	....	.....	..	100	
591	Camaxillo, Angola	......	..	4	....	....	....	....	....	.....	.....	52.6	.....	...	...	...	...	....	....	.....	..	100	
592	Elisabethville, S. E. Congo	......	..	14	1925	....	....	....	....	.....	.....	45.2	.....	...	...	...	...	....	....	.....	..	100	
593	Gazi, N. E. Congo	......	..	12	1922	....	....	....	....	.....	.....	72.7	.....	...	...	...	...	....	....	.....	..	100	
594	Eala, N. Congo	......	..	16	1922	....	....	....	....	.....	.....	70.5	.....	...	...	...	...	....	....	.....	..	100	
595	Boma, W. Congo	......	..	18	1922	....	....	....	....	.....	.....	35.7	.....	...	...	...	...	....	....	.....	..	100	

596	St. Helena Is.	1,983	..	29	1920	....	....	....	....	11.0	31.0	40.1	49.8	...	...	....	....	12.3	....	27.4	77	100	124
597	Calabar, Nigeria	42	..	24	1920	....	....	....	....	41.2	90.0	123.2	167.4	...	...	....	....	14.8	....	33.5	73	100	136
598	Lagos, Nigeria	7	..	29	1920	....	....	....	....	31.0	40.5	71.6	115.5	...	...	....	....	26.1	....	43.4	57	100	162
599	Akkra, Gold Coast	61	..	27	1920	....	....	....	....	21.1	13.1	27.2	44.2	...	...	....	....	25.8	....	77.9	48	100	163
600	Gambaga, W. Africa	350	..	20	1920	....	....	....	....	18.2	30.7	45.6	71.7	...	...	....	....	24.1	....	39.9	67	100	157
601	Timbuktu, W. Africa	......	..	15	1914	....	....	....	....	.....	.....	7.7	.....	...	...	....	....	....	....	.....	..	100	
602	Freetown, Sierra Leone	189	..	46	1920	....	....	....	....	61.1	102.5	157.2	206.6	...	...	....	....	23.3	....	38.9	65	100	132
603	Bathurst, Gambia	7	..	37	1920	....	....	....	....	36.6	24.6	47.6	80.8	...	...	....	....	41.1	....	77.0	52	100	170
604	La Laguna, Canary Is.	10	..	26	1920	....	....	....	....	20.6	10.3	23.4	41.7	...	...	....	....	16.3	....	87.8	44	100	178
605	Funchal, Madeira Is.	82	..	41	1920	....	....	....	....	19.0	10.7	27.2	55.8	...	...	....	....	14.6	....	69.6	39	100	204
606	Cape Spartel, Tangiers	193	..	28	1920	....	....	....	....	14.1	21.9	30.4	46.8	...	...	....	....	18.2	....	46.4	72	100	154
607	Algiers, Algeria	1,129	..	27	1920	....	....	....	....	11.0	12.7	27.4	42.7	...	...	....	....	14.6	....	40.2	46	100	156
		13	..	54	1894	....	....	....	....	.....	.....	26.9	.....	...	...	....	....	....	....	.....	..	100	
608	Tunis, Tunis	69	..	31	1920	....	....	....	....	7.1	11.0	16.5	22.8	...	...	....	....	11.3	....	43.0	67	100	138
609	Alexandria, Egypt	112	37	39	1922	....	....	....	11.0	15.4	3.2	8.0	23.0	...	...	....	....	32.6	137.5	192.7	40	100	288
610	Cairo (Abbassia), Egypt	112	34	36	1922	....	....	....	1.4	2.0	0.2	1.3	3.4	...	...	....	....	21.5	107.5	154.0	15	100	261
611	Aswan, Egypt	331	..	20	1920	....	....	....	....	.....	.....	0.0	.....	...	...	....	....	....	.....	.....	..	100	
612	Suez, Egypt	22	34	35	1920	....	....	....	1.2	2.5	.....	1.0	3.6	...	...	....	....	12.0	120.0	250.0	..	100	360
613	Ismaili, Suez Canal	39	18	35	1920	....	....	....	2.1	2.7	.....	1.8	4.1	2.5	...	....	....	23.8	116.4	149.5	..	100	227
614	Gaza, Palestine	66	..	10	1905	....	....	....	....	.....	.....	16.5	.....	...	...	....	....	....	....	.....	..	100	
615	Jerusalem, Palestine	1,968	..	53	1913	....	....	....	....	16.4	13.4	25.2	45.0	1.7	...	....	....	21.8	....	65.0	53	100	178
				44	1895	....	....	....	....	.....	.....	25.5	.....	...	...	....	....	....	....	.....	..	100	
616	Tiberias, Egypt	−653	..	18	....	....	....	....	....	.....	.....	18.1	.....	...	...	....	....	....	....	.....	..	100	
617	Le Krey, Egypt	3,330	..	11	1914	....	....	....	....	.....	.....	59.6	.....	...	...	....	....	....	....	.....	..	100	
618	Beirut, Syria	111	..	45	1920	....	....	....	....	15.7	23.2	35.5	51.1	...	...	....	....	16.2	....	43.2	65	100	144
619	Nicosia, Cyprus	525	14	14	1920	....	....	....	8.7	.....	.....	15.1	.....	...	...	....	....	12.1	57.5	.....	..	100	
620	Kyrenia, Cyprus	46	14	14	1920	....	....	....	5.2	.....	.....	21.1	.....	...	...	....	....	25.2	24.6	.....	..	100	
621	Candia, Crete	128	13	13	1920	....	....	....	4.4	.....	.....	21.1	.....	...	...	....	....	20.1	20.8	.....	..	100	
622	Athens, Greece	356	..	26	1920	....	....	....	....	8.1	4.6	15.5	22.4+	...	...	....	....	16.8	....	52.0	30	100	144

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No.	Station	Elevation above sea, ft.	Years of record			Maximum observed precipitation				Annual precipitation				Observed precipitation, percentages of mean annual rainfall									
			For day or less	For month and year	Last year included	Number of minutes			Day of 24 hr.	Month	Minimum	Mean	Maximum	Mean daily intensity on days of rainfall	Maximum during short periods, Number of minutes			Mean for month of maximum record	Maximum		Annual		
						5	15	60							5	15	60		In 24 hr.	In one calendar month	Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
623	Smyrna, Turkey	...	..	20	1882	...	...	...	...	...	...	25.6	...	1.4	...	...	...	...	...	...	..	100	
624	Constantinople, Turkey	246	..	48	1895	...	...	...	6.3	...	...	28.9	...	...	...	...	...	5.7	21.7	...	..	100	
625	Batak, S. W. Bulgaria	3,180	..	20	1918	...	...	...	...	...	13.9	26.6	36.5	...	...	...	...	...	...	...	52	100	137
626	Sofia, W. Bulgaria	1,803	..	20	1918	...	...	...	...	...	15.6	24.3	33.3	...	...	...	...	...	...	...	64	100	137
627	Petrohan, N. W. Bulgaria	4,584	..	15	1913	...	...	...	...	...	31.4	45.3	61.2	...	...	...	...	...	...	...	70	100	136
628	Ferdinand, N. W. Bulgaria	525	..	20	1918	...	...	...	...	...	8.1	21.4	31.8	...	...	...	...	...	...	...	38	100	148
629	Varna, N. E. Bulgaria	115	15	19	1918	...	...	...	3.9	...	14.8	20.3	26.2	...	...	...	...	8.3	19.2	...	73	100	129
630	Sulina, E. Roumania	7	..	58	1925	...	...	...	...	10.3	5.2	15.4	24.4	...	...	...	...	8.3	...	67.0	34	100	158
631	Braila, E. Roumania	82	20	32	1910	...	...	...	3.8	...	...	17.9	...	...	...	...	...	7.5	21.2	...	..	100	
632	Bucharest, S. Roumania	269	..	61	1924	...	...	...	...	11.7	13.5	23.1	34.0	...	...	...	...	15.0	...	50.8	58	100	147
633	Belgrade, Jugo-Slavia	452	..	35	1924	...	...	...	...	8.0	12.8	24.4	33.8	...	...	...	...	9.7	...	32.6	52	100	138
634	Lesina, S. W. Jugo-Slavia	66	..	61	1918	...	...	...	...	18.9	17.1	31.1	52.0	...	...	...	...	14.0	...	60.8	55	100	167
635	Obir, Austria	6,710	..	45	1923	...	...	...	...	21.1	36.1	60.2	91.5	...	...	...	...	5.7	...	35.0	60	100	152
636	Vienna, Austria	663	..	70	1920	...	...	...	...	9.0	16.5	25.5	35.6	...	...	...	...	10.7	...	35.2	65	100	140
637	Sonnblick, Austria	10,020	..	33	1923	...	...	...	...	15.5	41.2	65.0	79.7	...	...	...	...	7.9	...	23.9	63	100	123
637-A	Klagenfurt, Austria	1,476	..	88	1900	...	...	...	...	12.7	16.1	38.5	57.8	...	...	...	...	7.8	...	32.9	42	100	150
638	Santis, Switzerland	8,200	..	38	1920	...	...	...	...	27.5	53.3	100.3	154.2	...	...	...	...	8.6	...	27.4	53	100	154
639	St. Gallen, Switzerland	2,230	..	50	1920	...	...	...	...	...	...	53.7	...	...	...	...	...	...	...	...	..	100	
640	Zurich, Switzerland	1,565	..	57	1920	...	...	...	...	16.9	29.0	43.6	78.3	...	...	...	...	12.0	...	38.6	66	100	180
641	Chalet, Capt., Switzerland	4,433	..	50	1900	...	...	...	...	...	...	81.7	...	...	...	...	...	...	...	...	..	100	
642	Interlaken, Switzerland	1,862	..	26	1919	...	...	...	...	...	...	46.6	...	...	...	...	...	...	...	...	..	100	
643	Geneva, Switzerland	1,329	30	50	1920	...	...	...	4.9	...	...	35.3	...	...	...	...	...	11.6	13.9	...	..	100	
644	Gotthard, Switzerland	6,890	..	13	1920	...	...	...	...	...	...	80.1	...	...	...	...	...	...	...	...	..	100	
645	Locarno, Switzerland	785	..	20	1905	...	...	...	...	...	...	76.5	...	...	...	...	...	...	...	...	..	100	

646	Domodossola, Italy	984	..	26	1905	...	...	...	....	25.3	32.8	55.3	73.6	.:.	...	...	...	13.1	....	45.6	59	100	133
647	Aosta, N. W. Italy	1,912	..	66	1910	...	...	...	....	.....	.....	22.4	.....	...	...	...	...	....	....	.....	..	100	
648	Valdobria, N. W. Italy	8,356	..	20	1905	...	...	...	....	18.1	36.8	46.6	76.1	...	...	...	...	7.8	....	38.7	79	100	164
649	Milan, N. Italy	482	..	161	1924	...	...	...	....	14.8	24.9	39.4	61.8	...	...	...	...	12.0	....	37.6	63	100	157
650	Pavia, N. Italy	269	..	92	1908	...	...	...	....	11.3	23.0	31.1	51.0	...	...	...	...	10.3	....	36.3	74	100	164
651	Venice, N. E. Italy	154	..	26	1905	...	...	...	....	8.8	19.1	26.2	38.3	...	...	...	...	6.1	....	33.6	73	100	146
651-A	Padua, N. E. Italy	103	17	202	1926	...	...	...	3.5	12.3	18.9	33.8	61.6	1.0	...	...	...	9.8	10.7	36.4	56	100	182
652	Camaldoli, N. Central Italy	3,680	..	21	1905	...	...	...	....	20.3	44.7	71.7	100.0	...	...	...	...	11.7	....	28.4	62	100	140
653	Florence, N. Central Italy	363	..	26	1905	...	...	...	....	10.7	15.9	33.0	44.8	...	...	...	...	11.8	....	32.4	48	100	136
654	Rome, Italy	208	..	143	1924	...	...	...	....	13.9	12.6	32.7	57.9	...	...	...	...	15.4	....	42.6	39	100	177
655	Naples, Italy	532	..	26	1905	...	...	...	....	10.3	17.4	33.6	44.4	...	...	...	...	12.1	....	30.6	52	100	132
656	Oppido Mamertina, S. Italy	1,228	..	24	1905	...	...	...	....	25.9	33.8	52.6	73.0	...	...	...	...	13.3	....	49.1	64	100	139
657	Catania, Sicily	213	..	32	1923	...	...	...	....	19.8	12.6	24.6	43.7	...	...	...	...	18.4	....	80.3	51	100	177
658	Sassari, Sardinia	735	..	42	1924	...	...	...	....	10.6	17.0	23.6	38.2	...	...	...	...	15.8	....	44.9	72	100	162
659	Genoa, Italy	259	..	26	1905	...	...	...	....	18.7	34.3	52.9	75.1	...	...	...	...	14.1	....	35.4	65	100	142
660	Nice, France	1,140	30	30	1910	...	...	...	5.7	.....	.....	32.3	.....	...	...	...	...	18.4	17.6	.....	..	100	
661	Marseilles, France	246	30	50	1920	...	...	...	8.7	12.4	12.3	22.6	43.0	...	...	...	...	17.0	38.5	54.9	54	100	190
662	Lyon, France	981	30	80	1920	...	...	...	4.3	11.3	19.3	29.1	45.2	...	...	...	...	12.7	14.8	38.8	66	100	155
663	Besancon, France	1,020	30	30	1910	...	...	...	2.8	.....	.....	42.6	.....	...	...	...	...	9.2	6.6	.....	..	100	
664	Nancy, France	724	30	30	1910	...	...	...	2.6	.....	.....	28.8	.....	...	...	...	...	10.3	9.0	.....	..	100	
665	Paris, France	164	30	50	1923	...	...	...	2.0	6.2	10.9	22.7	29.8	...	...	...	...	10.1	8.8	27.3	48	100	131
665-A	Paris, France	164	30	214	1926	...	...	...	2.0	6.9	9.0	19.9	29.8	...	...	...	...	9.5	10.0	34.6	45	100	150
666	Clermont, France	1,309	30	30	1910	...	...	...	2.8	.....	.....	25.9	.....	...	...	...	...	10.1	10.8	.....	..	100	
667	Puy de Dome, France	4,812	30	30	1910	...	...	...	6.5	.....	.....	66.7	.....	...	...	...	...	9.9	9.8	.....	..	100	
668	Perpignan, S. E. France	106	30	50	1900	...	...	...	5.8	15.9	11.5	21.6	39.2	...	...	...	...	11.1	26.8	73.5	53	100	181
669	La Preste, S. E. France	3,680	..	17	1895	...	...	...	....	15.7	25.0	44.3	62.1	...	...	...	...	5.1	....	35.5	56	100	140
670	Puymorens, S. E. France	6,320	..	28	1900	...	...	...	....	14.0	20.1	31.4	47.4	...	...	...	...	10.5	....	44.5	64	100	151
671	Huesca, N. E. Spain	1,653	20	39	1900	...	...	...	3.4	11.4	11.2	22.2	33.7	...	...	...	...	11.5	15.3	51.3	50	100	152
672	Saragossa, N. E. Spain	672	19	40	1899	...	...	...	2.6	6.5	7.0	12.0	22.3	...	...	...	...	11.5	21.7	54.1	58	100	186
673	Barcelona, Spain	70	..	30	1890	...	...	...	....	13.3	10.7	21.1	38.5	...	...	...	...	15.6	....	63.0	51	100	182
674	Palma, Spain	.....	..	56	1921	...	...	...	....	9.3	9.4	18.9	30.6	...	...	...	...	11.6	....	49.2	50	100	162
675	Valencia, Spain	59	..	30	1890	...	...	...	....	13.8	7.5	19.1	50.7	...	...	...	...	15.6	....	72.0	39	100	265
676	Madrid, Spain	2,149	..	60	1919	...	...	...	....	7.3	10.2	16.6	27.5	...	...	...	...	9.9	....	44.0	61	100	166
677	Gibraltar, Gibraltar	53	..	67	1920	...	...	...	....	24.9	14.6	35.3	65.3	...	...	...	...	18.0	....	70.5	41	100	185
678	Seville, S. W. Spain	98	..	29	1890	...	...	...	....	12.3	6.3	18.6	34.7	...	...	...	...	11.7	....	66.0	34	100	186
679	Lisbon, Portugal	312	..	57	1920	...	...	...	....	15.1	17.3	29.3	55.9	...	...	...	...	13.9	....	51.4	59	100	190
680	Serra de Estrella, Portugal	4,756	19	19	1900	...	...	...	11.5	48.6	51.1	113.2	184.3	0.8	...	...	...	13.2	10.1	43.0	45	100	163
681	Oporto, Portugal	328	28	38	1900	...	...	...	6.5	20.4	23.8	51.4	89.0	0.8	...	...	...	10.8	12.7	39.8	46	100	173
682	Valladolid, Spain	2,345	20	40	1900	...	...	...	2.2	8.2	5.6	12.5	17.4	1.0	...	...	...	11.0	17.6	65.6	45	100	139
683	Oviedo, N. W. Spain	800	50	50	1900	...	...	...	3.2	16.4	25.2	36.0	49.8	1.4	...	...	...	11.5	8.9	45.5	70	100	138
684	Bilbao, N. Spain	56	20	46	1900	...	...	...	4.8	19.6	30.2	47.2	64.4	...	...	...	...	11.1	10.2	41.5	64	100	137
685	Pic du Midi, N. Spain	9,380	..	30	1910	...	...	...	4.9	.....	.....	65.5	.....	...	...	...	...	7.5	7.5	.....	..	100	

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of record			Maximum observed precipitation					Annual precipitation			Observed precipitation, percentages of mean annual rainfall									
			For day or less	For month and year	Last year included	Number of minutes			Day of 24 hr.	Month	Minimum	Mean	Maximum	Mean daily intensity on days of rainfall	Maximum during short periods, Number of minutes			Mean for month of maximum record	Maximum		Annual		
						5	15	60							5	15	60		In 24 hr.	In one calendar month	Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
686	Bayonne, S. W. France	.....	..	30	1890	....	....	....	....	....	....	45.3	....	...	...	....	....	....	....	....	..	100	
687	Toulouse, S. France	636	..	30	1910	....	....	....	3.6	....	....	26.1	....	...	...	....	....	12.3	13.8	....	..	100	
688	Bordeaux, S. W. France	243	30	30	1910	....	....	....	2.5	....	....	30.3	....	...	...	....	....	9.2	8.2	....	..	100	
689	Nantes, W. France	121	30	40	1920	....	....	....	2.2	10.0	22.1	30.9	41.1	...	...	....	....	12.1	7.1	32.3	72	100	133
690	Brest, W. France	200	30	30	1910	....	....	....	2.8	....	....	32.3	....	...	...	....	....	9.4	8.7	....	..	100	
691	Jersey Island	25	39	51	1921	....	1.59	....	2.4	10.8	15.9	33.5	45.6	...	...	....	....	12.2	7.2	32.2	47	100	136
692	Arras, N. France	207	30	30	1910	....	....	....	2.1	....	....	26.5	....	...	...	....	....	9.6	7.9	....	..	100	
693	Calais, N. France	23	..	38	1910	....	....	....	....	....	....	29.1	....	...	...	....	....	....	....	....	..	100	
694	Dungeness, S. E. England	21	30	50	1921	....	1.60	2.8	2.4	10.4	13.6	24.4	46.6	...	...	....	....	11.3	9.8	42.6	56	100	191
695	Greenwich, England	150	81	81	1921	....	....	3.6	3.7	7.6	12.5	24.3	35.5	...	...	....	....	11.0	15.2	31.3	51	100	146
696	London, Kew Obs., England	18	51	51	1921	0.7	1.4	3.28	2.4	7.2	12.2	23.8	38.2	...	2.9	5.9	13.7	9.0	10.1	30.1	51	100	160
697	Southampton, England	64	43	51	1921	....	....	....	2.9	11.1	16.1	30.9	43.3	...	...	....	....	12.8	9.4	36.0	52	100	140
698	Falmouth, S. W. England	167	51	51	1921	....	....	....	2.8	11.0	28.9	43.6	60.6	...	...	....	....	14.3	6.4	25.2	66	100	139
699	Scilly Island, England	131	41	51	1921	....	....	....	3.1	8.8	21.6	31.9	45.3	...	...	....	....	13.7	9.7	27.6	68	100	142
700	Cardiff, S. Wales	202	..	35	1915	....	1.02	....	....	....	....	41.4	....	...	...	....	....	....	....	....	..	100	
701	Lluchfa, N. Wales	2,500	..	35	1917	....	....	....	....	....	....	161.0	....	...	...	....	....	....	....	....	..	100	
702	Holyhead, England	15	46	51	1921	....	....	....	3.4	10.4	24.0	34.9	56.2	...	...	....	....	11.4	9.7	29.8	69	100	161
703	Dublin, Ireland	47	51	51	1921	....	....	....	3.4	7.6	16.6	27.4	38.5	...	...	....	....	11.1	12.4	27.7	61	100	140
704	Cork Harbor, Ireland	32	..	35	1915	....	....	....	....	....	....	41.9	....	...	...	....	....	....	....	....	..	100	
705	Valencia, S. W. Ireland	30	52	52	1921	....	....	....	2.8	11.4	42.5	56.8	69.1	...	...	....	....	11.5	4.9	18.4	75	100	122
706	Killarney, Ireland	174	..	35	1915	....	....	....	....	....	....	54.7	....	...	...	....	....	....	....	....	..	100	

707	Kilkenny, Ireland.............	212	..	35	1915	....	....	....	....	.....	.....	33.1	.....	...	...	....	....	....	....	.....	..	100	
708	Mullingar, Ireland.............	360	..	35	1915	....	....	....	....	.....	.....	36.3	.....	...	...	....	....	....	....	.....	..	100	
709	Crossmolina, N. W. Ireland......	67	..	35	1915	....	....	....	....	.....	.....	53.3	.....	...	...	....	....	....	....	.....	..	100	
710	Armagh, N. E. Ireland..........	204	51	51	1921	....	....	....	2.6	7.2	23.6	31.7	39.6	...	...	....	....	9.9	8.2	22.7	74	100	125
711	Belfast, N. E. Ireland..........	61	..	35	1915	....	....	....	....	.....	.....	34.5	.....	...	...	....	....	....	....	.....	..	100	
712	Douglas, Isle of Man...........	277	..	46	1921	....	....	....	4.7	10.4	27.7	41.8	54.5	...	...	....	....	11.4	11.2	25.0	66	100	130
713	Stonyhurst, N. W. England......	375	52	52	1921	1.25	....	....	2.5	12.7	30.6	46.5	60.5	...	...	....	....	8.2	5.4	27.3	66	100	130
714	Seathwaite, N. W. England......	422	..	35	1915	....	....	....	....	.....	.....	129.5	.....	...	...	....	....	....	....	.....	..	100	
715	The Stye, N. W. England.......	1,070	..	35	1915	....	....	....	....	.....	.....	176.9	.....	...	...	....	....	....	....	.....	..	100	
716	Glasgow, Scotland.............	180	51	51	1920	....	....	....	2.7	9.8	29.0	37.2	61.6	...	...	....	....	8.0	7.3	26.4	78	100	166
717	Ben Nevis, Scotland...........	4,405	20	20	1904	....	....	....	7.3	48.3	107.8	160.8	240.1	...	...	....	....	11.9	4.5	30.0	67	100	150
718	Port William, W. Scotland......	171	46	46	1921	....	....	....	4.5	23.1	19.1	77.8	108.5	...	...	....	....	13.0	5.8	29.7	25	100	140
719	Stornoway, Hebrides Is.........	51	49	49	1921	....	....	....	2.8	14.9	32.2	49.8	71.6	...	...	....	....	12.6	5.6	30.0	65	100	144
720	Deerness, Orkney Is............	160	49	49	1921	....	....	....	3.6	8.2	29.1	35.4	44.6	...	...	....	....	11.1	10.2	23.2	82	100	128
721	Inverness, N. Scotland..........	98	..	35	1915	....	....	....	....	.....	.....	26.9	.....	...	...	....	....	....	....	.....	..	100	
722	Aberdeen, Scotland.............	48	36	50	1920	....	....	....	2.8	9.4	23.5	30.4	43.8	...	...	....	....	10.7	9.2	31.0	77	100	144
723	Dundee, Scotland..............	160	..	35	1915	....	....	....	....	.....	.....	27.4	.....	...	...	....	....	....	....	.....	..	100	
724	Crieff, Scotland...............	432	..	35	1915	....	....	....	....	.....	.....	40.9	.....	...	...	....	....	....	....	.....	..	100	
725	Edinburgh (Leith), Scotland.....	30	50	50	1920	....	....	....	2.8	7.2	15.9	23.3	37.5	...	...	....	....	10.3	12.0	30.9	68	100	161
726	Wolfelee, Cheviot Hills, Scotland	537	..	35	1915	....	....	....	....	.....	.....	37.6	.....	...	...	....	....	....	....	.....	..	100	
727	Tynemouth, N. E. England.....	96	51	51	1921	....	....	....	3.2	7.8	18.6	24.5	40.9	...	...	....	....	12.3	13.0	31.8	76	100	167
728	Liverpool, W. England..........	188	..	35	1915	....	....	....	....	.....	.....	27.9	.....	...	...	....	....	....	....	.....	..	100	
729	Derby, Central England.........	987	47	47	1921	....	....	....	2.8	10.3	32.4	48.4	65.5	...	...	....	....	10.2	5.8	21.3	67	100	135
730	Yarmouth, E. England..........	17	51	51	1921	....	....	....	4.8	8.6	19.4	24.5	44.2	...	...	....	....	12.0	19.6	35.0	79	100	180
731	Vlieland, N. Holland...........	......	26	26	1910	....	....	....	2.1	7.5	10.4	22.8	33.4	...	...	....	....	14.4	9.2	33.0	46	100	147
732	Leeghwater, Holland...........	......	45	45	1911	....	....	....	2.3	10.2	21.8	32.1	46.6	...	...	....	....	13.0	7.2	31.8	68	100	145
733	Utrecht-De Bilt, Holland.......	15	62	72	1920	....	....	....	2.9	8.5	17.7	28.7	41.1	...	...	....	....	10.1	10.2	29.7	63	100	144
734	Ostend, Belgium...............	12	..	50	1900	....	....	....	....	.....	.....	25.7	.....	...	...	....	....	....	....	.....	..	100	
735	Brussels, Belgium..............	187	..	50	1900	....	....	....	....	.....	.....	28.8	.....	...	...	....	....	....	....	.....	..	100	
736	Bouillon, Belgium..............	774	..	50	1900	....	....	....	....	.....	.....	47.4	.....	...	...	....	....	....	....	.....	..	100	
737	Treves, W. Germany...........	460	..	101	1924	....	....	....	....	9.4	15.1	26.9	39.2	...	...	....	....	8.3	....	34.9	56	100	146
738	Freiburg, S. W. Germany.......	925	..	50	1900	....	....	....	....	.....	.....	33.4	.....	...	...	....	....	....	....	.....	..	100	
739	Frankfort-am-Main, Germany...	335	..	88	1924	....	....	....	....	8.4	14.1	24.3	36.8	...	...	....	....	10.9	....	34.6	58	100	151
740	Gutersloh, Germany............	249	..	87	1924	....	....	....	....	8.2	19.1	30.0	38.4	...	...	....	....	9.3	....	27.3	64	100	128
741	Bremen, Germany..............	......	..	37	....	....	....	....	....	.....	.....	27.9	.....	...	...	....	....	....	....	.....	..	100	
742	Berlin, Germany...............	115	..	74	1924	....	....	....	....	9.0	14.2	22.7	30.0	...	...	....	....	13.1	....	39.6	63	100	132

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(*Continued*)

Station No.	Station	Elevation above sea, ft.	Years of record			Maximum observed precipitation					Annual precipitation			Observed precipitation, percentages of mean annual rainfall									
			For day or less	For month and year	Last year included	Number of minutes			Day of 24 hr.	Month	Minimum	Mean	Maximum	Mean daily intensity on days of rainfall	Maximum during short periods, Number of minutes			Mean for month of maximum record	Maximum		Annual		
						5	15	60							5	15	60		In 24 hr.	In one calendar month	Minimum	Mean	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
743	Dresden, Germany	.....	27	58	1885	....	....	....	4.0	8.2	11.7	23.0	34.1	...	...	....	....	13.6	17.5	35.8	51	100	148
744	Breslau, Germany	482	..	66	1924	....	....	....	....	9.0	15.9	22.6	32.6	...	...	....	....	12.6	....	39.8	70	100	145
745	Prague, Czechoslovakia	.....	..	50	....	....	....	....	....	.....	.....	18.5	.....	...	...	....	....	....	....	.....	..	100	
746	Krynica, Poland	1,923	..	44	1920	....	....	....	....	12.0	21.2	33.4	48.4	...	...	....	....	11.1	....	35.9	63	100	145
747	Lemberg, Poland	978	..	45	1920	....	....	....	....	10.6	20.3	27.0	37.1	...	...	....	....	15.0	....	39.2	75	100	138
748	Warsaw, Poland	436	40	109	1920	....	....	....	3.4	6.8	14.6	21.2	46.5	0.8	...	....	....	14.9	16.2	32.3	69	100	220
749	Konigsberg, E. Prussia	10	..	84	1924	....	....	....	....	9.8	12.9	20.5	33.3	...	...	....	....	13.3	....	48.0	63	100	163
750	Vilna, E. Poland	486	..	43	1915	....	....	....	....	8.1	10.6	20.4	28.7	...	...	....	....	15.7	....	39.6	54	100	141
751	Riga, Latvia	66	30	31	1882	....	....	....	1.7	7.4	12.5	20.0	30.4	0.7	...	....	....	10.5	8.5	37.0	62	100	152
752	Tallinn, Esthonia	46	27	40	1891	....	....	....	2.0	7.5	7.1	19.2	27.7	...	...	....	....	14.3	10.4	39.1	37	100	144
753	Leningrad, Russia	16	44	95	1920	....	....	....	2.3	7.3	12.1	19.2	29.3	0.6	...	....	....	14.7	12.0	38.0	63	100	153
754	Moscow, Russia	538	27	64	1916	....	....	....	1.7	8.0	13.7	22.7	32.8	0.6	...	....	....	9.6	7.5	35.3	61	100	144
755	Gorki, Russia	679	32	42	1891	....	....	....	3.1	8.1	12.8	19.9	38.5+	0.6	...	....	....	13.7	15.6	40.7	64	100	194+
756	Kiev, Russia	600	23	62	1915	....	....	....	4.1	8.8	13.0	22.2	33.5	0.8	...	....	....	14.1	18.5	39.7	59	100	151
757	Odessa, Russia	213	20	69	1915	....	....	....	3.1	6.6	9.0	15.5	24.9	1.3	...	....	....	14.1	19.9	42.4	58	100	160
758	Taganrog, Russia	115	..	30	1908	....	....	....	5.5	6.8	9.4	16.1	26.4	...	...	....	....	12.8	34.1	42.2	58	100	164
759	Sevastopol, Crimea	132	16	34	1882	....	....	....	2.1	4.7	8.4	15.2	25.3	1.0	...	....	....	8.2	13.8	30.8	55	100	166
		76	30	30	1908	....	....	....	4.6	.....	.....	14.1	.....	...	...	....	....	7.8	32.5	.....	..	100	
760	Novorossisk, Russia	122	34	40	1914	....	....	....	4.0	10.2	11.7	27.1	45.5	...	...	....	....	12.8	14.8	37.6	43	100	168
761	Batum, Trans-Caucasia	10	..	26	1908	....	....	....	10.3	20.5	66.2	98.3	111.0	...	...	....	....	9.5	10.5	20.9	68	100	113
762	Tiflis, Trans-Caucasia	1,325	..	70	1914	....	....	....	....	9.0	11.3	19.6	30.6	1.0	...	....	....	15.9	....	45.9	58	100	156

763	Lenkoran, Trans-Caucasia	—66	..	27	1891	....	....	....	....	19.6	29.3	46.5	66.5	1.0	...	...	...	17.2	....	42.2	63	100	143
764	Baku, Trans-Caucasia	7	20	39	1888	....	....	....	4.0	6.6	5.6	9.7	19.2	1.6	...	...	...	13.7	41.3	68.1	58	100	198
765	Astrakhan, Russia	—46	33	68	1915	....	....	....	2.2	6.0	1.0	6.3	12.4	2.2	...	...	...	8.3	35.0	95.5	16	100	196
766	Lugan, S. Russia	164	43	54	1891	....	....	....	2.5	5.7	8.8	14.9	24.5	0.9	...	...	...	12.7	16.8	48.2	59	100	164
767	Nikolaewskoe, Russia	633	..	36	1915	....	....	....	....	6.0	10.2	14.5	22.6	1.2	...	...	...	14.5	....	41.3	70	100	155
768	Kasan, E. Russia	266	..	54	1916	....	....	....	....	5.9	9.9	16.4	24.7	...	...	...	...	14.8	....	36.0	60	100	151
769	Slatousk, E. Russia	1,476	44	55	1891	....	....	....	3.5	10.2	8.6	18.6	31.4	0.6	...	...	...	15.8	18.8	54.8	46	100	168
770	Orenburg, E. Russia	374	32	64	1916	....	....	....	1.7	6.5	9.2	15.0	24.0	0.9	...	...	...	12.5	11.3	43.3	61	100	160
771	Irgis, W. Siberia	367	18	26	1891	....	....	....	1.2	3.5	2.8	6.9	9.6	2.0	...	...	...	9.9	17.4	50.8	41	100	139
772	Khiva, Turkestan	328	..	17	1891	....	....	....	....	2.3	0.8	2.1	3.4	...	...	...	...	26.6	....	109.4	38	100	162
773	Tashkent, Turkestan	3,378	..	20	1891	....	....	....	....	5.3	9.4	13.1	18.0	1.8	...	...	...	19.0	....	40.4	72	100	137
774	Barnaul, S. Siberia	479	43	54	1891	....	....	....	2.2	5.8	4.2	11.5	17.7	1.2	...	...	...	14.3	19.1	50.4	37	100	154
775	Irkutsk, S. Siberia	1,509	..	17	1891	....	....	....	....	7.1	9.6	14.5	18.8	1.6	...	...	...	15.8	....	49.0	66	100	130
776	Nertschinsk, S. Siberia	2,155	42	51	1891	....	....	....	6.2	11.2	7.2	16.0	25.9	1.2	...	...	...	26.9	38.7	69.8	45	100	162
777	Nikolaewsk-am-Amur, S. Siberia	108	..	60	1919	....	....	....	2.5	9.0	7.8	17.6	30.2+	1.2	...	...	...	15.2	14.2	51.0	44	100	172+
778	Ajan, E. Siberia	33	12	12	1912	....	....	....	3.6	19.1	16.2	35.0	83.0	1.3	...	...	...	22.2	10.3	54.5	46	100	237
779	Okhotsk, E. Siberia	20	23	23	1919	....	....	....	1.9	11.2	6.4	11.2	16.5	...	...	...	...	20.6	16.9	100.0	57	100	147
780	Gishiga, E. Siberia	40	..	24	1919	....	....	....	....	3.9	5.8	9.2	12.2+	...	...	...	...	18.0	....	42.5	63	100	133
781	Petropavlovsk, E. Siberia (S. Kamchatka)	52	..	8	1891	....	....	....	....	16.1	36.4	47.0	76.0	...	...	...	...	12.4	....	34.3	78	100	162
782	Novo-Mariinski, N. E. Siberia	16	..	21	1919	....	....	....	....	4.3	4.7	7.4	10.3	...	...	...	...	20.6	....	58.1	64	100	139
783	Markoyo, N. E. Sibera	66	25	25	1919	....	....	....	1.6	6.1	5.4	7.9	10.0	...	...	...	...	24.1	20.3	77.2	68	100	127
784	Verkhoyansk, N. Siberia	328	34	34	1919	....	....	....	0.8	3.4	2.0	5.0	9.1	...	...	...	...	17.6	16.0	68.0	40	100	182
785	Yakutsk, Siberia	354	36	36	1923	....	....	....	1.4	4.6	4.0	7.4	12.8	0.7	...	...	...	14.5	18.9	62.2	54	100	173
786	Viluisk, N. Siberia	394	26	26	1923	....	....	....	1.8	4.8	5.7	8.9	11.5	...	...	...	...	18.0	20.3	54.0	64	100	129
787	Turuchansk, N. W. Siberia	131	39	39	1922	....	....	....	1.4	5.7	5.0	13.2	20.8	0.7	...	...	...	14.9	10.6	43.2	38	100	158
788	Obdorsk, N. W. Siberia	59	..	27	1909	....	....	....	....	5.8	4.5	10.1	14.4	1.1	...	...	...	20.0	....	57.3	44	100	142
789	Beresov, N. W. Siberia	134	28	28	1909	....	....	....	2.6	6.8	5.6	13.8	21.9	...	...	...	...	16.5	18.8	49.3	41	100	158
790	Surgut, N. W. Siberia	138	24	24	1909	....	....	....	2.7	6.0	11.8	15.8	23.0	...	...	...	...	16.4	17.0	37.8	74	100	145
791	Bogoslovsk, W. Siberia	623	42	54	1891	....	....	....	2.6	7.7	5.4	16.4	28.4	0.8	...	...	...	17.4	15.8	47.0	33	100	173
792	Ekaterinburg, W. Siberia	921	45	80	1915	....	....	....	3.7	8.4	6.9	15.2	24.4	1.0	...	...	...	19.0	24.3	55.4	45	100	160
793	Perm, E. Russia	534	..	33	1915	....	....	....	....	7.2	17.6	23.2	31.4	0.7	...	...	...	12.3	....	31.0	76	100	135
794	Ust-Zylma, N. E. Russia	82	..	21	1916	....	....	....	....	4.6	12.2	16.6	21.9	...	...	...	...	15.9	....	27.7	73	100	132
795	Archangel, N. Russia	22	..	51	1916	....	....	....	....	5.1	9.5	16.8	29.8	1.0	...	...	...	14.3	....	30.2	56	100	177
796	Kem, N. W. Russia	20	18	28	1891	....	....	....	1.8	4.8	10.2	14.5	21.7	...	...	...	...	13.6	12.4	33.0	70	100	150
797	Helsingfors, Finland	38	..	80	1924	....	....	....	....	7.6	14.3	24.4	33.9	...	...	...	...	9.2	....	31.1	59	100	139
798	Abo, S. W. Finland	66	..	49	1800	....	....	....	....	7.3	15.7	23.8	33.0	...	...	...	...	12.5	....	30.6	66	100	138
799	Upsala, Sweden	79	..	74	1924	....	....	....	....	7.9	12.3	21.2	32.0	...	...	...	...	12.6	....	37.4	58	100	151
800	Ostersund, Sweden	1,017	..	51	1924	....	....	....	....	6.3	11.2	18.6	26.4	...	...	...	...	16.4	....	33.9	60	100	142
801	Haparanda, Sweden	30	..	65	1924	....	....	....	....	7.4	9.2	19.2	30.1	...	...	...	...	11.6	....	38.6	48	100	157
802	Karesuanda, Sweden	1,092	..	46	1924	....	....	....	....	6.1	6.8	12.4	20.2	...	...	...	...	18.9	....	49.1	55	100	163

TABLE V.—RAINFALL CHARACTERISTICS.[1]—(Continued)

Station No.	Station	Elevation above sea, ft.	Years of record: For day or less	Years of record: For month and year	Years of record: Last year included	Maximum observed precipitation: Number of minutes: 5	Maximum observed precipitation: Number of minutes: 15	Maximum observed precipitation: Number of minutes: 60	Maximum observed precipitation: Day of 24 hr.	Maximum observed precipitation: Month	Annual precipitation: Minimum	Annual precipitation: Mean	Annual precipitation: Maximum	Observed precipitation, percentages of mean annual rainfall: Mean daily intensity on days of rainfall	Maximum during short periods: Number of minutes: 5	Maximum during short periods: Number of minutes: 15	Maximum during short periods: Number of minutes: 60	Mean for month of maximum record	Maximum: In 24 hr.	Maximum: In one calendar month	Annual: Minimum	Annual: Mean	Annual: Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
803	Kola, Lapland	33	..	14	1891	....	....	....	....	2.4	4.9	6.9	10.6	...	...	....	....	15.5	....	34.8	71	100	154
804	Gjesvar, N. Norway	26	..	37	1920	....	....	....	....	10.4	17.3	29.1	46.1	...	...	....	....	11.4	....	35.6	59	100	158
805	Bodo, N. Norway	75	..	53	1920	....	....	....	....	12.4	20.1	35.6	58.0	...	...	....	....	9.4	....	34.8	56	100	162
806	Christiania (Oslo), S. Norway	85	..	55	1920	....	....	....	....	8.1	14.5	22.1	31.7	...	...	....	....	10.6	....	36.6	65	100	143
807	Copenhagen, Denmark	16	..	100	1920	....	....	....	....	8.0	10.4	22.5	30.3	...	...	....	....	7.9	....	35.5	46	100	134
808	Lerwick, Shetland Is.	112	50	50	1921	....	....	....	2.8	10.8	27.6	38.0	57.3	...	...	....	....	11.2	7.3	28.4	72	100	150
809	Thorshavn, Faroes Is.	89	..	48	1920	....	....	....	....	13.7	37.6	57.2	77.1	...	...	....	....	11.5	....	33.9	66	100	134
810	Berufjord, E. Iceland	64	..	51	1923	....	....	....	....	15.1	22.5	45.9	68.4	...	...	....	....	10.8	....	32.9	49	100	149
811	Vestmanno S. W. Iceland	27	..	43	1923	....	....	....	....	13.9	37.3	53.7	70.7	...	...	....	....	10.8	....	25.8	69	100	131
812	Stykkisholm, N. W. Iceland	88	..	67	1923	....	....	....	....	8.6	13.8	25.7	40.5	...	...	....	....	10.0	....	33.4	54	100	157
813	Grimsey, N. Iceland	78	..	41	1923	....	....	....	....	7.3	2.6	11.7	27.1	...	...	....	....	11.6	....	62.0	22	100	231
814	Angmagsalik, E. Greenland	109	..	22	1920	....	....	....	....	18.6	21.8	35.6	58.4	...	...	....	....	17.5	....	52.2	61	100	164
815	Upernivik, N. W. Greenland	68	..	47	1920	....	....	....	....	5.3	3.4	9.0	17.8	...	...	....	....	10.1	....	59.0	38	100	198
816	Jacobshavn, W. Greenland	48	..	47	1920	....	....	....	....	5.9	5.2	9.1	17.8	...	...	....	....	15.2	....	65.0	57	100	196
817	Godthaab, S. W. Greenland	35	..	37	1920	....	....	....	....	11.0	12.2	23.7	35.5	...	...	....	....	13.1	....	46.5	51	100	150
818	Ivigtut, S. Greenland	21	..	45	1920	....	....	....	....	20.6	20.9	44.8	72.5	...	...	....	....	9.8	....	46.0	47	100	161
819	Horta, Azores	210	..	22	1923	....	....	....	....	9.9	30.5	42.0	64.0	...	...	....	....	11.2	....	23.0	71	100	149
820	Ponta-Delgada, Azores	123	..	30	1923	....	....	....	....	9.5	19.6	28.3	42.2	...	...	....	....	5.2	....	33.5	69	100	149

[1] JARVIS, "Rainfall Characteristics and Their Relation to Soils and Runoff," *Proc. Am. Soc. Civil Eng.*, pp. 3 to 47, January, 1930.

APPENDIX II

EFFECTS OF HOLES IN DAMS ON STRESSES

STRESSES AROUND CIRCULAR HOLES IN DAMS[1]

Let Fig. 18*a*, page 109, represent a section of the theoretical profile of a gravity dam with a circular sluiceway through it near the base. Let p be the computed vertical mass stress per square foot at the center of the hole, calculated on the basis of a solid dam. The stress p will be a maximum at the heel for reservoir empty and at the toe for reservoir full.

Let Fig. 18*b* be taken as a right section of the sluiceway shown in Fig. 18*a*, and let it be assumed that the disturbed stresses lie mainly within the larger circle there shown. The stresses at the boundary of the outer circle are essentially the same as in the dam without the hole. The radial stress s_r perpendicular to the elementary volume will be $s_r\,rd\theta$ and will be balanced by the vertical force pdx resolved onto the opposite side of this volume; therefore, $s_r\,rd\theta = pdx \cos\theta$, whence $s_r = \dfrac{p\cos\theta\,dx}{rd\theta}$ and therefore

$$s_r = p\cos^2\theta = \frac{p}{2} + \frac{p}{2}\cos 2\theta. \tag{356}$$

In like manner the shearing stress s_s would be opposed by the force

$$s_s = -\tfrac{1}{2}p\sin 2\theta, \tag{357}$$

and the tangential stress is

$$s_\theta = p\sin^2\theta = \frac{p}{2} - \frac{p}{2}\cos 2\theta. \tag{358}$$

The stress distribution may now be considered to consist of two parts, one the constant components $\tfrac{1}{2}p$ and the other the components $\tfrac{1}{2}p\cos 2\theta$ and the shearing force $\tfrac{1}{2}p\sin 2\theta$, both containing functions of θ. The stresses due to these two groups of forces can best be determined separately and their sum taken for their total effect.

[1] HANNA, F. W., "Stresses in Circular Holes in Dams," *Proc. Am. Soc. Civil Eng.*, pp. 745–751, April, 1937.

The first group of forces acts symmetrically around the center of the hole in the dam and is represented by the stress distribution in a thick hollow cylinder subjected to external normal pressure.

By adding Eqs. (356) and (358),

$$s_r' + s_\theta' = p, \text{ a constant.} \tag{359}$$

Now from Fig. 1*d*, $(s_r' + ds_r')(r + dr) - s_r'r = s_\theta' \, dr$ by equilibrating the acting pressure and the resisting stress. Dropping differentials of higher orders,

$$r ds_r' + s_r' \, dr = s_\theta' \, dr. \tag{360}$$

Solving for s_θ in Eq. (359) and substituting the resulting value in this expression, it is found that

$$r ds_r' + s_r' \, dr = (p - s_r') \, dr. \tag{361}$$

Rearranging,

$$\frac{r ds_r'}{s_r' - \frac{p}{2}} = -\frac{2dr}{r}. \tag{362}$$

Integrating,

$$\log\left(s_r' - \frac{p}{2}\right) = \log \frac{C}{r^2}. \tag{363}$$

Finding the antilogs of both sides of this equation,

$$s_r' - \frac{p}{2} = \frac{C}{r^2}. \tag{364}$$

When $r = r_1$ in this expression, $s_r' = 0$; and, therefore,

$$-\frac{p}{2} = \frac{C}{r^2}. \tag{365}$$

Solving for C,

$$C = -\frac{pr_1^2}{2}. \tag{366}$$

Substituting the value of C just found in Eq. (364),

$$s_r' - \frac{p}{2} = -\frac{pr_1^2}{2r^2}. \tag{367}$$

Solving for s_r' in this equation, the following important equation is derived:

$$s_r' = \frac{p}{2}\left(1 - \frac{r_1^2}{r^2}\right). \tag{368}$$

Eliminating s_r' between Eqs. (368) and (359) and solving for s_θ', the companion important equation is found,

$$s_\theta' = \frac{p}{2}\left(1 + \frac{r_1^2}{r^2}\right). \tag{369}$$

Since functions of θ are not involved in this group of forces, a third important result is obtained,

$$s_s' = 0. \tag{370}$$

Addressing the computations to the solution for the stresses produced by the second group of forces involving functions of both r and θ, it is found that this can best be done by the use of the Airy stress functions and the incidental equations, all expressed in polar coordinates. First it can be shown from Fig. 18*b* that the equations of equilibrium on the elementary volume there shown are

$$\frac{ds_r}{dr} + \frac{ds_s}{rd\theta} + \frac{s_r - s_s}{r} = 0, \tag{371}$$

$$\frac{ds_\theta}{rd\theta} + \frac{ds_s}{dr} + \frac{2s_s}{r} = 0. \tag{372}$$

The differential equation to be solved is

$$\left(\frac{d^2}{dr^2} + \frac{d}{rdr} + \frac{d^2}{r^2d\theta^2}\right)\left(\frac{d^2F}{dr^2} + \frac{dF}{rdr} + \frac{d^2F}{r^2d\theta^2}\right) = 0. \tag{373}$$

The Airy stress function F is defined as

$$s_r = \frac{d^2F}{dr^2} + \frac{1}{r}\frac{dF}{dr}, \tag{374}$$

$$s_\theta = \frac{d^2F}{dr^2}, \tag{375}$$

and

$$s_s = -\frac{d^2F}{dr\,r\,d\theta}. \tag{376}$$

From Eqs. (356), (357), and (358) it is evident that F should be assigned the following value as being a function of r and θ:

$$F = f(r) \cos \theta. \tag{377}$$

Differentiating Eq. (377) twice with respect to θ and noting that the result must hold for $\theta = 0$ as well as other values around the center of the hole,

$$\frac{d^2F}{r^2 d\theta^2} = -\frac{4f(r)}{r^2}. \tag{378}$$

Substituting this value in Eq. (373), the differential equation becomes

$$\left(\frac{d^2}{dr^2} + \frac{d}{rdr} - \frac{4f(r)}{r^2}\right)\left(\frac{d^2f(r)}{dr^2} + \frac{df(r)}{rdr} - \frac{4f(r)}{r^2}\right) = 0. \tag{379}$$

The general solution of this equation is

$$f(r) = C_1r^2 + C_2r^4 + \frac{C_3}{r^2} + C_4. \tag{380}$$

Substituting the value of $f(r)$ in Eq. (377),

$$F = \left(C_1r^2 + C_2r^4 + \frac{C_3}{r^2} + C_4\right) \cos 2\theta; \tag{381}$$

performing the differentiations on Eq. (381) required by Eqs. (374), (375), and (376) and substituting the results in these equations, the following equations are obtained:

$$s_r'' = -\left(2C_1 + \frac{6C_3}{r^4} + \frac{4C_4}{r^2}\right) \cos 2\theta, \tag{382}$$

$$s_\theta'' = \left(2C_1 + 12C_2r^2 + \frac{6C_3}{r^4}\right) \cos 2\theta, \tag{383}$$

and

$$s_s'' = \left(2C_1 + 6C_2r^2 - \frac{6C_3}{r^4} - \frac{2C_4}{r^2}\right) \sin 2\theta. \tag{384}$$

The constants in Eqs. (382), (383), and (384) are to be determined from the external boundary conditions at a distance as required by Eqs. (356), (357), and (358) and from the fact that the stresses are 0 at the edge of the hole. The following four equations result from these conditions:

$$2C_1 + \frac{6C_3}{r^4} + \frac{4C_4}{r^2} = -\frac{1}{2}p, \tag{385}$$

$$2C_1 + \frac{6C_3}{r_1^4} + \frac{4C_4}{r_1^2} = 0, \tag{386}$$

$$2C_1 + 6C_2r^2 - \frac{6C_3}{r^2} - \frac{2C_4}{r^2} = -\frac{1}{2}p, \tag{387}$$

$$2C_1 + 6C_2r_1^2 - \frac{6C_3}{r_1^4} - \frac{2C_4}{r_1^2} = 0. \tag{388}$$

Solving these equations for values of C_1, C_2, C_3, and C_4 and assuming $r_1/r = 0$ to fit the conditions for distant points, it will be found that $C_1 = -p/4$, $C_2 = 0$, $C_3 = -pr_1^4/4$, and $C_4 = pr_1^2/2$. Substituting these values in Eqs. (382), (383), and (384), there result the following equations:

$$s_r'' = \frac{p}{2}\left(1 + \frac{3r_1^4}{r^4} - \frac{4r_1^2}{r^2}\right) \cos 2\theta, \tag{389}$$

$$s_\theta'' = -\frac{p}{2}\left(1 + \frac{3r_1^4}{r^4}\right) \cos 2\theta, \tag{390}$$

$$s_s'' = -\frac{p}{2}\left(1 - \frac{3r_1^4}{r^4} + \frac{2r_1^2}{r^2}\right) \sin 2\theta. \tag{391}$$

Now adding the latter three equations to Eqs. (368), (369), and (370), respectively, and noting that $s_r' + s_r'' = s_r$, $s_\theta' + s_\theta'' = s_\theta$, and $s_s' + s_s'' = s_s$, the following final important equations are obtained:

$$s_r = \frac{p}{2}\left[1 - \frac{r_1^2}{r^2} + \left(1 - \frac{4r_1^2}{r^2} + \frac{3r_1^4}{r^4}\right) \cos 2\theta\right], \tag{392}$$

$$s_\theta = \frac{p}{2}\left[1 + \frac{r_1^2}{r^2} - \left(1 + \frac{3r_1^4}{r^4}\right) \cos 2\theta\right], \tag{393}$$

$$s_s = -\frac{p}{2}\left(1 + \frac{2r_1^2}{r^2} - \frac{3r_1^4}{r^4}\right) \sin 2\theta. \tag{394}$$

These equations give, respectively, the radial compression, the tangential tension, and the radial shear in the vicinity of the hole. If r is very large, s_r, s_s, and s_θ approach the values given in Eqs. (356), (357), and (358). If $r = r_1$, $s_r = s_s = 0$, and $s_\theta = p - 2p \cos 2\theta$. For $\theta = 90°$ and $270°$, $s_\theta = 3p$, the maximum compression located at the ends of the horizontal diameter. For $\theta = 0°$ and $180°$, $s_\theta = -p$, the maximum tension located at the end of the vertical diameter. In general, the compressive stresses

are not serious in a dam although they may be quite large, but the tensional stresses at the top and bottom of the hole should be provided for with appropriate steel reinforcement.

Equations similar to Eqs. (392), (393), and 394) have been developed by several investigators[1] for tension in thin plates. Here they have been developed for compression in plates in order to make them apply to dams.

STRESSES AROUND ELLIPTICAL HOLES IN DAMS

Drainage and inspection galleries are not often circular in shape but generally may be made to approach ellipses (see Fig. 91*a*). The resultant of mass and water loads on circular sluice-

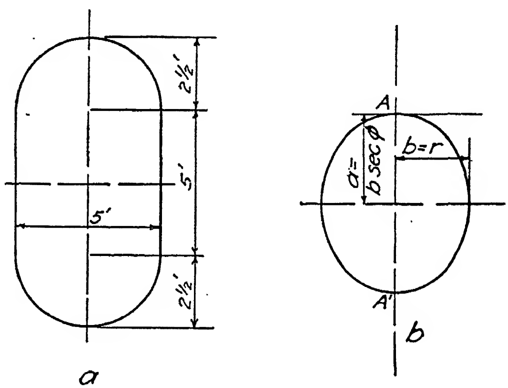


FIG. 91.

ways acts on elliptical sections of these openings (see Fig. 91*b*). Consequently, stresses around elliptical holes are important in dam design.

The development of formulas for the determination of stresses caused by elliptical holes in dams involves the use of curvilinear coordinates and hyperbolic functions. Formulas have been developed by Inglis for elliptical plates,[2] and they may be applied

[1] KIRSH, G., *Zeitschr. Ver. deut. Ing.*, Vol. 42, 1898. SUYEHIRO, K., "Stresses in Plates Having Discontinuities and Some Problems Connected with Them," *Engineering*, Sept. 1, 1911. FÖPPLE, A., *Drang and Zwang*, Vol. I, pp. 314–319; *Vorlesungen über Technische Mechaniks*, Vol. 5, p. 352. TIMOSHENKO, S., "Theory of Elasticity," pp. 75–78.

[2] INGLIS, C. E., "Stresses in a Plate Due to the Presence of Cracks and Sharp Corners," *Trans. Inst. Naval Architecture*, 1913, Part I, pp. 219–230.

to holes in dams by making the proper assumptions. The resultant Inglis formulas applicable to the case will only be summarized here, as the reader can find their development in available engineering literature.

If the major axis of the elliptical inspection gallery is vertical, it will be parallel to the acting force due to the mass load, which is the only force when the reservoir is empty. For this case there would be compressive stress developed tangentially at the ends of the horizontal diameter. This stress is given by the equation

$$s_\theta = p\left(1 + \frac{2b}{a}\right), \tag{395}$$

and tangential tensile stress at the ends of the vertical diameter is given by the equation

$$s_\theta = -p; \tag{396}$$

Eqs. (395) and (396) apply also to combined mass and water loads on elliptical sections of circular sluiceways.

For the water pressure acting horizontally against this gallery and vicinity, there would be tangential compressive stress developed at the ends of the major axis of the ellipse represented by the equation

$$s_\theta = p'\left(1 + \frac{2a}{b}\right), \tag{397}$$

and tangential tensile stress at the ends of the minor axis represented by the equation

$$s_\theta = -p', \tag{398}$$

where p' is the mean water pressure.

Radial stresses, both for the mass load and for the water load, would be 0 at the ends of the axes of the ellipse. Those at right angles to the acting force would increase and then die down to 0 again as distance from the hole increased; those parallel to the acting force would gradually approach p as the distance from the hole increases.

If the acting force is inclined to the major axis of the ellipse, the tangential compressive stress at the ends of the axis is given by the equation

$$s_\theta = p\left[\frac{a}{b} - \cos 2\theta\left(1 + \frac{a}{b}\right)\right], \tag{399}$$

where θ is the angle of inclination. For $\theta = 45°$, $s_\theta = p\frac{a}{b}$; the tangential stress at the point of tangency of the tangent parallel to the acting force is $s_\theta = p\left(1 + \frac{a}{b} + \frac{b}{a}\right)$; and that at the point of tangency perpendicular to the acting force is $s_\theta = -p$. The maximum tangential stress is somewhat larger and is approximated by the equation

$$s_\theta = \frac{pa}{2b}\left(1 + \frac{\sqrt{2a^2 + 2b^2}}{a - b}\right). \tag{400}$$

Equations (399) and (400) are useful in finding the stress around an elliptical hole in a dam where the major axis of the ellipse is vertical and the resultant of the mass and water load is inclined at the angle θ with this axis.

It is of interest and value to note the relation of the stresses at the ends of the diameters of elliptical holes to the radii of curvatures at these points. The radius of curvature at the ends of the major axis is $r = b^2/a$; and, therefore, $b = \sqrt{ar}$. Likewise, the minor axis is $r = a^2/b$; and then $a = \sqrt{br}$. Substituting $\sqrt{br}$ for a in Eq. (395), there is obtained the equation

$$s_\theta = p\left(1 + \sqrt{\frac{b}{r}}\right). \tag{401}$$

Substituting ar for b in Eq. (397), the following equation results:

$$s_\theta = p\left(1 + \sqrt{\frac{a}{r}}\right). \tag{402}$$

Both Eqs. (401) and (402) show the stress s_θ to decrease as r increases. As r approaches infinity, *i.e.*, as the curve approaches a straight line, s_θ approaches p; and, as r approaches 0, *i.e.*, as the curve approaches a crack, s_θ approaches infinity. This is why cracks are sometimes very dangerous. These conclusions show that the two foregoing equations may be applied to a great variety of cavities where the length is $2a$ or $2b$ and the ends have a radius of curvature of r, provided that the cavity merges quite smoothly into an ellipse at its ends. In other words, it may be stated generally that the stresses at the ends of such a cavity depend almost entirely on the length of the cavity and the form of its ends.

SQUARE HOLES

Square holes in dams should be avoided, especially if rectangular corners are employed. Rounding the corners with liberal radii helps the stress conditions. The injunction against the use of square holes applies to the customary flat bottoms in otherwise well-designed drainage and inspection galleries.

LIMITATION OF APPROXIMATE FORMULAS

It is appreciated that the foregoing approximate formulas are not applicable where there is too much variation in the pressure in the vicinity of the hole in the dam, which is the case in a low dam. However, inspection and drainage galleries are not often needed in such structures. In high dams where such openings are needed, the pressures are fairly uniform, and only slight error is involved in the assumption that uniformity of pressure exists in the vicinity of the hole both for mass and water pressure. It should be observed that mass pressure is the predominant one and is at a maximum with the absence of water pressure. When the reservoir is full, the mass pressure nearly all disappears, and the horizontal water pressure predominates.

Little valid objection can be made to the assumption that a narrow vertical slice of a gravity dam can be considered as a plate. Gravity dam sections are designed on the basis of the existence of such solitary plates, and this in spite of the effects of Poisson's ratio on their sides. Corrections can be made for the effects of this ratio on the sides of the slice by making a separate computation for a stress equal to 25 per cent of the major force. The effect of Poisson's ratio is to lessen that of the major force.

APPENDIX III

EARTHQUAKE STRESSES IN ARCH DAMS DUE TO WEIGHT

GENERAL STATEMENT

In those parts of the world where earthquakes are prevalent, consideration should be given to stresses produced by earthquake shocks. In the United States this applies particularly to the Pacific Coast region. Stresses due to the inertia of the water acting on the face of a dam have been treated in Chap. V. Here only the stresses due to the mass or weight of the masonry of an arch dam will be considered. The discussion covers both single- and multiple-arch dams of the symmetrical type.

EARTHQUAKE STRESSES IN SINGLE-ARCH DAMS

First, let the earthquake impulse be assumed to act horizontally in a vertical plane parallel to the vertical section through the crown and arch centers. This impulse may act either in a downstream or an upstream direction. In this connection see Fig. 92, representing a semiarch ring in a thin, symmetrical, single-arch dam of uniform thickness. Let the coordinates, signs, symbols, crown-bracket support, redundant forces, and dummy unit loads used in Chaps. XII and XIII be used as there described and shown in Figs. 39 and 41.

Owing to the circularity of the arch, the center of gravity of any elementary volume of the arch ring will be slightly upstream from the central axis. The distance from the center of the arch ring to the center of gravity of such a volume will be designated by r_1, and the distance from the arch-ring center axis will be designated by r. Now the relation of these two radii may be expressed as follows:

$$r_1 = \int_{r-\frac{t}{2}}^{r+\frac{t}{2}} rdr\ rd\phi \div \int_{r-\frac{t}{2}}^{r+\frac{t}{2}} dr\ rd\phi. \qquad (403)$$

Taking $d\phi$ as constant for a single elementary volume and integrating, it is found that

$$r_1 = \left[\frac{r^3}{3}\right]_{r-\frac{t}{2}}^{r+\frac{t}{2}} \div \left[\frac{r^2}{2}\right]_{r-\frac{t}{2}}^{r+\frac{t}{2}}. \tag{404}$$

Substituting the limits and reducing,

$$r_1 = r\left(1 + \frac{t^2}{12r^2}\right), \tag{405}$$

$$\frac{r_1}{r} = 1 + \frac{t^2}{12r^2}. \tag{406}$$

Now for an extreme case where $r = 2t$, it is found that $r_1/r = 1.002$ and, therefore, the use of r for r_1 in the calculations will not introduce serious error. It will thus be permissible to assume that the earthquake impulse acts at the mid-point instead of at the center of gravity of each elementary volume of the arch ring.

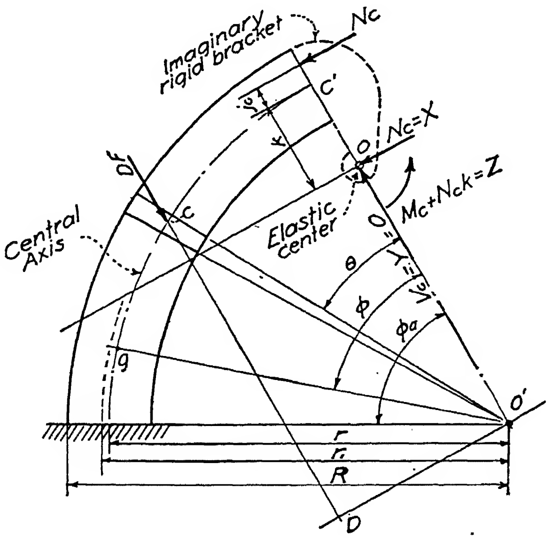


FIG. 92.

In Fig. 92, let DF, an elementary earthquake impulse, act either downstream or upstream on an elementary volume of the arch ring at the point C and in a vertical plane through CD parallel to the plane passing through the crown and arch center. Now

$$DF = \pm\frac{a}{g}\,dW = \pm c\,dW = \pm cw'tr\,d\theta, \tag{407}$$

where a is the earthquake acceleration on the elementary volume whose weight is dW at C. The effect of DF on the arch-ring section at g in thrust, moment, and shear will be expressed by the following equations:

$$N_0 = \pm \int_0^{\phi} cw't \sin \phi \; r \, d\theta, \tag{408}$$

$$M_0 = \pm \int_0^{\phi} -cw'tr(\sin \phi - \sin \theta) r \, d\theta, \tag{409}$$

$$N_0 = \pm \int_0^{\phi} -cw't \cos \phi \; r \, d\theta. \tag{410}$$

On integrating and substituting the limits,

$$N_0 = \pm cw'tr\phi \sin \phi, \tag{411}$$

$$M_0 = \mp cw'tr^2(\phi \sin \phi + \cos \phi - 1), \tag{412}$$

$$V_0 = \mp cw'tr\phi \cos \phi. \tag{413}$$

From Fig. 41 and paragraph 13-3, it will be noted that $ds = r\,d\phi$, $\theta = \phi$, $y = r \cos \phi - h$, and $h = r \sin \phi_a \div \phi_a$. Therefore,

$$y = r\left(\cos \phi - \frac{\sin \phi_a}{\phi_a}\right). \tag{414}$$

Substituting the values of N_0, M_0, V_0, y, ds, $I = t^3/12$, $A = t$, and θ_1 in Eqs. (192), (193), and (194), expanding, integrating between the limits ϕ_a and 0, and rearranging, the following values of X, Z, and Y are found:

$$X = \pm cw'tr \left[\frac{\dfrac{4 \sin^2 \phi_a}{\phi_a} - \dfrac{7}{4} \sin 2\phi_a + \dfrac{\phi_a \cos 2\phi_a}{2} + \dfrac{2k^2}{r^2}\left(\sin^2 \phi_a - \dfrac{\phi_a \cos 2\phi_a}{2}\right)}{\left(1 + \dfrac{k^2}{r^2}\right)\phi_a + \dfrac{\sin 2\phi_a}{2} - \dfrac{2 \sin 2\phi_a}{\phi_a} + \dfrac{3k^2}{r^2}\left(\phi_a - \dfrac{\sin 2\phi_a}{2}\right)}\right] = \pm cw'tr \; C_1, \tag{415}$$

$$Z = \pm cw'tr\left(1 + \cos \phi_a - \frac{2 \sin \phi_a}{\phi_a}\right) = cw'tr^2 C_2, \tag{416}$$

$$Y = 0. \tag{417}$$

From Fig. 39 and paragraph 12-4, it will be seen that the thrust and moment at the crown are, respectively,

$$N_c = \pm X = \pm cw'tr \; C_1, \tag{418}$$

$$M_c = \pm(Z - N_c k) = \pm(Z - N_c y). \tag{419}$$

From Eq. (211),

$$y = r\left(1 - \frac{\sin \phi_a}{\phi_a}\right). \tag{420}$$

Substituting the values of Z, N_c, and y from the foregoing equations in Eq. (419),

$$M_c = \pm cw'tr^2\left[C_2 - C_1\left(1 - \frac{\sin \phi_a}{\phi_a}\right)\right], \tag{421}$$

$$V_c = Y = 0. \tag{422}$$

To find the thrust, moment, and shear at any point in the semiarch ring, use Eqs. (198), (199), and (200), and note that θ_1 in these equations corresponds with ϕ_1 in the foregoing equations. N, M, and V being thus found for as many points as desired in the arch ring, the unit compressive stresses for each of them may be obtained by means of Eq. (174) and the unit shearing stress from Eq. (177), both of Chap. XI. As many rings of the arch as necessary to give the results desired should be analyzed.

Since the algebraic signs of the thrust, moment, and shear are reversed for a change of downstream and upstream earthquake impulses, it will be observed that an arch ring designed for customary water pressure is well suited to resist stresses due to downstream impulses but not for those in an upstream direction. Steel reinforcement should be introduced to take the tensional stresses produced by this latter kind of shock.

In case of an earthquake acceleration parallel to the canyon walls, the value of F for each arch ring of the arch barrel will be different [see Eq. (26) paragraph 5-19]. For an arch dam relatively high for its length, the effect of the canyon walls is of considerable consequence. The impulses conveyed by the side walls of the canyon may even exceed that conveyed by the canyon floor. The stresses in such an arch barrel may be more nearly approximated by considering it to be a plate supported on three edges.

Next, let consideration be given to the effects of an earthquake impulse acting perpendicularly to the canyon walls. In this case the load will not be symmetrical about the arch crown, and it will be necessary to consider the entire arch ring. Let Fig. 93 represent a symmetrical arch ring of uniform thickness fixed at the ends loaded with an earthquake impulse as shown. Assume the left abutment removed and the arch ring held in equilibrium by an imaginary rigid bracket connected to the elastic center and held there by the reacting forces of the abutment consisting of thrust, moment, and shear at that point. Now let the thrust N, moment M, and the shear V at the left abutment

be replaced by three redundant forces applied at the elastic center by means of the imaginary rigid bracket and acting positively in the directions as shown.

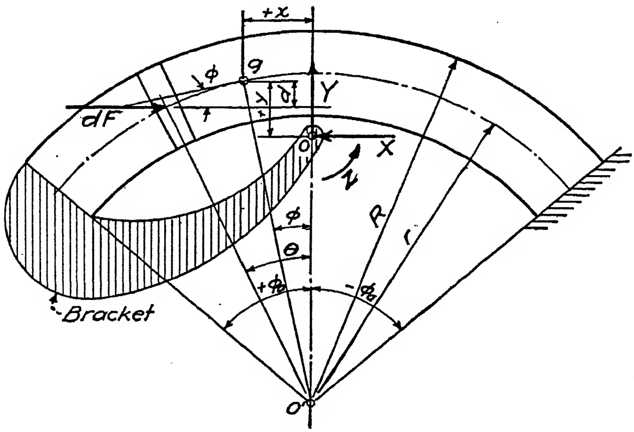


FIG. 93.

The general deflection Eq. (234), derived from Eq. (169), may now be applied and the specific deflection Eqs. (239), (240), and (241) found as stated in paragraph 16-7, noting that symbols in them apply to the earthquake load instead of the water load. In these equations it is now necessary to find values of n, m, v and N, M, and V applicable to the problem.

Referring to Fig. 93, it will be seen that $n_x = \cos\phi$, $n_y = \sin\phi$, $n_z = 0$, $m_x = y$, $m_y = x$, $m_z = 1$, $v_x = \sin\phi$, $v_y = -\cos\phi$, $v_z = 0$. Also from this figure it is evident that, where N_0, M_0, and V_0 represent the thrust, moment, and shear on the radial section through g,

$$DF = cw'tr\,d\theta, \tag{423}$$
$$DN_0 = DF\cos\phi, \tag{424}$$
$$DM_0 = DFd = DF(r\cos\phi - r\cos\theta), \tag{425}$$
$$DV_0 = DF\sin\phi. \tag{426}$$

From the preceding equations, it follows that

$$N_0 = \int_{\phi}^{\phi_a} cw'tr\cos\phi\,d\theta = cw'tr(\phi_a - \phi)\cos\phi, \tag{427}$$

$$M_0 = \int_{\phi}^{\phi_a} cw'tr^2(\cos\phi - \cos\theta)d\theta = cw'tr^2[(\phi_a - \phi)\cos\phi - \sin\phi_a + \sin\phi], \tag{428}$$

$$V_0 = \int_{\phi}^{\phi_a} cw'tr(\phi_a - \phi)\sin\phi. \tag{429}$$

Substituting the values of n_x, n_y, n_z, m_x, m_y, m_z, v_x, v_y, v_z, N_0, M_0, and V_0 in Eqs. (239), (240), and (241), the deflections Δ_x, Δ_y, and Δ_z are found. These are the deflections at the pole O due to the earthquake impulse. Now to find the deflections due to the dummy unit loads applied at O, substitute n_x, etc., in Eqs. (239), (240), and (241). From paragraph 12-6 it is evident that

$$X = -\frac{\Delta_x}{\Delta_{x'}},\ Y = -\frac{\Delta_y}{\Delta_{y'}},\ Z = -\frac{\Delta_z}{\Delta_{z'}}, \tag{430}$$

where Δ_x, Δ_y, and Δ_z are the deflections due to the earthquake load and $\Delta_{x'}$, $\Delta_{y'}$, and Δ_z are those due to the dummy unit loads.

Substituting the deflections due to the earthquake load and those due to the dummy unit loads in Eq. (430), it is found that

$$X = -cw'tr\ \phi_a, \tag{431}$$

$$Y = -\frac{cw'tr\left[\dfrac{1}{2} - \dfrac{\sin 2\phi_a}{4\phi_a} - \dfrac{1-\cos 2\phi_a}{2}\right]\left[\left(1+\dfrac{2k^2}{r^2}\right)\left(1-\dfrac{\sin 2\phi_a}{2\phi_a}\right)\right]}{\left(1+\dfrac{k^2}{r^2}\right)\left(1-\dfrac{\sin 2\phi_a}{2\phi_a}\right)+3\dfrac{k^2}{r^2}\left(1+\dfrac{\sin 2\phi_a}{2\phi_a}\right)} = -cw'tr\ Q, \tag{432}$$

$$Z = 0. \tag{433}$$

At any radial section in the arch ring,

$$N = N_0 + X\cos\phi + Y\sin\phi, \tag{434}$$
$$M = M_0 + Xy + Yx + Z, \tag{435}$$
$$V = V_0 + X\sin\phi - Y\cos\phi. \tag{436}$$

It should be observed that $x = 0$ at the crown and $y = r\left(\cos\phi - \dfrac{\sin\phi_a}{\phi_a}\right) = r\left(1 - \dfrac{\sin\phi_a}{\phi_a}\right)$ since $\phi = 0$ and $\cos\phi = 1$. Using Eqs. (427), (428), (429), (431), (432), and (433), it can be readily shown that, at the crown,

$$N = 0, \tag{437}$$
$$M = 0, \tag{438}$$
$$V = -cw'tr\ Q. \tag{439}$$

At the left abutment, $\phi = \phi_a$; and, therefore,

$$N_1 = cw'tr(-\phi_a\cos\phi_a - Q\sin\phi_a), \tag{440}$$
$$M_1 = cw'tr^2[(1-Q)\sin\phi_a - \phi_a\cos\phi_a], \tag{441}$$
$$V_1 = cw'tr(-\phi_a\sin\phi_a + Q\cos\phi_a). \tag{442}$$

At the right abutment, $\phi = -\phi_a$, whence

$$N_2 = -N_1, \tag{443}$$

$$M_2 = M_1, \tag{444}$$

$$V_2 = V_1, \tag{445}$$

Stresses at other points in the arch ring may be found by assigning other values to ϕ and x and y. Unit stresses at any point may be found by the use of the foregoing values and by employing Eqs. (174) and (177).

The foregoing calculations apply to arch rings of uniform thickness, but the same general process applies to arch rings of variable thickness by following the procedure outlined in the latter part of paragraphs 12-10 and 12-15, using the form of layout given in Fig. 51 and the form of recording the computations given in Fig. 40.

In case the earthquake impulse acts at an angle to the axis of curvature of the dam, the impulse on each elementary volume may be resolved into two components, one parallel to the axis of the ring and the other perpendicular to this axis. The summation of each of these components forms a separate problem that may be solved by the two procedures just outlined in the foregoing discussion.

EARTHQUAKE STRESSES IN MULTIPLE-ARCH DAMS

The determination of stresses in an arch ring of a multiple-arch dam due to its weight is almost identical with the procedure just outlined for single-arch dams. In the case of a multiple-arch dam, a part of the impulse of the earthquake is carried into the arch barrel and a part to the piers or abutments, as may be seen by referring to Figs. 53 and 54 and to Eqs. (285), (286), and (287). The portion effective in producing stresses in the arch ring then becomes for an elementary volume (see Figs. 94*a* and 94*b*),

$$DR = DF \sin a; \tag{446}$$

and, therefore,

$$DR = \pm cw't \sin a \; r \; d\theta, \tag{447}$$

where a is the angle between the plane of the arch ring and the vertical. With substitution of the right member of Eq. (447) for the right member of Eq. (423) hereof, the procedure is identical with that for single-arch dams.

In order to clarify further the effect of the inclination of the arch rings with the horizontal, let Fig. 94*a* represent a semisection of a symmetrical ring of uniform thickness of a multiple-arch dam, the plane of the ring being inclined at an angle of a with the vertical. Let CD be the trace, on the plane of the arch ring, of a vertical plane parallel to the vertical plane through the crown section of the arch ring. Assume the impulse of an earthquake shock to act horizontally either downstream or upstream on the arch in a direction parallel to this vertical plane. In Fig. 94*b* are shown to the left of the point C a downstream impulse dF on an elementary volume of the arch ring, depicted in section in Fig. 94*a*, resolved into its component forces: dR

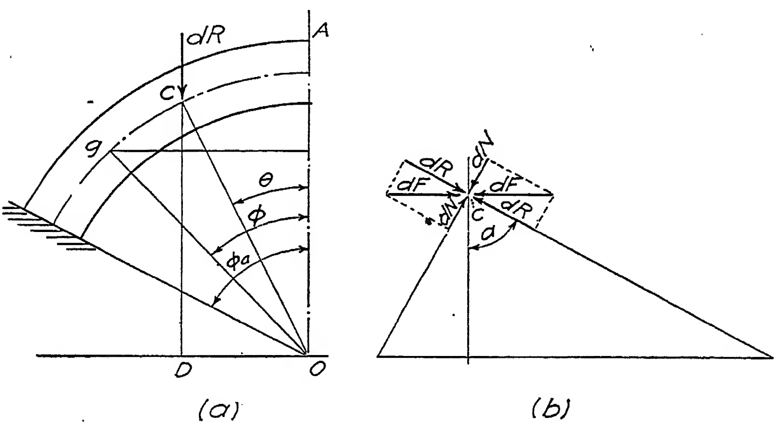


FIG. 94.

acting along CD in the arch ring, and dN acting upward along the arch barrel, in the vertical plane. Now, dR is resisted by arch action of the arch ring, and dN is carried mainly by beam action of the arch ring to the adjacent piers and tends to overturn them downstream. Also in Fig. 94*b* are shown to the right of the point C an upstream impulse of dF on the elementary volume and its components dR and dN. In this case also dR is resisted by the arch ring, and dN is carried mainly by beam action to the adjacent piers and tends to weaken their resistance against overturning upstream. From these considerations it is evident that the algebraic signs of dF and dR and, therefore, of the thrust, moment, and shear in the arch ring are reversed for an earthquake acceleration acting upstream from those of one acting down-

stream. It also follows that an arch ring designed for customary water pressure downstream is well suited to resist the stresses due to an earthquake shock with a downstream acceleration; but that an arch ring so designed may not be suitable to resist stresses due to an earthquake shock with an upstream acceleration. Steel should be introduced to take the tensional stresses produced by this latter kind of shock.

In the case of an empty reservoir, the effects of earthquake shocks must be applied only to the masonry of the structure; but for a full or partly full reservoir the influence of the movement of the reservoir water on the masonry stresses must be taken into consideration (see pages 57 to 61).

APPENDIX IV

STRESSES DUE TO ARCH-RING WEIGHT IN A SYMMETRICAL ARCH RING OF UNIFORM THICKNESS IN A MULTIPLE-ARCH DAM

Let Fig. 95 represent the left half of an arch ring of uniform thickness in a multiple-arch dam; then where t is the ring thick-

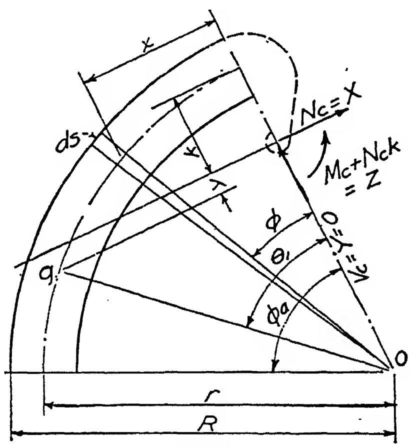


FIG. 95.

ness, w' the weight per cubic foot of concrete, and a the angle made by the plane of the arch ring with the horizontal,

$$N_0 = \int_0^{\theta_1} w'trd\phi \sin a \sin \theta_1 = w'tr \sin a\, \theta_1 \sin \theta_1, \qquad (448)$$

$$M_0 = \int_0^{\theta_1} -w'tr^2 d\phi \sin a(\sin \theta_1 - \sin \phi)$$
$$= -w'tr^2 \sin a(\theta_1 \sin \theta_1 + \cos \theta_1 - 1), \qquad (449)$$

$$V_0 = \int_0^{\theta_1} -w'trd\phi \cos \theta_1 = -w'tr \sin a\, \theta_1 \cos \theta_1. \qquad (450)$$

Substituting these values of N_0, M_0, and V_0 in Eqs. (192), (193), and (194); noting that $ds = r\, d\phi$, $\theta_1 = \phi_a$ at the abutments, $I = t^3 \div 12 = tk^2$, $A = t$, $x = r \sin \phi$, and $y = r \cos \phi - r \sin$

$\phi_a \div \phi_a$; expanding and integrating between the limits of ϕ_a and 0, the following values of X, Z, and Y are found:

$$X = w'tr \sin a$$
$$\left[\frac{\dfrac{4\sin^2\phi_a}{\phi_a} - \dfrac{7}{4}\sin 2\phi_a + \dfrac{\phi_a \cos 2\phi_a}{2} + \dfrac{2k^2}{r^2}\left(\sin^2\phi_a - \dfrac{\phi_a\cos 2\phi_a}{2}\right)}{\left(1 + \dfrac{k^2}{r^2}\right)\left(\phi_a + \dfrac{\sin 2\phi_a}{2}\right) - \dfrac{2\sin^2\phi_a}{\phi_a} + \dfrac{3k^2}{r^2}\left(\phi_a - \dfrac{\sin 2\phi_a}{2}\right)}\right]$$
$$= w'tr \sin a\, C_1, \quad (451)$$

$$Z = w'tr^2 \sin a\left(1 + \cos\phi_a - \frac{2\sin\phi_a}{\phi_a}\right) = w'tr^2 \sin a\, C_2, \quad (452)$$

$$Y = 0. \quad (453)$$

Using Eqs. (451) and (452) and Fig. 39 or 41, it will be seen that the thrust and moment at the crown are, respectively,

$$N_c = X = w'tr \sin a\, C_1, \quad (454)$$
$$M_c = Z - N_c k. \quad (455)$$

From Fig. 41, it is evident that $k = y$ and that at the crown

$$y = r\left(1 - \frac{\sin\phi_a}{\phi_a}\right). \quad (456)$$

From Eqs. (452), (454), (455), and (456), it is found that

$$M_c = w'tr^2 \sin a\left[C_2 - C_1\left(1 - \frac{\sin\phi_a}{\phi_a}\right)\right]. \quad (457)$$

Also from Eq. (194),

$$V_c = Y = 0. \quad (458)$$

To find the thrust, moment, and shear at any point in the semi-arch ring, use Eqs. (198), (199), and (200) or Eqs. (288), (289), and (290), and note that θ_1 in these equations correspond with ϕ_1 in the foregoing equations. N, M, and V being thus found for as many points as desired in the arch ring, the unit compressive stresses for each of them can be ascertained by means of Eq. (174) and the unit shearing stress from Eq. (177), both of Chap. XI.

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